Using learners' responses to inform the teaching of mathematics

Resource materials based on the Annual National Assessments

Grade 9

basic education
Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

JET EDUCATION SERVICES

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Foreword from the Minister of Basic Education

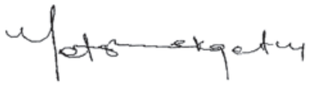
It is my pleasure to present this set of materials to be part of the range of resources made available for our teachers to improve the quality of their teaching. Since 2011, the Department of Basic Education (DBE) has been conducting the Annual National Assessment (ANA) on Grades 1-6 and 9 learners in Language and Mathematics.

The diagnostic reports produced after the administration of the ANA point to areas where individual teachers need specific support in terms of effective methods of facilitating learning. One of these areas is the utilisation of assessment data in a manner which will inform improved teaching.

This set of materials has therefore been developed to build the capacity of teachers to analyse the ANA and other tests in order to identify typical errors made by learners and thereafter select appropriate teaching strategies to correct these errors and improve the teaching of Mathematics. The materials give precise and useful guidelines, with regards to accurate identification of the challenging content and conceptual areas shown through the errors as well as using these errors to create opportunities for learners to improve their mathematical abilities.

I wish to express my sincere gratitude to our partners, the United Nations International Children's Emergency Fund (UNICEF) and JET Education Services for their invaluable contribution in making this resource available.

I am confident that teachers will find the materials useful and that this intervention will make a meaningful contribution to their teaching and professional development.



MRS ANGIE MOTSHEKGA, MP
MINISTER OF BASIC EDUCATION

Table of Content

Introduction

All South African learners in Grades 1 to 6 and 9 annually write the Annual National Assessments, which have become known colloquially as 'the ANAs'. The Department of Basic Education's diagnostic reports on the ANA results identified a need to strengthen the formative use of assessment data to support teaching. This resource book has been created to meet this need.

The material presented here focuses on mathematics topics covered in both the 2013 and 2014 ANAs and deals with the errors made by learners as shown in their responses to the ANA questions. In addition to highlighting learners' misconceptions, analysis of the learners' responses also reveals the correct understanding of the topics tested.

Following the error analysis of responses in the ANAs, teaching strategies to address these mistakes and misconceptions are presented. Both the error analysis and the teaching strategies in this resource book can be applied to other assessment questions that test the same topic, or used in the normal course of teaching the topic.

The materials presented here are, however, not meant to be exhaustive in terms of content covered in the mathematics curriculum, but are rather a living and growing resource that can be added to in response to future ANAs. Teachers are encouraged to add their own examples of learners' responses to assessment questions as well as to the teaching strategies which address the misconceptions noted in the learners' responses.

The following section of this book explains the concept of error analysis and how to conduct an analysis of learner's responses in an assessment or test. Next, guidelines on how to use these materials in conjunction with error analysis are presented. Finally, actual error analysis of learners' responses in the ANAs and ideas for teaching strategies to address the identified errors and misconceptions are presented per topic.

What is error analysis?

The formative use of assessment data to support teaching can be achieved by what researchers call “error analysis”.

Error analysis, also referred to as error pattern analysis, is a multifaceted activity involving the study of errors in learners' work with a view to finding explanations for learners' reasoning errors.

It is important to note that not all errors are reasoning faults; some are simply careless errors which educational researchers have termed “slips”. Slips can easily be corrected if the faulty process is pointed out to the learner. Slips are random errors that learners may make (e.g. reversing a number) and do not indicate systematic misconceptions or conceptual problems.

Error analysis is concerned with identifying and addressing the common errors (or 'bugs') which learners make due to their lack of conceptual or procedural understanding. These types of mathematical errors occur when the learner believes that what has been done is correct – showing that the learner's reasoning is faulty. Researchers have termed these errors systematic and persistent errors. Unless the misconceptions that cause the learners to make these errors are corrected, learners will repeat them over and over again and not be aware that they are using incorrect procedures to solve problems. These systematic errors can be said to be the result of the use of algorithms that lead to incorrect answers or the use of procedures that have not been fully understood.

Error analysis is important as it does not just mean analysing learners' steps in finding the solutions to a problem, but also involves finding the best ways to remediate the misconceptions the analysis shows. Rectifying learners' misconceptions enables teachers to make sure that learners can move on to the next step in the curriculum with a sound base.

Many writers on education have pointed out that the ability of teachers to understand and remediate common learner errors and misconceptions is an important part of what teachers should know, that is, of the pedagogical content knowledge that teachers should have. In order to conduct error analysis efficiently, teachers need to have a good knowledge of mathematical content, a good grasp of their learners' levels of mathematical understanding and a well-grounded understanding of the learner and how a learner learns.

Shulman (1986)[1] was a writer who developed a theory of teacher knowledge. He

[1] Shulman, L.S. 1986. Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2):4–14.

included teachers' knowledge of learners' levels of understanding as an important part of teacher knowledge and explained that this knowledge helps teachers to become aware of the process of learning mathematics as well as to understand the mathematical concepts that learners struggle to grasp. Other authors describe how error analysis and efforts to understand learners' levels of mathematical understanding build teacher's own knowledge of the underlying cognitive processes involved.

In conclusion, error analysis is a valuable activity that helps teachers to understand some of the thinking of their learners. Understanding the way learners think can assist teachers to adjust their teaching strategies and classroom and assessment practices and may ultimately lead to improvement in learner achievement.

How to analyse your learners' responses to assessment questions?

This section contains a fictional ten-question test which is used to illustrate how to design and use an assessment grid to analyse your class's results in a test or assessment. This is done in a step-by-step fashion.

The assessment grid, as the example given shows, reflects:

- The content addressed in each question;
- The question number;
- The marks allocated;
- The total of correct answers for each learner;
- The percentage each learner achieved;
- The learners' names; and
- The average score per question.

Step 1: Drawing up an assessment grid

- The first step in drawing up the grid is to identify the content addressed in each question in the assessment. The descriptions of this content make up the first heading row of the grid. It helps to be as specific as possible when identifying the content or skill being tested, as in the descriptions in blue in the example.
- You also need to record the marks (see red) allocated for each question and the total possible marks for the test (see green).

Content addressed in question	Simplification of expressions involving division	Equation involving exponents	Factorisation of expression	n^{th} term of a sequence	Simple interest	Compound interest	Congruent triangles	Drawing the reflected image	Surface area and volume of triangular prism	Determining gradient of line	**Total correct**	**Percentage**
Question number	1	2	3	4	5	6	7	8	9	10		
Mark	3	2	2	4	5	4	1	2	6	1	30	
Learners' name												
Average per question												

Step 2: Recording the results of each learner

In the next step you have to record the results per learner per question.

- First list all the learners in your class. We have 5 learners in our fictional class (see red).
- We have recorded the mark each learner got for each question (see blue).
- We also recorded the total marks each learner got (see green) and each learner's percentage (see pink).
- You might want to an additional column for the level code.

- Looking at the grid you will start to notice individual differences, e.g. learners that got the same total mark (see purple) did not get the same questions right or wrong.
- Also, you can identify which learners are doing well and which need additional support, e.g. Rosi and Lizzy are doing well, but Themba needs additional support.

Content addressed in question	Simplification of expressions involving division	Equation involving exponents	Factorisation of expression	n^{th} term of a sequence	Simple interest	Compound interest	Congruent triangles	Drawing the reflected image	Surface area and volume of triangular prism	Determining gradient of line	**Total correct**	**Percentage**
Question number	1	2	3	4	5	6	7	8	9	10		
Mark	3	2	2	4	5	1	1	2	6	1	30	%
Learners' name												
Rosi Thladi	2	1	2	2	3	3	1	1	4	1	20	67
Themba Hlambelo	1	1	1	0	2	2	1	0	2	0	10	33
Luneta Petersen	3	2	2	1	5	2	0	0	0	0	15	50
Johan De Wit	1	2	1	1	2	2	1	1	3	1	15	50
Lizzy Gregory	3	2	2	2	4	3	1	2	4	1	24	80
Average per question												

Step 3: Working out the class average

The third step is to work out and record the class average for each question.

- To calculate your class average, add up all the learners' averages and then divide by the number of learners in the class, e.g, Class average = 67 + 33 + 50 + 50 + 83 (see pink) ÷ 5 learners in class x 100 = 57%

Content addressed in question	Simplification of expressions involving division	Equation involving exponents	Factorisation of expression	n^{th} term of a sequence	Simple interest	Compound interest	Congruent triangles	Drawing the reflected image	Surface area and volume of triangular prism	Determining gradient of line	**Total correct**	**Percentage**
Question number	1	2	3	4	5	6	7	8	9	10		
Mark	3	2	2	4	5	4	1	2	6	1	30	%
Learners' name												
Rosi Thladi	2	1	2	2	3	3	1	1	4	1	20	67
Themba Hlambelo	1	1	1	0	2	2	1	0	2	0	10	33
Luneta Petersen	3	2	2	1	5	2	0	0	0	0	15	50
Johan De Wit	1	2	1	1	2	2	1	1	3	1	15	50
Lizzy Gregory	3	2	2	2	4	3	1	2	4	1	24	80
Average per question												60

Step 4: Working out the class average per question

In this step you want to determine the average your class got for each question. This will allow you to see the areas your learners are doing well in and the areas in which they need additional support.

- First work out the maximum possible marks the class could get for each question by multiplying the marks allocated to the question by the number of learners in the class, e.g., question 1 counts out of 3 marks, so the maximum possible total for the class would be 3 marks x 5 learners = 15 marks.
- Now add up all the learners' marks for question 1 (see red) to get the total marks achieved by the class for question 1, i.e. 2 + 1 + 3 + 1 + 3 = 10.
- To determine the class average for question 1 (see blue) divide the total marks achieved by the maximum possible marks and multiply the answer by 100, i.e. 10 ÷ 15 x 100 = 67% (see blue).

Content addressed in question	Simplification of expressions involving division	Equation involving exponents	Factorisation of expression	n^{th} term of a sequence	Simple interest	Compound interest	Congruent triangles	Drawing the reflected image	Surface area and volume of triangular prism	Determining gradient of line	**Total correct**	**Percentage**
Question number	1	2	3	4	5	6	7	8	9	10		
Mark	3	2	2	4	5	4	1	2	6	1	30	%
Learners' name												
Rosi Thladi	2	1	2	2	3	3	1	1	4	1	20	67
Themba Hlambelo	1	1	1	0	2	2	1	0	2	0	10	33
Luneta Petersen	3	2	2	1	5	2	0	0	0	0	15	50
Johan De Wit	1	2	1	1	2	2	1	1	3	1	15	50
Lizzy Gregory	3	2	2	2	4	3	1	2	4	1	24	80
Average per question	67	80	80	30	64	60	80	40	43	60		60

- Once you have done the same for all the questions, note which questions your class did well in (e.g. questions 2, 3 and 7) and which they did not do so well in (i.e. questions 8 and 9).
- Try to identify why this might be the case.

- Have you, for example, not spent enough time on these content areas?
- Do learners have a specific misunderstanding or misconception of this content area?
- Or are learners maybe making careless mistakes?

Step 5: Determine what learners should know to answer a specific question correctly

Let's consider what learners need to know to, for example, calculate the surface area of a triangular prism (question 9 in our fictional test). Learners should be able to:

- Deconstruct the prism into its net;
- Calculate the area of 2-D shapes, for example, triangles and rectangles;
- Write down the formulae for calculating the surface area of prisms;
- Substitute correctly within the formulae for calculating the surface area of prisms and cylinders.

You will note some of the content a learner needs to know in order to solve the problem correctly might have been covered in previous grades.

Step 6: Identify the typical errors learners make

Look at a selection of learners' responses and identify the typical errors they make. For example for question 9, you might note that learners:

- Neglected to calculate all the surface areas of the prism;
- Calculated the surface area of a specific face incorrectly; or
- Calculated all the surface areas of the prism correctly, but then added them incorrectly.

- It is useful here to have a discussion with specific learners or with the class to identify the reasoning learners used to solve the problem. By asking a learner to talk you through the process he/she followed to solve the problem, you could identify where the learner's logic went wrong or if the learner just made a careless calculation error.

Step 7: Determine appropriate teaching strategies to address learner errors and misconceptions

Once you have determined the reasoning or calculation errors learners made in a particular topic, you need to take steps to address these misconceptions or errors.

How to use these materials

The main section of this resource is organised according to topic in the following way:

- The topic and question used in the ANA to test it;
- The knowledge and skills required to answer the question and where the topic is located in the CAPS;
- Examples of learners' responses showing full, partial or no understanding of the question;
- Statistics showing the percentage of learners countrywide that answered the question correctly;
- Reasons the learners may have found it difficult to answer the question correctly;
- Teaching strategies to rectify the misconceptions that caused the errors; and (in some instances)
- Additional examples of how to test the topic.

To use the material

1. First determine which topic your class is experiencing difficulties with, then refer to the topic in the resource book.
   - Take note of what learners need to know to answer the question correctly and where the topic is found in the CAPS.
   - You should reflect critically on your teaching practice and the learners' knowledge with regard to the content and skills needed to answer the question. For instance, ask yourself:
     - Has sufficient time been spent on the required content and skills?
     - Have learners had sufficient time and practice to master the requisite skills?
   - You might need to revise the content and skills needed for a particular topic before attempting to remediate the learners' performance.
   - The content or skills might have been taught in a previous grade, but it is essential to make sure the learners have a solid foundation on which to build conceptual understanding of new topics.

2. Secondly, establish what kinds of slips and errors your learners make by either:
   - Looking at their answers to the ANA question (remember, you could give the learners the question in class to solve as part of class work); or
   - Looking at their answers to similar questions found in textbooks or class exercises.
   - You can refer to the examples of typical learner responses to assist you.

3. Take note of the statistics that follow the examples of learner responses.
   - The statistics will give you an indication of how difficult the questions should be for your class.

- If your class is doing better than the national sample, then it means you have started to lay a solid foundation and should continue your systematic teaching of the topic.

4. Lastly, look at and use the recommended teaching strategies to remedy the learners' errors and misconceptions as shown by the error analysis.
   - The teaching strategies are linked to the particular errors and misconceptions shown in this book – you may identify others made by your own learners.
   - The strategies are not meant as an exhaustive list of teaching strategies or exercises to do with learners on a specific topic, but as a starting point for remediation.
   - You will need to supplement the teaching strategies with your own ideas.

Note: Remember to teach **all the topics specified in the CAPS**, not just the ones featured in this resource.

Happy teaching!

Extra reading on error analysis

If you would like to know more about the theory of error analysis and how it can be used to aid you in your teaching, the following articles and books are suggested reading.

Allsopp, D.H., Kuger, M.H. & Lovitt, L.H. (2007). *Teaching mathematics meaningfully: Solutions for reaching struggling learners*. Baltimore: Paul H. Brooks.

Ashlock, R.B. (2006). *Error patterns in computation: Using error patterns to improve instruction. 9th edition*. New Jersey: Pearson.

Franke, M.L. & Kazemi, E. (2001). *Learning to teach mathematics: Focus on student thinking. Theory into Practice*, 40(2):102–109.

Herholdt, R & Sapire, I. (2014). An error analysis in early grades mathematics – A learning opportunity? *South African Journal of Childhood Education*, 4(1), 42-60.

Hill, H.C., Ball, D.L. & Schilling, S.C. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal of Research in Mathematics Education*, 39(4):372–400.

Ketterlin-Geller, L.R. & Yovanoff, P. (2009). Diagnostic assessments in mathematics to support instructional decision making. *Practical Assessment, Research &*

Evaluation, 14(16). Retrieved from http://pareonline.net/getvn.asp?v=14&n=16 (accessed on 19 April 2014).

McGuire, P. (2013). Using online error analysis items to support pre-service teachers' pedagogical content knowledge in mathematics. *Contemporary Issues in Technology and Teacher Education*, 13(3). Retrieved from http://www.citejournal.org/vol13/iss3/mathematics/article1.cfm (accessed on 19 April 2014).

Nesher, P. (1987). Towards an instructional theory: The role of students' misconceptions. *For the Learning of Mathematics*, 7(3):33–39.

Olivier, A. (1996). Handling pupils' misconceptions. *Pythagoras*, 21:10–19.

Radatz, H. 1979. Error analysis in mathematics education. *Journal for Research in Mathematics Education*, 10(3):163–172.

Riccomini, P.J. (2005). Identification and remediation of systematic error patterns in subtraction. *Learning Disability Quarterly*, 28(3):233–242.

Russell, M. & Masters, J. (2009). *Formative diagnostic assessment in algebra and geometry*. Paper presented at the annual meeting of the American Education Research Association. San Diego, California.

Sapire, I., Shalem, Y., & Reed, Y. (2013). *Assessment for Learning. Johannesburg: University of Witwatersrand and Saide.*

Shalem, Y., Sapire, I., & Sorto, M.A. (2014). Teachers' explanations of learners' errors in standardised mathematics assessments. *Pythagoras,* 35(1), Art. #254, 11 pages. http://dx.doi.org/10.4102/pythagoras.v35i1.254

Shulman, L.S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2):4–14.

Sousa, D.A. (2008). *How the brain learns mathematics*. California: Corwin Press.

Yang, C.W., Sherman, H. & Murdick, N. (2011). Error pattern analysis of elementary school-aged students with limited English proficiency. *Investigations in Mathematics Learning*, 4(1):50–67.

Topics

Algebraic expressions and fractions

ANA 2013 Grade 9 Mathematics Items 1.4, 2.1 and 2.4

1.4 Given the expression $\frac{x-y}{3} + 4 - x^2$

Circle the letter of the <u>incorrect</u> statement:

A The expression consists of 3 terms.

B The coefficient of x is 1.

C The coefficient of x^2 is -1.

D The expression contains 2 variables. [1]

2 Simplify each of the following expressions:

2.1 $\frac{6x^5}{x^4} - \frac{15x^3}{3x^2}$ [3]

2.4 $\frac{2x+1}{4} - \frac{x+2}{2} - \frac{1}{4}$ [4]

What should a learner know to answer these questions correctly?

Learners should be able to:

Item 1.4

- Differentiate between an expression and a term;
- Correctly identify the number of terms in an expression;
- Correctly identify coefficients of variables in an expression;
- Correctly identify variables in an expression.

Item 2.1 and item 2.4

- Write fractions in the simplest form;
- Subtract common fractions;
- Simplify algebraic fractions;
- Subtract terms in an algebraic expression;
- Apply the laws of exponents.

Where is the topic located in the curriculum? Grade 9 Terms 1 and 3

Content area: Patterns, Functions and Algebra.

Topic: Algebraic expressions (including fractions).

Concepts and skills:

- Identify variables and constants in given formulae and equations;
- Recognise and identify coefficients and exponents in algebraic expressions;
- Use knowledge of multiples and factors to write fractions in the simplest form before or after calculations;
- Use knowledge of equivalent fractions to add and subtract common fractions;
- Simplify algebraic expressions using division of monomials;
- Add and subtract like terms in an algebraic expression.

What would show evidence of full understanding?

Item 1.4

If the learner chose the correct option, B:

- The learner knows the correct number of terms in the expression;
- The learner knows the coefficient of each term in the expression; and
- The learner correctly identified the number of the variables in the expression.

Item 2.1 and 2.4

If the learner obtained the correct answers by:

- Correctly dividing and subtracting the monomials;
- Correctly determining the LCM of the denominators and correctly subtracting the fractions; and
- Correctly adding and subtracting like terms, as shown in the answers that follow.

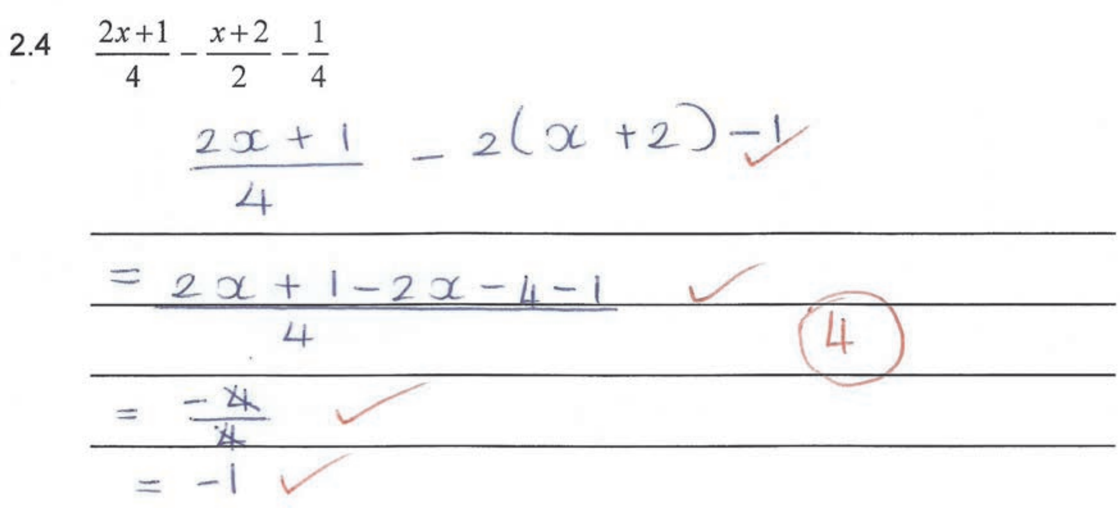
QUESTION 2

Simplify each of the following expressions:

2.1 $\frac{6x^5}{x^4} - \frac{15x^3}{3x^2}$

$= 6x - 5x$

$= x$



2.4 $\frac{2x+1}{4} - \frac{x+2}{2} - \frac{1}{4}$

$\frac{2x+1}{4} - 2(x+2) - 1$

$= \frac{2x+1-2x-4-1}{4}$

$= \frac{-4}{4}$

$= -1$

(4)

What would show evidence of partial understanding?

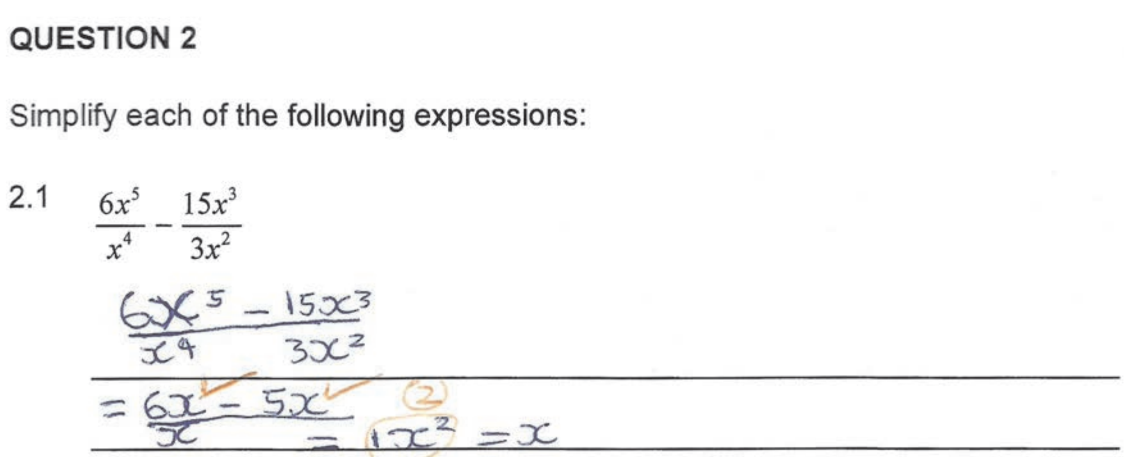
Item 1.4

If the learner chose option A, C or D, this indicates a partial understanding of algebraic expression theory:

- The choice of A indicates that the learner did not know how to identify the number of terms in the expression;
- The choice of C indicates that the learner did not know how to identify the coefficient of each term in the expression;
- The choice of D indicates that the learner did not know how to identify the number of the variables in the expression.

Items 2.1 and 2.4

- If the learner simplified the fractions partially correctly but subtracted incorrectly:



QUESTION 2

Simplify each of the following expressions:

2.1 $\frac{6x^5}{x^4} - \frac{15x^3}{3x^2}$

$\frac{6x^5}{x^4} - \frac{15x^3}{3x^2}$

$= \frac{6x}{x} - 5x = 1x^2 = x$

(2)

- If the learner simplified the fractions incorrectly but subtracted correctly:

QUESTION 2

Simplify each of the following expressions:

2.1 $\frac{6x^5}{x^4} - \frac{15x^3}{3x^2}$

$$\frac{6x^5}{x^4} - \frac{15x^3}{3x^2} = 2x - 5x$$

$$\frac{2x^5 - 5x^3}{x^4} = -3x$$

- If the learner identified the LCM of the denominators correctly but applied the LCM incorrectly and inconsistently:

2.4 $\frac{2x+1}{4} - \frac{x+2}{2} - \frac{1}{4}$

$$\times 4 \left(\frac{2x+1}{4}\right)\left(\frac{4}{1}\right) - \left(\frac{x+2}{2}\right)\left(\frac{4}{1}\right) - \left(\frac{1}{4}\right)\left(\frac{4}{1}\right)$$

$$2x+1 - (x+2)^2 - 1$$

$$2x+1 - (x^2+4) - 1 = 2x+1-(x-2)(x+2)-1$$

What would show evidence of no understanding?

Item 1.4

- If the learner did not answer the question;
- If the learner chose more than one option.

Items 2.1 and 2.4

- If the learner incorrectly simplified the fractions and also incorrectly subtracted;
- If the learner added unlike terms in the numerator;
- If the learner subtracted the fractions incorrectly:

QUESTION 2

Simplify each of the following expressions:

2.1 $\frac{6x^5}{x^4} - \frac{15x^3}{3x^2}$

$\frac{6x \times 6x \times 6x \times 6x \times 6x}{x \times x \times x \times x} - \frac{15x \times 15x \times 15x}{3x \times 3x}$ (3)

$=$ ~~29292~~ $9x$ ✗

2.4 $\frac{2x+1}{4} - \frac{x+2}{2} - \frac{1}{4}$

$\frac{3x}{4} - \frac{3x}{2} - \frac{1}{4}$

$= \frac{1}{-2}$

What do the item statistics tell us?

Item 1.4

16% of the learners chose the correct option.

Item 2.1

10% of learners answered the question correctly.

Item 2.4

3% of learners answered the question correctly.

Factors contributing to the difficulty of the items

- Learners cannot correctly divide monomials.
- Learners are unable to correctly subtract algebraic terms.
- Learners cannot correctly determine the LCM of algebraic terms.
- Learners confuse addition and subtraction of unlike algebraic terms.
- Learners are unable to subtract fractions.
- Learners have difficulty working with algebraic fractions.

Teaching strategies

Working with algebraic expressions

Coefficients and variables

- An algebraic **expression** is made up of terms which are separated by an operation sign. These terms comprise numerals and letters of the alphabet, for example, $3xy + 2y - 5$
- When learners first start to work with algebraic expressions, they need to practice identifying the parts of an expression so that they feel comfortable working with expressions, which are not just simple numbers with constant values.
- The **variable** (which can change) is often represented by a letter of the alphabet, for example, x. This letter stands in the place of a number in the expression.
- The **coefficient** of a term is a number with a constant value that is usually written in front of a variable in an algebraic expression. For example: Learners should know that in the term $5xy$, 5 is the numerical **coefficient**, x and y are **variables**.
- A **constant** term has no variable in it, for example, 3.

- Here are three activities that you can use to revise and consolidate your learners' familiarity with algebraic expressions. In each case, when your learners have completed the table you should discuss the solutions together.

Activity 1: Terminology

- Give your learners a list of algebraic terms. Write this table on the board. You could include more terms if you wish.

Complete the table as indicated in the shaded row.

	Term	Numerical coefficient	Variables
	ab^2	1	a, b^2
a).	$-3pq^2$		
b).	$4xy$		
c).	7		
d).	$5ab^2$		
e).	$-9xy$		
f).	$7ab$		
g).	$\frac{pq}{5}$		
h).	$\frac{-3xy}{4}$		

Solutions

	Term	Number coefficient	Variables
a).	$-3pq^2$	-3	p and q^2
b).	$4xy$	4	x and y
c).	7	7 (constant)	
d).	$5ab^2$	5	a and b^2
e).	$-9xy$	-9	x and y
f).	$7ab$	7	a and b
g).	$\frac{pq}{5}$	$\frac{1}{5}$	p and q
h).	$\frac{-3xy}{4}$	$\frac{-3}{4}$	x and y

Activity 2: Revise like and unlike terms with the learners

- Like terms have identical variables. For example: $3xy$ and $7xy$ are like terms and the variable in both of the expressions is xy.

Put a tick to indicate whether Term 1 and Term 2 are like terms or not. See shaded rows for examples.

	Term 1	Term 2	Like terms	Unlike terms
	$5ab^2$	ab^2	✓	
	ab^2	$a^2 b$		✓
a).	$7ab$	ab^2		
b).	$-5p$	$5p^2$		
c).	$4xy$	$-9xy$		
d).	$7ab$	$2ab^2$		
e).	$5x^2$	$5x$		
f).	$-3xp^2$	$2p^2x$		
g).	$12ax^2y$	$5axy^2$		

Solutions: Activity 2

	Term 1	Term 2	Like terms	Unlike terms
a).	$7ab$	ab^2		✓
b).	$-5p$	$5p^2$		✓
c).	$4xy$	$-9xy$	✓	
d).	$7ab$	$2ab^2$		✓
e).	$5x^2$	$5x$		✓
f).	$-3xp^2$	$2p^2x$	✓	
g).	$12ax^2y$	$5axy^2$		✓

Activity 3: Identify the number of terms in an expression, the number of variables and the coefficients of the variables

- Once learners are familiar with the terminology they should be able to identify how many terms there are in an algebraic expression, which of them are variable and which are constant.
- Remind them that a constant term is purely numeric and variable terms have variables in them.

- They should also be able to identify the coefficients of the terms in the expression.

Complete the following table

	Expression	Number of terms	Number of variables	Coefficient of variable is__
a).	$5x - 3$	2	1	x *is* 5
b).	$4 - 3m$			m *is* __
c).	$3y - 4 + k$			y *is* __ k *is* __
d).	$\frac{y}{3} - 2x + 1$			y *is* __ x *is* __
e).	$\frac{p-4}{5}$			p *is* __
f).	$2(p+3) - 8$			p *is* __
g).	$3y - \frac{4x-1}{3}$			y *is* __ x *is* __
h).	$\frac{x-y}{3} + 4 - x^2$			x *is* __ y *is* __ x^2 *is* __

Solutions

	Expression	Number of terms	Number of variables	Coefficient of variable is __
a).	$5x - 3$	2	1	x *is* 5
b).	$4 - 3m$	2	1	m *is* -3
c).	$3y - 4 + k$	3	2	y *is* 3 k *is* 1
d).	$\frac{y}{3} - 2x + 1$	3	2	y *is* $\frac{1}{3}$ x *is* -2
e).	$\frac{p-4}{5}$	1	1	p *is* $\frac{1}{5}$
f).	$2(p+3) - 8$	2	1	p *is* 2
g).	$3y - \frac{4x-1}{3}$	2	2	y *is* 3 x *is* $\frac{-4}{3}$
h).	$\frac{x-y}{3} + 4 - x^2$	3	3	x *is* $\frac{1}{3}$ y *is* $\frac{-1}{3}$ x^2 *is* -1

Activity 4: Revise addition and subtraction rules with the learners

- Working with algebraic expressions also involves doing operations. Learners need to be able to add and subtract algebraic expressions.
- The key rule they should remember to guide them is that like terms can be added or subtracted.
- Here are some examples to revise working with like terms. The first few examples in the activity involve working with units of measurement. This kind of question emphasises the need for like units when we add or subtract.
- When the units are not the same we can convert them if they are related.

- For example, 1 kg = 1 000 g. Be sure to revise this with your learners before they do the activity, if necessary.
- Like terms are similar to like units – they can be added or subtracted.

Calculate

1. a). 4 kg + 25 g

   b). 5 cm - 3 cm

   c). 125 ml – 50 ml

   d). 5 cm - 3 mm

   e). 3 fifths - 1 fifths

   f). 2 tenths + 5 tenths

   g). 2 fifths – 1 tenths

   h). $5x-3x$

   I). $5x-3$

   j). $5x-3y$

Solutions

1. a). 4 kg + 25 g = 4 kg + 0,025 kg = 4,025 kg

   b). 5 cm – 3 cm = 2 cm

   c). 125 ml – 50 ml = 75 ml

   d). 5 cm – 3 mm = 5 cm – 0,3 cm = 4,7 cm

   e). 3 fifths – 1 fifths = 2 fifths

   f). 2 tenths + 5 tenths = 7 tenths

   g). 2 fifths – 1 tenth = 4 tenths – 1 tenth = 3 tenths

   h). $5x-3x=2x$

   I). $5x-3=5x-3$

   j). $5x-3y=5x-3y$

- Possible ideas that may come from discussions:
  - In b), c), e) and f) we add because we have like units, like fractions and like terms.
  - In a), d) and g) we can convert to the same unit and fractions of the same name and then add.
  - In i) and j) we cannot do anything unless we know the numbers x and y.
  - In h) there are like terms and so we can subtract.

Activity 5: Substitution

- Revise substitution with the learners – remind them that substitution means replacing a variable in an expression with a given constant value.
- When you have made a substitution into an expression, you find the particular value of the expression for the given value.

Complete the table by substituting for x as shown in shaded rows

	x	$2x-1$	$2x-x$	$3x$	$2x^2$	$2x^2-x$	$2x^3$
	3	5	3	9	18	15	54
	−5	−11	−5	−15	50	55	−250
a).	4						
b).	5						
c).	10						
d).	100						
e).	0,25						

- Learners should be asked questions such as:
  - Is $2x+1=3x$? Explain your answer.
  - Are we allowed to add $2x$ and 1? Explain your answer.
  - Are x and x^2 the same? Why/why not?

- Possible ideas that may come from discussions are:
  - The table shows that $2x+1$ is not equal to $3x$;
  - x and x^2 are not like terms, etc.

Using exponential laws and expanding terms

Learners should be reminded how terms can be expanded by using multiplication and prime factors. They should then be shown how expansion can be used to divide monomials and polynomials.

Examples of expansion of terms

1). $12=2\times2\times3$

2). $p^3=p\times p\times p$

3). $12p^3=2\times2\times3\times p\times p\times p$

Examples of the use of expansion to divide monomials

1). $\frac{15y^4}{3y^3}=\frac{3\times5\times y\times y\times y\times y}{3\times y\times y\times y}$

$=5\times y$ [Dividing like factors in the numerator by those in the denominator]

$=5y$

2). $\frac{6x^5}{x^4}-\frac{15x^3}{3x^2}=\frac{2\times3\times x\times x\times x\times x\times x}{x\times x\times x\times x}-\frac{3\times5\times x\times x\times x}{3\times x\times x}$

$=6x-5x$ [Dividing like factors in the numerator by those in the denominator]

$=x$

Using exponential laws

- Learners should be reminded that algebraic fractions can be factored into a product of the numerical coefficient and variables, for example:

$$\frac{15x^3}{3x^2} = \frac{15}{3} \times \frac{x^3}{x^2}$$

- Learners should then be shown that each factor can be simplified through division. They should be shown that division of variables requires the use of exponential laws.

Using the example above:

$$\frac{15}{3} = 5$$

and

$$\frac{x^3}{x^2} = x^3 \div x^2 = x^{3-2} = x$$

Therefore, $\frac{15x^3}{3x^2} = \frac{15}{3} \times \frac{x^3}{x^2}$
$= 5 \times x = 5x$

Practice Exercise

1). Expand each of the following:

a). $x^4 =$
b). $3x^2 =$
c). $6k^5 =$
d). $15x^3 =$
e). $8a^5b^5 =$

2). Simplify each of the following by expanding terms:

a). $\frac{12x^5}{6x^4} =$

b). $\frac{15x^3}{5x^2} =$

c). $\frac{15x^3}{5x^2} - \frac{12x^5}{6x^4} =$

d). $\frac{14p^2q^5}{2pq^3} + \frac{20pq^3}{5q} =$

e). $\frac{10m^2n - 6m^3}{2m^2} =$

3). Simplify each of the following using exponential laws:

a). $\frac{8a^5b^3}{2a^3b^2} =$

b). $\frac{30p^3k^2}{6p^2} - pk^2 =$

c). $\frac{12xyz^3}{4yz} + \frac{20x^2z^5}{4xz^3} =$

d). $\frac{-3u^3t^4}{u^2t^2} - \frac{15u^4t^2}{3u^3} =$

e). $\frac{16p^2q + 8pq^2}{4pq} =$

Solutions

Note that when we write out strings of terms multiplied by one another we can use a multiplication sign OR a dot to show that terms are being multiplied.

For example, $k \cdot k \cdot k \cdot k \cdot k = k \times k \times k \times k \times k$

1. a). $x^4 = x \cdot x \cdot x \cdot x$

   b). $3x^2 = 3 \times x \times x$

   c). $6k^5 = 2 \times 3 \times k \cdot k \cdot k \cdot k \cdot k$

   d). $15x^3 = 3 \times 5 \times x \cdot x \cdot x$

   e). $8a^5b^5 = 2 \times 2 \times 2 \times a \cdot a \cdot a \cdot a \cdot a \cdot b \cdot b \cdot b \cdot b \cdot b$

2. a). $\frac{12x^5}{6x^4}$

   $= \frac{2 \times 2 \times 3 \times x \cdot x \cdot x \cdot x \cdot x}{2 \times 3 \times x \cdot x \cdot x \cdot x}$

   $= 2x$

   b). $\frac{15x^3}{5x^2}$

   $= \frac{3 \times 5 \times x \cdot x \cdot x}{5 \times x \cdot x}$

   $= 3x$

   c). $\frac{15x^3}{5x^2} - \frac{12x^5}{6x^4}$

   $= \frac{3 \times 5 \times x \cdot x \cdot x}{5 \times x \cdot x} - \frac{2 \times 2 \times 3 \times x \cdot x \cdot x \cdot x \cdot x}{2 \times 3 \times x \cdot x \cdot x \cdot x}$

   $= 3x - 2x$

   $= x$

d). $\frac{14p^2q^5}{2pq^3} + \frac{20pq^3}{5q}$
$= \frac{2 \times 7 \times p \cdot p \cdot q \cdot q \cdot q \cdot q \cdot q}{2 \times p \cdot q \cdot q \cdot q} + \frac{4 \times 5 \times p \cdot q \cdot q \cdot q}{5 \times q}$
$= 7pq^2 + 4pq^2$
$= 11pq^2$

e). $\frac{10m^2n - 6m^3}{2m^2}$
$= \frac{10m^2n}{2m^2} - \frac{6m^3}{2m^2}$
$= \frac{2 \times 5 \times m \cdot m \cdot n}{2 \times m \cdot m} - \frac{2 \times 3 \times m \cdot m \cdot m}{2 \times m \cdot m}$
$= 5n - 3m$

3. a). $\frac{8a^5b^3}{2a^3b^2}$
$= \frac{8}{2} \times \frac{a^5}{a^3} \times \frac{b^3}{b^2}$
$= 4a^2b$

b). $\frac{30p^3k^2}{6p^2} - pk^2$
$= \frac{30}{6} \times \frac{p^3}{p^2} \times k^2$
$= 5pk^2$

c). $\frac{12xyz^3}{4yz} + \frac{20x^2z^5}{4xz^3}$
$= 3xz^2 + 5xz^2$
$= 8xz^2$

d). $\frac{-3u^3t^4}{u^2t^2} - \frac{15u^4t^2}{3u^3}$
$= -3ut^2 - 5ut^2$
$= -8ut^2$

e). $\frac{16p^2q + 8pq^2}{4pq}$
$= 4p + 2q$

Using equivalence and partial fractions to simplify expressions with fractions

- First you should give your learners fraction calculations that have no variables to simplify, i.e. fractions with numerators and denominators that are integers. This will help them to revise the algorithms for working with fractions.
- Learners should be shown that fractions are added if they have the same denominator. The denominator is the LCD (Lowest Common Denominator).
- The LCD is a special form of an LCM (Lowest Common Multiple) and so some people just call the LCD an LCM.

- Remember that the LCM is the smallest number into which other numbers can be divided. For example, 8 is the LCM of 4 and 8; 30 is the LCM of 5 and 6.

Examples

1). $\frac{2}{5}+\frac{1}{2}$

2). $\frac{2}{10}+\frac{2}{4}$

Solutions

1). $\frac{2}{5}+\frac{1}{2}=\frac{4}{10}+\frac{5}{10}=\frac{9}{10}$

[LCM of 5 and 2 is 10. Subtracted fractions are written as their equivalent fractions with denominator 10]

2). $\frac{2}{10}+\frac{2}{4}=\frac{1}{5}+\frac{1}{2}$ [Fractions first simplified]

$=\frac{2}{10}+\frac{5}{10}=\frac{7}{10}$ [Equivalence used]

OR

$\frac{2}{10}+\frac{2}{4}=\frac{4}{20}+\frac{10}{20}=\frac{14}{20}=\frac{7}{10}$

[Equivalence used first, answer obtained then simplified using HCF]

- Learners can next be given algebraic fractions with variables in the numerator only.
- Work through examples like those given below to demonstrate how to work with fractions which have algebraic terms in them.
- While you work through the examples you should show all of your working and allow the learners opportunities to discuss the working with you.
- Notice how the examples build on each other as they become more complicated.

Examples

1). $\frac{2x}{5}-\frac{x}{2}=\frac{4x}{10}-\frac{5x}{10}=\frac{-x}{10}$

[Equivalence has been used to find a common denominator making it possible to add the two fractions].

2). $\frac{3x}{5}-\frac{x+1}{2}=\frac{6x}{10}-\frac{5(x+1)}{10}=\frac{6x-5x-5}{10}=\frac{x-5}{10}$

[Equivalence has again been used to find a common denominator making it possible to add the two fractions. The next example works in the same way, but uses more complex fractions. The common denominator in all three examples is the same to link the examples that you are showing the class].

3). $\frac{3x-1}{5} - \frac{x-1}{2}$

$= \frac{2(3x-1)}{10} - \frac{5(x-1)}{10}$

$= \frac{6x-2-(5x-5)}{10}$

$= \frac{6x-5x-2+5}{10}$

$= \frac{x+3}{10}$

- Another strategy you can demonstrate to your class is the use of partial fractions when you simplify. This emphasises that there are different ways to write fractions.
- Learners should be able to write fractions using the different forms. It will help them to work with fractions (and algebraic expressions) more confidently.
- Show learners that the coefficient can be manipulated to make the fraction appear to "look different" – but it is in fact the same fraction:

Examples

1). $\frac{2x}{5} = \frac{2}{5}x$

The x is in the numerator – but this can be re-written as it has been here with an x-term of $\frac{2}{5}$.

2). $\frac{3x-1}{5} = \frac{1}{5}(3x-1)$

The binomial $(3x-1)$ Is an expression in the numerator. All of it is over the denominator which is 5. Hence we can re-write this expression as the binomial expression with a coefficient of $\frac{1}{5}$.

- Fractions can also be expanded to make them "look different": In the two examples below the fractions have been rewritten as two separate fractions instead of a fraction with a common denominator.

Examples

1). $\frac{4+11}{15} = \frac{4}{15} + \frac{11}{15}$

2). $\frac{3x-1}{5} = \frac{3x}{5} - \frac{1}{5}$

- You can also use factorisation to re-write fractions in different ways:
- Learners should revise factorisation using the highest common factor.

Examples

1). $\frac{2x}{5} - \frac{x}{2} = x(\frac{2}{5} - \frac{1}{2})$

which could also be written as $(\frac{2}{5} - \frac{1}{2})x$

2). $\frac{3x-1}{5}$

$= \frac{3x}{5} - \frac{1}{5}$

$= \frac{1}{5}(3x-1)$

3). $\frac{2x+1}{4} - \frac{x+2}{2} - \frac{1}{4}$

$= \left(\frac{2x}{4} + \frac{1}{4}\right) - \left(\frac{x}{2} + \frac{2}{2}\right) - \frac{1}{4}$

$= \frac{1}{4}(2x+1) - \frac{1}{2}(x+2) - \frac{1}{4}$

Practice exercise

1). Copy and complete

	Expression	Number of terms	Variables	Coefficients of variables
a).	$3x^2 + 5x - 3$			
b).	$6xy^2 - 5x^2y$			
c).	$\frac{x-y}{3} + 4 - x^2$			
d).	$\frac{2p-5}{3} + p^2$			

2). Evaluate the expressions when $b = 4, a = 3, c = -2$:

a). $b - ac$

b). $\frac{a-b}{c}$

c). $b^3 - 3ab + c$

3). Simplify the following

a). $\frac{2x}{5} - \frac{x}{2}$

b). $\frac{3x}{5} - \frac{x+1}{2}$

c). $\frac{3x-1}{5} - \frac{x-1}{2}$

d). $\frac{y+2}{4} - \frac{y-6}{3} + \frac{1}{2}$

e). $\frac{2m-3}{2} + \frac{m+1}{3} - \frac{3m-1}{2}$

Solutions

1). Copy and complete

	Expression	Number of terms	Variables	Coefficients of variables
a).	$3x^2 + 5x - 3$	3	x^2 and x	3 and 5
b).	$6xy^2 - 5x^2y$	2	xy^2 and x^{2y}	6 and -5
c).	$\frac{x-y}{3} + 4 - x^2$	3	x, y and x^2	$\frac{1}{3}$; $-\frac{1}{3}$ and -1
d).	$\frac{2p-5}{3} + p^2$	2	p and p^2	$\frac{2}{3}$ and 1

2. a). $b - ac$
$= 4 - 3(-2)$
$= 10$

b). $\frac{a-b}{c}$
$= \frac{3-4}{-2}$
$= \frac{-1}{-2} = \frac{1}{2}$

c). $b^3 - 3ab + c$
$= (4)^3 - 3(3)(4) + (-2)$
$= 64 - 36 - 2 = 26$

3. a). $\frac{2x}{5} - \frac{x}{2}$
$= x(\frac{2}{5} - \frac{1}{2})$
$= (\frac{2}{5} - \frac{1}{2})x$
$= \frac{-x}{10}$

b). $\frac{3x}{5} - \frac{x+1}{2}$
$= \frac{3x}{5} - (\frac{x}{2} + \frac{1}{2})$
$= \frac{3x}{5} - \frac{x}{2} - \frac{1}{2}$
$= \frac{6x}{10} - \frac{5x}{10} - \frac{5}{10} = \frac{x-5}{10}$

OR

$\frac{3x}{5} - \frac{x+1}{2}$
$= \frac{3x}{5} - (\frac{x}{2} + \frac{1}{2})$
$= \frac{3x}{5} - \frac{x}{2} - \frac{1}{2}$
$= (\frac{3}{5} - \frac{1}{2})x - \frac{1}{2}$
$= \frac{x}{10} - \frac{5}{10} = \frac{x-5}{10}$

c). $\frac{3x-1}{5} - \frac{x-1}{2}$

$= \frac{3x}{5} - \frac{1}{5} - (\frac{x}{2} - \frac{1}{2})$

$= \frac{3x}{5} - \frac{x}{2} - \frac{1}{5} + \frac{1}{2}$

$= \frac{6x}{10} - \frac{5x}{10} - \frac{2}{10} + \frac{5}{10} = \frac{x+3}{10}$

OR

$\frac{3x-1}{5} - \frac{x-1}{2}$

$= \frac{3x}{5} - \frac{1}{5} - (\frac{x}{2} - \frac{1}{2})$

$= (\frac{3}{5} - \frac{1}{2})x - \frac{1}{5} + \frac{1}{2}$

$= (\frac{6}{10} - \frac{5}{10})x - \frac{2}{10} + \frac{5}{10} = \frac{x+3}{10}$

d). $\frac{y+2}{4} - \frac{y-6}{3} + \frac{1}{2}$

$= \frac{3(y+2)}{12} - \frac{4(y-6)}{12} + \frac{6}{12}$

$= \frac{3y+6-4y+24+6}{12}$

$= \frac{-y+36}{12}$

e). $\frac{2m-3}{2} + \frac{m+1}{3} - \frac{3m-1}{2}$

$= \frac{3(2m-3)}{6} + \frac{2(m+1)}{6} - \frac{3(3m-1)}{6}$

$= \frac{-m-4}{6}$

Other examples of how to test algebraic expressions and fractions

ANA 2014 Grade 9 Mathematics Item 1.2

3.4 $\frac{x^2}{2} + \frac{2x^2}{3} - \frac{7x^2}{6}$	[3]

Notes:

Simplification of algebraic expressions

ANA 2013 Grade 9 Mathematics Item 2.2

Simplify each of the following expressions:		
2.2	$x\,(x + 2) - (x - 1)(x - 3)$	[4]

What should a learner know to answer this question correctly?

Learners should be able to:

- Multiply a binomial by a monomial;
- Multiply a binomial by a binomial; and
- Add or subtract terms within an algebraic expression.

Where is this topic located in the curriculum? Grade 9 Term 1 and 3

Content area: Patterns, Functions and Algebra.

Topic: Algebraic expressions.

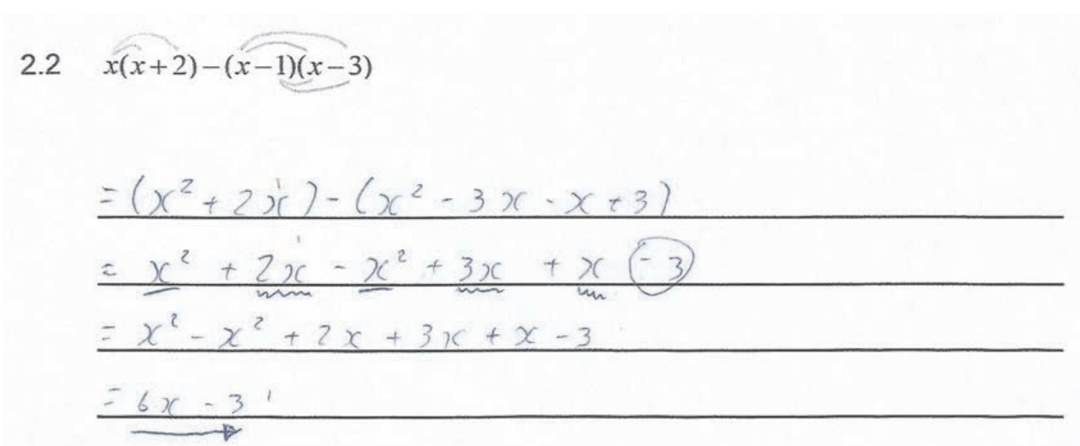
Concepts and Skills:

- Multiply integers and monomials by binomials;
- Expand and simplify algebraic expressions involving the product of binomials.

What would show evidence of full understanding?

If the learner obtained the correct answer through:

- The correct multiplication of a binomial by a monomial;
- The correct multiplication of a binomial by a binomial; and
- The correct addition and subtraction of algebraic terms:



2.2 $x(x+2)-(x-1)(x-3)$

$= (x^2 + 2x) - (x^2 - 3x - x + 3)$

$= x^2 + 2x - x^2 + 3x + x \; (-3)$

$= x^2 - x^2 + 2x + 3x + x - 3$

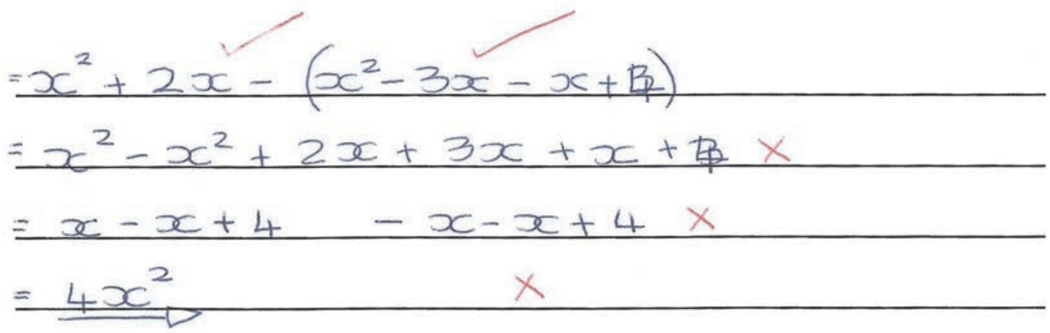
$= 6x - 3$

What would show evidence of partial understanding?

If the learner obtained an incorrect answer because:

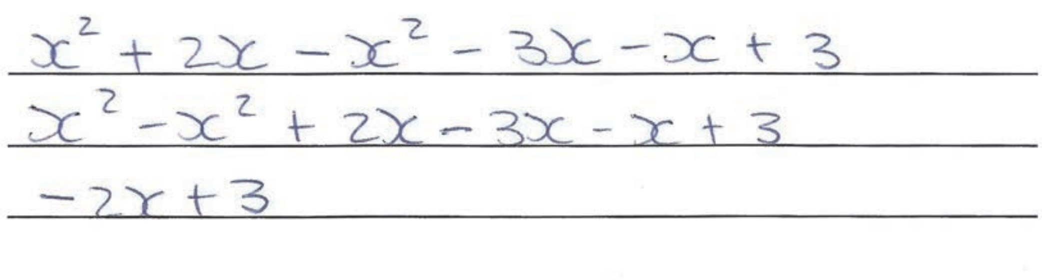
- The learner performed one or both multiplications correctly, but added or subtracted terms incorrectly:

2.2 $x(x+2)-(x-1)(x-3)$



- The learner incorrectly multiplied one or both products, but added and/or subtracted algebraic terms correctly:

2.2 $x(x+2)-(x-1)(x-3)$

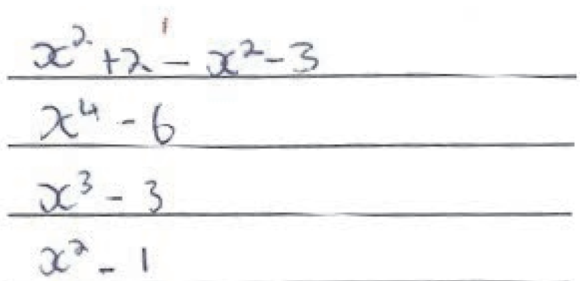


What would show evidence of no understanding?

If the learner obtained an incorrect answer because:

- The learner incorrectly multiplied both terms and then incorrectly added or subtracted the incorrect products:

2.2 $x(x+2)-(x-1)(x-3)$



What do the item statistics tell us?

9% of learners answered Item 2.2 correctly.

Factors contributing to the difficulty of the item

- Learners do not know how to correctly multiply expressions;
- Learners do not know how to correctly add or subtract algebraic terms.

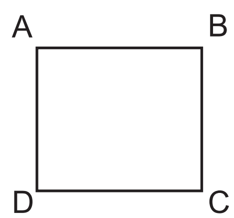
Teaching strategies

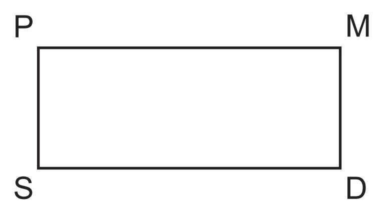
Using area of squares and rectangles to introduce multiplication

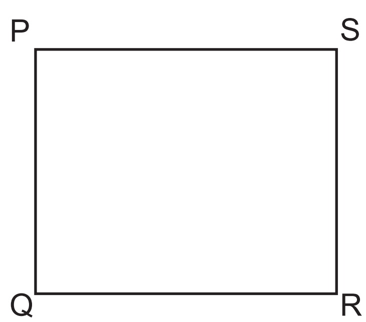
- The area of a rectangle equals its length multiplied by its breadth.
- Thus when you calculate area you are finding a product of terms.
- You can find areas of rectangles using different lengths and breadths to demonstrate multiplication.
- First, learners should calculate the areas of squares and rectangles where the lengths of the sides are constants.

Example

ABCD is a square and PMDS is a rectangle:







Calculate

1). The area of square ABCD if $AB = 5\,units$.

2). The area of rectangle PMDS if $PM = 12\,units$ and $PS = 3\,units$.

3). The area of rectangle PQRS with $PQ = 12\,units$ and $QR = 14\,units$.

Solutions

1). Area of ABCD =

$5\,units \times 5\,units = 5^2\,square\,units = 25\,square\,units$

2). Area of PMDS =

$12\,units \times 3\,units = 3\,units \times 12\,units = [3 \times (10+2)]\,square\ units$

$= [(3 \times 10) + (3 \times 2)]\,square\ units$

which is the same as

$[(3 \times 2) + (3 \times 10)] = [6 + 30] = 36\,square$ units

The process shows that $3 \times (10+2) = (3 \times 10) + (3 \times 2)$

This suggests a rule for multiplying a binomial by a monomial.

3). Area of PQRS

$= 12\,units \times 14\,units$

$=[(12 \times 4) + (10 \times 12)]\,square\ units$

$=[4 \times (10+2) + 10 \times (10+2)]\,square\ units$ - $168\,square\ units$

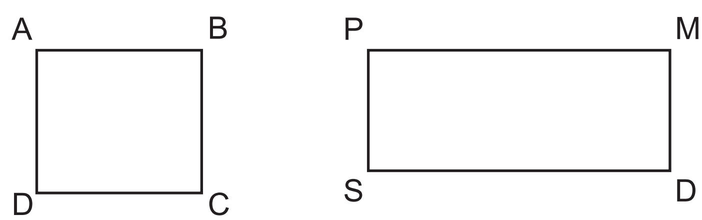
- Learners should discuss how they arrive at their answers.

Calculating area of rectangles where one side is a variable

Next allow your learners to calculate area of rectangles where one side is a variable.

Example

ABCD and PMDS are rectangles



1). Calculate the area of ABCD if $AB = 5$ and $BC = 2p$

2). Calculate the area of PMDS if $PM = \mathrm{x} + 3$ and $PS = 5$

Solutions

1). Area of ABCD $= 5 \times 2p = 10p$

2). Area of PMDS $= 5 \times (x+3)$ which could be written as

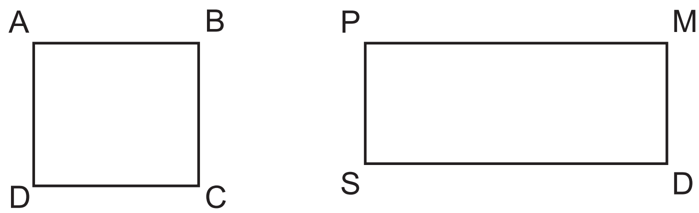
$5(x+3) = 5x + 15$ (Applying the process of multiplying a 2-digit number by a 1-digit number)

Calculating area of squares where both sides are variables

Learners should ultimately work out the area of squares and rectangles where both sides are variables.

Example

ABCD is a square and PMDS is a rectangle



1). Calculate the area of ABCD if $AB = 3y$
2). Calculate the area of PMDS if $PS = m$ and $SD = m + 2$
3). Calculate the area of ABCD if $BC = 2 + 3x$
4). Calculate the area of PMDS if $PM = t + 5$ and $PS = t - 3$

Solutions

1). Area of ABCD $= 3y \times 3y = 9y^2$
2). Area of PMDS $= m(m + 2) = m \times m + m \times 2 = m^2 + 2m$
3). Area of ABCD $= (2 + 3x)^2 = (2 + 3x)(2 + 3x) = 4 + 12x + 9x^2$
4). Area of PMDS $= (t + 5)(t - 3)$

$$= t(t - 3) + 5(t - 3)$$
$$= t^2 - 3t + 5t - 15$$
$$= t^2 + 2t - 15$$

- These activities are for demonstration and discussion.
- While you work through them you should discuss the multiplication process and allow learners to ask questions about it and use their own words to explain how they multiply the terms.

Linking products of expressions to products of numbers

Binomial × monomial

- Learners should multiply a 2-digit number by a 1-digit number.
- They should show details of what they did to arrive at the answer.
- They should also write an explanation of the process.
- The process should be linked to the multiplication of a binomial by a monomial (this is possible because a two-digit number can be split into two parts in order to multiply it).

Example

$23 \times 4 = (20 + 3) \times 4$

$= (4 \times 3) + (4 \times 20)$

$= 12 + 80$

$= 92$

The order of the process does not matter:

- $4 \times 3 + 4 \times 20 = 4 \times 20 + 4 \times 3 = 4 \times (20 + 3)$ [Taking out 4 as the common factor].
- Working backwards: $4 \times (20 + 3) = 4 \times 20 + 4 \times 3$
- Whichever way you calculate it, you can check that the answer you get is 92.

Thus:

$$x(x + 2) = x \times x + x \times 2 = x^2 + 2x$$

Binomial × Binomial

- Learners should multiply a 2-digit number by a 2-digit number.
- They should show details of what they did to get the answer.
- They should also write an explanation of the process.
- The process should be linked to multiplication of a binomial by a binomial.

Example

$25 \times 13 = (20 + 5) \times (10 + 3)$

So, $(20 + 5) \times (10 + 3) = 3 \times (20 + 5) + 10 \times (20 + 5)$

$= 10 \times (20 + 5) + 3 \times (20 + 5)$

$= (20 \times 10) + (5 \times 10) + (20 \times 3) + (5 \times 3)$

$= 200 + 50 + 60 + 15$

$= 200 + 110 + 15$

$= 310 + 15$

$= 325$ [Cross check using a calculator that $25 \text{ x } 13 = 325$]

Thus: $(x + 3)(x + 2) = x(x + 2) + 3(x + 2) = x^2 + 5x + 6$

- Splitting numbers into parts and breaking down the multiplication is another way to demonstrate how the distributive law works when we multiply binomials.
- Use this to lead into the next two strategies where simplification of algebraic expressions is spoken about in more detail.

Simplifying algebraic expressions

- Learners should start with simplifying expressions where there will be no sign changes.
- Learners should be reminded that we do multiplication before addition and/or subtraction.
- Once we have multiplied all of the terms, we add like terms together to get the final answer.

Example

1). $p(p+1)+(p+1)(p+2)$

First calculate $p(p+1)=p^2+p$

Next calculate $(p+1)(p+2)=p^2+3p+2$

This means $p(p+1)+(p+1)(p+2)=(p^2+p)+(p^2+3p+2)$

Lastly, add like terms: $(p^2+p)+(p^2+3p+2)=2p^2+4p+2$

Simplifying expressions where signs change

Example

$(a+2)(a-2)-(a+2)(a-3)$

$=(a^2-4)-(a^2-a-6)$

$= a^2 - 4 - a^2 + a + 6$ [Note that signs change once brackets are removed. This is because you have multiplied the terms inside the brackets by –1].

$=a+2$

- Work through many examples with your class to allow them to become familiar with the procedures.
- Here are some further examples you could use:

1). $(a+b)(a+2b)=a(a+2b)+b(a+2b)$

$=a^2+2ab+ab+2b^2$

$=a^2+3ab+2b^2$

2). $(3m+4)(m-2)=3m(m-2)+4(m-2)$

$=3m^2-6m+4m-8$

$=3m^2-2m-8$

3). $(3x-2y)^2=(3x-2y)(3x-2y)$

$=3x(3x-2y)-2y(3x-2y)$

$=9x^2-6xy-6xy+4y^2$

$=9x^2-12xy+4y^2$

4). $(x+8)(x-1)-(x-8)(x+8)$

$=x^2+8x-x-8-(x^2-64)$

$=x^2+7x-8-x^2+64)$

$=7x+56$

- Once your class is ready to work independently you could use the following activity to consolidate the learners' understanding of the simplification of algebraic expressions.

Practice exercise

Simplify each of the following expressions:

1). $3p(1-4p)$
2). $(m-5)(m+5)$
3). $(x+5)^2$
4). $(n-5)^2$
5). $(x-4)(2x+3)$
6). $3y(y+2)+(y-2)(y+3)$
7). $(t-7)(t+7)+(t+7)^2$
8). $(x+3)(x-4)-(x-3)(x+3)$
9). $3(p-2)(p+5)-2(p-3)^2+2(p+2)(p-2)$
10). $(a+b)(b-c)-(a-c)(b+c)-(a-b)^2$

Solutions

1). $3p(1-4p)=3p-12p^2$

2). $(m-5)(m+5)=m^2-25$

3). $(x+5)^2=x^2+10x+25$

4). $(n-5)^2=n^2-10n+25$

5). $(x-4)(2x+3)=2x^2-5x-12$

6). $3y(y+2)+(y-2)(y+3)$

$=3y^2+6y+y^2+y-6$

$=4y^2+7y-6$

7). $(t-7)(t+7)+(t+7)^2$

$=t^2-49+t^2+14t+49$

$=2t^2+14t$

8). $(x+3)(x-4)-(x-3)(x+3)$

$=x^2-x-12-x^2+9$

$=-x-3$

9). $3(p-2)(p+5)-2(p-3)^2+2(p+2)(p-2)$

$=3(p^2+3p-10)-2(p^2-6p+9)+2(p^2-4)$

$=3p^2+9p-30-2p^2+12p-18+2p^2-8$

$=3p^2+21p-56$

10). $(a+b)(b-c)-(a-c)(b+c)-(a-b)^2$

$=ab-ac+b^2-bc-ab-ac+bc+c^2-a^2+2ab-b^2$

$=-a^2+c^2+2ab-2ac$

Another example of how to test simplification of algebraic expressions

ANA 2014 Grade 9 Mathematics Item 3.1

Simplify each of the following expressions. The denominators in the fractions are not equal to zero.

3.1 $2(x+2)^2-(2x-1)(x+2)$ [4]

Notes:

Square roots and cube roots of monomials

ANA 2013 Grade 9 Mathematics Item 2.3

Simplify each of the following expressions:

2.3 $\sqrt{225x^4} - \sqrt[3]{125x^6}$ [5]

What should a learner know to answer this question correctly?

Learners should be able to:

- Determine correct square roots and cube roots of whole number coefficients;
- Determine correct square roots and cube roots of monomials;
- Subtract terms in an algebraic expression.

Where is this topic located in the curriculum? Grade 9 Term 1 and 3

Content area: Patterns, Functions and Algebra.

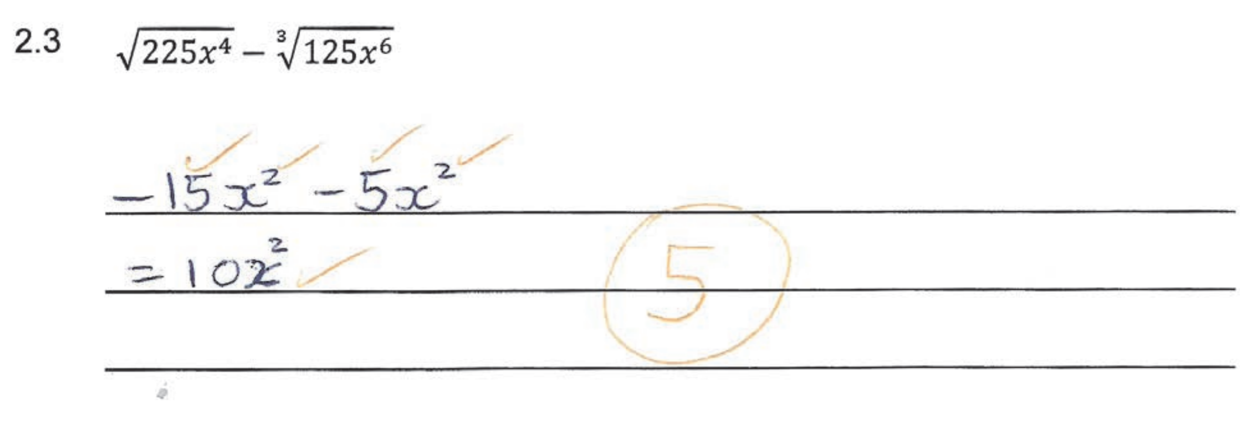
Topic: Algebraic expressions.

Concepts and skills: Determine square roots and cube roots of algebraic terms.

What would show evidence of full understanding?

If the learner obtained the correct answer through:

- Correct determination of the square roots and cube roots of the given terms; and
- Correct subtraction of the square roots obtained.


2.3 $\sqrt{225x^4} - \sqrt[3]{125x^6}$

$= 15x^2 - 5x^2$

$= 10x^2$

5

What would show evidence of partial understanding?

If the learner obtained an incorrect answer because

- The square roots and cube roots of the numerical coefficients were correctly determined, but the roots of variables were incorrect:

2.3 $\sqrt{225x^4} - \sqrt[3]{125x^6}$

$15x^4 - 5x^6$

$= 10x^2$

- The square roots and the cube roots of the variables were correctly determined, but the roots of the coefficients were incorrect:

2.3 $\sqrt{225x^4} - \sqrt[3]{125x^6}$

$225x^2 - 125x^2$

$= 100x^2$

- The learner correctly determined the square roots only and not the cube roots:

2.3 $\sqrt{225x^4} - \sqrt[3]{125x^6}$

$= 15x^2 - 33{,}54x^4$ ✓ ✓ (2) ✗

$= -18{,}54x^2$ ✗

- The learner interpreted the cube root sign as a square root and disregarded the variables. The learner used a calculator to find the approximate numeric value of the surd, √ 125. (A surd is a square root of number that that is not a perfect square.):

2.3 $\sqrt{225x^4} - \sqrt[3]{125x^6}$

15 − 11,2

3,8

What would show evidence of no understanding?

- If the learner gave an incorrect answer and showed no awareness of how to find square or cube roots:

In the following example the square root signs and cube root signs are interpreted as squaring or cubing the given monomial:

2.3 $\sqrt{225x^4} - \sqrt[3]{125x^6}$

$\sqrt{15x^4} - 25^3$

$= 50625 - 15625$

$= 35000$

What do the item statistics tell us?

8% of learners answered item 2.3 correctly.

Factors contributing to the difficulty of the item

- Learners are unable to determine the correct square roots or cube roots of monomials;
- Learners cannot correctly subtract algebraic terms.

Teaching strategies

Use of factoring and grouping to find square and cube roots

- An investigative approach is recommended. That is:
- Allow learners to work out several calculations involving a similar procedure;
- Let them write down observed patterns;
- The learners should then come up with generalisations for each set of calculations.
- If necessary, revise the basic theory needed for calculating square and cube roots with the learners.

- A square number is a number that can be written in the form x^2 and this can be expanded as $x \times x$.
- When we want to find the square root of a number we want to find the number x, so that $x \times x$ is equal to the number we have.
- For example,
  - $4 \times 4 = 16$, so 4 is the square root of 16.
  - $5x \times 5x = 25x^2$, so $5x$ is the square root of $25x^2$.

- A cubed number is a number that can be written in the form x^3 and this can be expanded as $x \times x \times x$.
- When we want to find the cube root of a number we want to find the number x, such that $x \times x \times x$ is equal to the number we have.
- For example,
  - $4 \times 4 \times 4 = 64$, so 4 is the cube root of 64.
  - $5x \times 5x \times 5x = 125x^3$, so $5x$ is the cube root of $125x^3$.

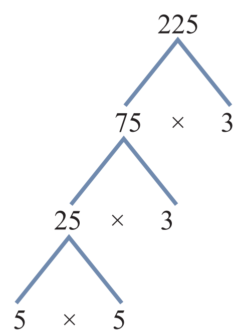
- NOTE: It is a good idea to suggest to your learners that they take time to learn the first 6 (or so) square and cubed numbers off by heart – this will help them to recognise these numbers when they see them in a mathematics question. The more numbers the learners know, the better it is. Doing a lot of repeated activities with your learners will help them to learn these numbers as they go along.
  - First 6 square numbers: $1, 4, 9, 16, 25, 36$, and so on;
  - First 5 cubed numbers: $1, 27, 64, 125, 216$, and so on.

- Learners should revise the use of prime factors to determine square roots and cube roots of whole numbers which are bigger than the basic square and cubed numbers that they may know off by heart.

Examples

a). Calculating $\sqrt{225}$

We start by factorising 225 as shown:



$225 = 5 \times 5 \times 3 \times 3$

$\sqrt{225} = 5 \times 3 = 15$

b). Calculating $\sqrt[3]{64}$

64 is factorised:

$2 \times 2 \times 2 \times 2 \times 2 \times 2$

$\sqrt[3]{64} = 2 \times 2 = 4$

- Learners should use expansion of variables to determine the roots of monomials. Let learners expand variables, pair factors or put them in groups of 2 or 3 to find square and cube roots as needed.
- To find square roots we need pairs of like factors.
- To find cube roots we need triplets of like factors.

Examples

Calculating $\sqrt{x^4}$

Expanding: $x^4 = x \times x \times x \times x$
Pairing: $x^4 = (x \times x) \times (x \times x)$
Thus, $\sqrt{x^4} = \sqrt[2]{x^2 x^2} = x \times x = x^2$

Calculating $\sqrt[3]{x^6}$

Expanding: $x^6 = x \times x \times x \times x \times x \times x$

Grouping: $x^6 = (x \times x \times x) \times (x \times x \times x)$
Thus, $\sqrt[3]{x^6} = \sqrt[3]{x^3 x^3} = x \times x = x^2$

- Using their ability to find square or cube roots of whole numbers and variables, learners will be able to find the roots of these monomials.
- You should work through plenty of similar examples to help your learners consolidate this process.
- Here are two such examples:

Example

1). $\sqrt{225x^4}$

Possible solution

$$225x^4 = 3 \times 3 \times 5 \times 5 \times x \times x \times x \times x$$

$$\sqrt{225x^4} = 3 \times 5 \times x \times x = 15x^2$$

2). $\sqrt[3]{125x^6}$

Possible solution

$125x^6 = 5 \times 5 \times 5 \times x \times x \times x \times x \times x \times x$

$\sqrt[3]{125x^6} = \sqrt[3]{125x^3x^3} = 5 \times x \times x$ (From grouping factors)

$= 5x^2$

- Learners should ultimately be able to simplify expressions which include roots, as in the ANA item discussed here.

Example: $\sqrt{225x^4} - \sqrt[3]{125x^6} = 15x^2 - 5x^2 = 10x^2$

- Once you have spent sufficient time explaining and demonstrating this method of algebraic simplification, you should allow your learners to work though many more similar examples on their own to consolidate this method. You could use the following activities for that purpose.

Activities

Using factoring and grouping to find square and cube roots

Determine: $\sqrt{324}$

Solution

$\sqrt{324} = \sqrt{2 \times 2 \times 3 \times 3 \times 3 \times 3}$

$= 2 \times 3 \times 3$

$= 18$

- Using rules of exponents when working with algebraic expressions
- Learners should be reminded of the following exponential laws:
- You should write them on the board and explain them.
- Give examples of how the laws work and allow learners to give you their own examples of the laws at work.

1). The law of multiplication: If terms that have the same base are multiplied, you add the powers to simplify.

$$a^m \cdot a^n = a^{m+n}$$

Example

$5^2 \times 5^3 = 5^{2+3} = 5^5$

2). When you raise a term with an exponent to a power, you multiply the two exponents by each other to simplify.

$$(a^m)^n = a^{m \times n}$$

Example

$(3^2)^5 = 3^{2 \times 5}$

3). The square root of a number can be written as a number to the exponent $\frac{1}{2}$.

$\sqrt{a} = a^{\frac{1}{2}}$

Example

$\sqrt{9} = 9^{\frac{1}{2}} = (3^2)^{\frac{1}{2}} = 3^{2 \times \frac{1}{2}} = 3$

4). The n[th] root of a number can be written as a number to the exponent $\frac{1}{n}$.

$\sqrt[n]{a} = a^{\frac{1}{n}}$

Example

$\sqrt[n]{16} = 16^{\frac{1}{n}}$

5). The square root of a product of terms is equal to the product of the square roots of the terms.

$\sqrt{ab} = \sqrt{a} \times \sqrt{b}$

Example

$\sqrt{16 \times 25} = \sqrt{16} \times \sqrt{25} = 4 \times 5 = 20$

6). The square root of the sum of two terms cannot be simplified to the sum of the square roots.

$\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$

Example

$\sqrt{25+144} \neq \sqrt{25} + \sqrt{144}$

$\sqrt{25+144} = \sqrt{169} = 13$

and

$\sqrt{25} + \sqrt{144} = 5 + 12 = 17$

Therefore $\sqrt{25+144} \neq \sqrt{25} + \sqrt{144}$

since $13 \neq 17$

- Give the learners further problems that require application of these rules.
- Learners should be able to write numbers as powers or products of powers.

Examples

$225 = 3^2 \times 5^2$

$125m^3 = 5^3 m^3$

- Learners should be able to apply the rules $\sqrt{x} = x^{\frac{1}{2}}$ and $\sqrt[3]{x} = x^{\frac{1}{3}}$ as shown below.

$\sqrt{225} = (3^2)^{\frac{1}{2}} \times (5^2)^{\frac{1}{2}} = 3 \times 5 = 15$

$\sqrt[3]{x^6} = (x^6)^{\frac{1}{3}} = x^2$

- Before you allow learners to work through the practice activity make sure they are confident in using the laws.
- Use as many examples from your textbook or other resources as you can find to give your learners the opportunity practice applying the laws efficiently.

Activity: Using rules of exponents when working with algebraic expressions

1). Determine:

a). $\sqrt{729}$

b). $\sqrt{p^6 q^{10}}$

c). $\sqrt{81x^8 y^2}$

d). $\sqrt[3]{512}$

e). $\sqrt[3]{64m^{12} y^9}$

2). Simplify each of the following expressions:

a). $\sqrt{256x^6} + \sqrt[3]{64x^9}$

b). $\sqrt[3]{216t^3} - \sqrt{144t^2}$

c). $\sqrt{9x^6 + 16x^6}$

d). $\sqrt{9x^6} + \sqrt{16x^6}$

e). $\sqrt{9x^6 \times 16x^6}$

f). $\sqrt{100n^8 - 36n^8}$

g). $\sqrt[3]{\dfrac{512b^3}{27}}$

h). $\sqrt{169p^4} - \sqrt[3]{729p^6}$

I). $\sqrt[3]{8m^{15}} - \sqrt{196m^4} + 7m^5$

j.) $\sqrt{400p^8} + \sqrt[3]{-125p^{12}} - \sqrt{5p^3 \times 20p^5}$

Solutions

1). a). $\sqrt{729} = 27$

b). $\sqrt{p^6q^{10}} = p^3q^5$

c). $\sqrt{81x^8y^2} = 9x^4y$

d). $\sqrt[3]{512} = 8$

e). $\sqrt[3]{64m^{12}y^9} = 4m^4y^3$

2). a). $\sqrt{256x^6} + \sqrt[3]{64x^9} = 16x^3 + 4x^3 = 20x^3$

b). $\sqrt[3]{216t^3} - \sqrt{144t^2} = 6t - 12t = -6t$

c). $\sqrt{9x^6 + 16x^6} = \sqrt{25x^6} = 5x^3$

d). $\sqrt{9x^6} + \sqrt{16x^6} = 3x^3 + 4x^3 = 7x^3$

e). $\sqrt{9x^6 \times 16x^6} = \sqrt{9x^6} \times \sqrt{16x^6} = 3x^3 \times 4x^3 = 12x^6$

or

$\sqrt{9x^6 \times 16x^6} = \sqrt{144x^{12}} = 12x^6$

f). $\sqrt{100n^8 - 36n^8} = \sqrt{64n^8} = 8n^4$

g). $\sqrt[3]{\dfrac{512b^3}{27}} = \dfrac{\sqrt[3]{512b^3}}{\sqrt[3]{27}} = \dfrac{8b}{3}$

h). $\sqrt{169p^4} - \sqrt[3]{729p^6} = 13p^2 - 9p^2 = 4p^2$

i). $\sqrt[3]{8m^{15}} - \sqrt{196m^4} + 7m^5$
$= 2m^5 - 14m^2 + 7m^5$
$= 9m^5 - 14m^2$

j). $\sqrt{400p^8} + \sqrt[3]{-125p^{12}} - \sqrt{5p^3 \times 20p^5}$
$= 20p^4 + (-5p^4) - \sqrt{100p^8}$
$= 15p^4 - 10p^4$
$= 5p^4$

Other examples of how to test square roots and cube roots of monomials

ANA 2014 Grade 9 Mathematics Item 1.1

1.1 $\sqrt{16x^{16}} =$

A $8x^8$

B $8x^4$

C $4x^4$

D $4x^8$

[1]

ANA 2014 Grade 9 Mathematics Item 2.2.1

2.2 Calculate without using a calculator. Show in each case all the calculation steps.

2.2.1 $\sqrt[3]{73-(-3)^2}$

[2]

Notes:

Factorisation of algebraic expressions

ANA 2013 Grade 9 Mathematics Items 3.1 and 3.2

Factorise fully:		
3.1	$6a^3 - 12a^2 + 18a$	[2]
3.2	$7x^2 - 28$	[2]

What should a learner know to answer these questions correctly?

Learners should be able to:

- Factorise a trinomial;
- Factorise algebraic expressions by removing the highest common factors;
- Factorise algebraic expressions with a difference of squares.

Where is this topic located in the curriculum? Grade 9 Term 1 and 3

Content area: Patterns, Functions and Algebra.

Topic: Algebraic expressions.

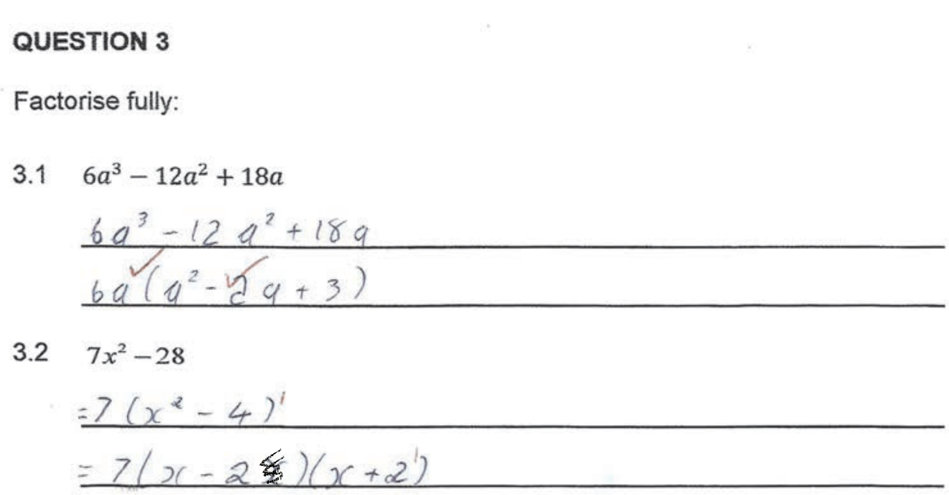
Concepts and skills:

- Factorise algebraic expressions that involve common factors;
- Factorise algebraic expressions that involve the difference of squares.

What would show evidence of full understanding?

If the learner obtained the correct answer by:

- Correctly factorising algebraic expressions by removing the highest common factor; and
- Correctly factorising algebraic expressions involving the difference of squares.


QUESTION 3

Factorise fully:

3.1 $6a^3 - 12a^2 + 18a$

$6a^3 - 12a^2 + 18a$

$6a(a^2 - 2a + 3)$

3.2 $7x^2 - 28$

$= 7(x^2 - 4)$

$= 7(x - 2)(x + 2)$

What would show evidence of partial understanding?

- If the learner obtained an incorrect answer because the expressions were incorrectly or not fully factorised, but showed some knowledge of factorisation.

QUESTION 3

Factorise fully:

3.1 $6a^3 - 12a^2 + 18a$

$6a(a^2 - 6a + 12)$ ✓ ①

3.2 $7x^2 - 28$

$7(x^2 - 4)$ ✓ ②

$7(x$

3.2 $7x^2 - 28$

①

✓ $7(x^2 - 21)$ ✓

What would show evidence of no understanding?

If the learner obtained an incorrect answer because:

- The algebraic expressions were incorrectly factorised;
- The learner gave no evidence of knowing the correct steps in factorising.

QUESTION 3

Factorise fully:

3.1 $6a^3 - 12a^2 + 18a$

$6a^3 - 12a^2 + 18a$

$= (3a^2 + 3a)(2a - 6a)$

What do the item statistics tell us?

Item 3.1

11% of learners answered correctly.

Item 3.2

7% of learners answered correctly.

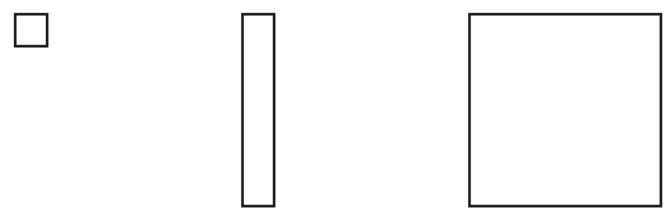
Factors contributing to the difficulty of the items

- The highest common factor in item 3.1 has 2 factors.
- Item 3.2 requires learners to factorise twice.

Teaching strategies

Using squares and rectangles to visualise factorisation, 1

- You can use the area of rectangular shapes to help your learners visualise the connection between algebraic expressions and their factors.
- Draw a diagram with a square and rectangles with the dimensions $1 \times 1; 1 \times x;$ and $x \times x$, as shown:



1×1
Area $= 1$

$1 \times x$
Area $= x$

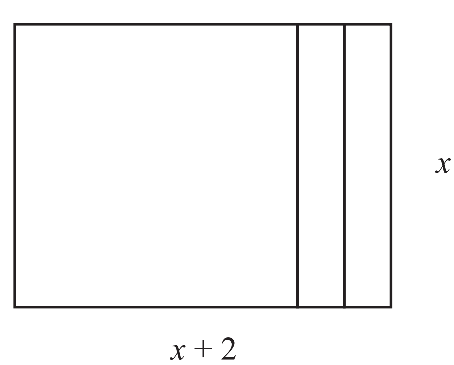
$x \times x$
Area $= x^2$

- Learners should use the drawing to make a visual representation of the expressions in the following way:
  - First, use the areas of all the shapes that make up the whole drawing to write an expression for the area.
  - Then use the length and breadth of the new shape to work out the area of the shape.
- In this way you get two different expressions for the area of the shape – one with a sum/difference of expressions and one with factors.

Examples

1). The drawing below is made up of one square with an area $x \times x$ and two rectangles each with an area of $1 \times x = x$.

a). The area of the whole shape, found by adding its component parts, is $x^2 + 2x$.



b). The length of the vertical side of the shape is x units. The length of the horizontal side of the shape (which has been extended by 2 units) is $x+2$ units.

c). The area of the rectangle with these side lengths is $x(x+2)$.

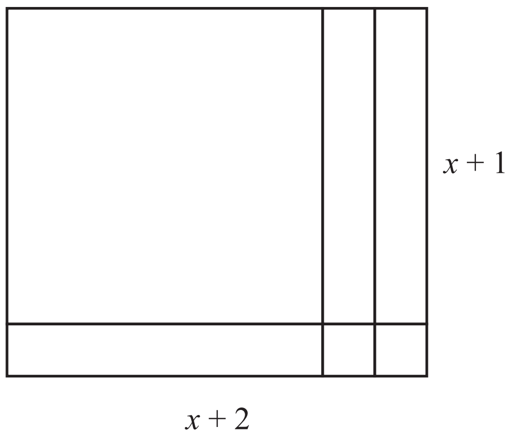
d). These two expressions for the area of the shape show us that: $x^2+2x=x(x+2)$

2). x^2+3x+2 can be depicted as follows:

a). One large square with an area of x^2;

b). 3 rectangles with an area of x; and

c). Two small squares with an area of 1.



d). Using the lengths of the sides of the extended square we get another expression for the area of the rectangle we have made: $(x+2)(x+1)$

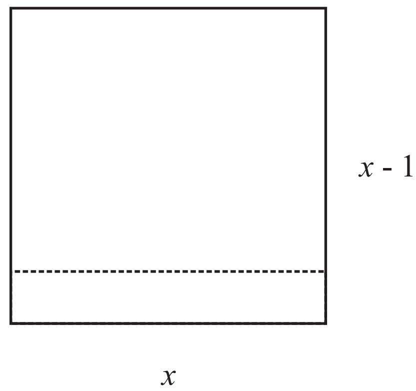
e). If we equate these two expressions for the area of the shape we can see that: $x^2+3x+2=(x+2)(x+1)$

- In the first two examples we increased the size of the shapes by adding rectangular or square pieces.
- We now look at what happens when we decrease the lengths of the sides of the shape.

- Agree with learners that diagrams with dashed lines as shown in the illustration will be used for subtraction.

x-1 can then be drawn as follows:

$x^2 - x$ can then be drawn as follows:



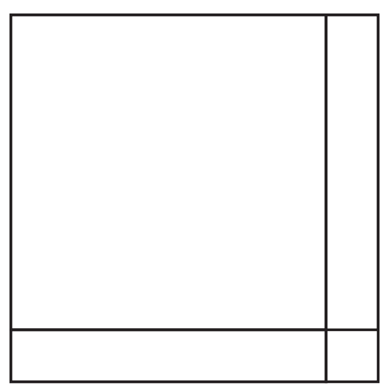
Therefore $x^2 - x = x(x - 1)$

Activity

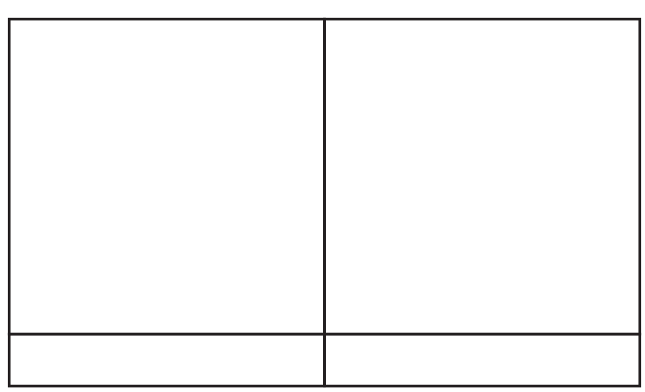
Using squares and rectangles to visualise factorisation

- Write down a product presented by each of the diagrams below.
- In each case the length of the sides of the squares is x units. When the length is increased or decreased it is by 1 unit of length each time.

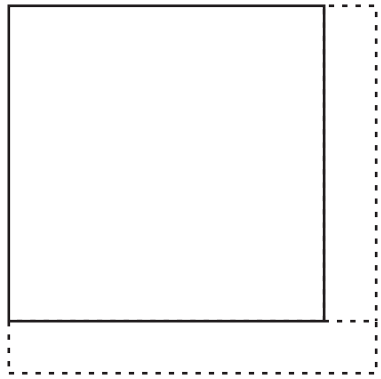
1). Increase the lengths of both sides.



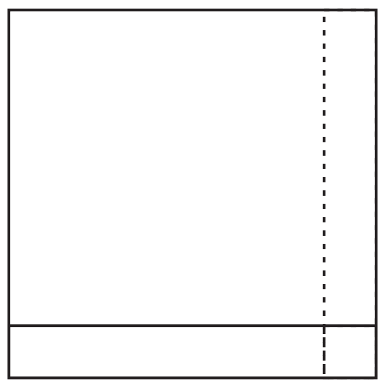
2). Double the number of squares and increase the length as shown.



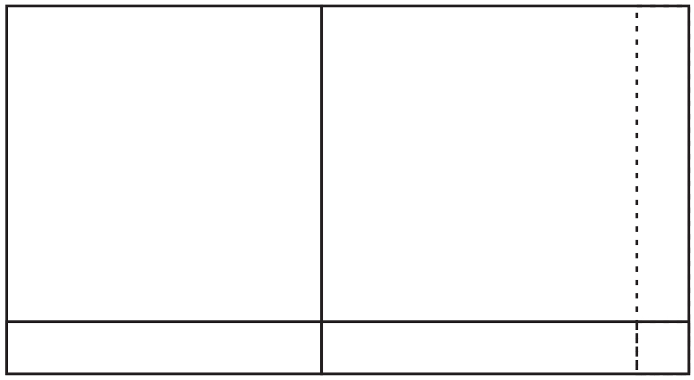
3). Decrease the lengths of both sides.



4). Increase the length of one side and decrease the length of the other side.



5). Double the number of squares, increase the length across the doubled edge and decrease the length of one of the squares.



Solutions:

1). $x^2+x+x+1=x^2+2x+1=(x+1)(x+1)=(x+1)^2$

2). $x^2+x^2+x+x=2x^2+2x=2x(x+1)$

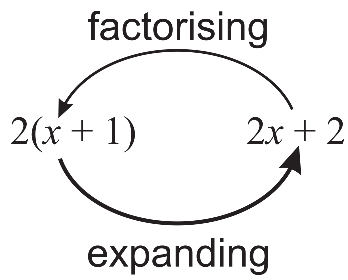
3). $x^2-2x+1=(x-1)(x-1)=(x-1)^2$

4). $x^2+x-x-1=x^2-1=(x+1)(x-1)$

5). $2x^2+2x-x-1=2x^2+x-1=(2x-1)(x+1)$

Relating products to factors

- Factorising reverses what multiplying does. When we simplify a factorised expression, we get an algebraic expression in which the brackets have been removed.
- This means that the factorisation process is the opposite of expanding brackets. For example, expanding brackets requires $2(x+1)$ to be written as $2x+2$. In factorisation we start with $2x+2$ and end up with $2(x+1)$.



- The two expressions $2(x+1)$ and $2x+2$ are equivalent: they have the same value for all values of x.

- Learners need to know how to multiply and to factorise. It is good for them to feel comfortable with the relationship between multiplication and factorisation so that they can reverse these operations mentally, where possible.
- First use simple expressions so that learners understand the link between factorising and expanding. Once they have understood this link they will be able to work more effectively with longer and more complicated algebraic expressions.
- Learners need to realise that there are different kinds of algebraic expressions which factorise in different ways and that they can learn to recognise these types of expressions and methods of factorisation.

Factorising monomials

Let learners factorise monomials.

Examples

a). $12 = 1 \times 12 = 2 \times 6 = 3 \times 4$

b). $3ab = 3 \times a \times b$

c). $10x^2y^3 = 10 \times x \times x \times y \times y \times y$

- Let the learners identify the HCF of pairs of monomials and then factorise related expressions.
- Write the following table on the board so that you can discuss it with your class.

Monomials	HCF	Possible expressions	Factorised expression	Conclusion
$3a$ and 6	3	$3a+6$	$3(a+2)$	$3a+6=3(a+2)$
		$3a-6$	$3(a-2)$	$3a-6=3(a-2)$
		$6-3a$	$3(2-a)$	$6-2a=3(2-a)$
$2a^2$ and $5a$				

- Fill in the second part of the table in discussion with the class.
- Let the learners tell you about the relationship between the expressions and the factorised expressions as you complete the table.

The completed table should be as follows:

Monomials	HCF	Possible expressions	Factorised expression	Conclusion
$3a$ and 6	3	$3a+6$	$3(a+2)$	$3a+6=3(a+2)$
		$3a-6$	$3(a-2)$	$3a-6=3(a-2)$
		$6-3a$	$3(2-a)$	$6-2a=3(2-a)$
$2a^2$ and $5a$		$2a^2+5a$	$a(2a+5)$	$2a^2+5a=a(2a+5)$
		$2a^2-5a$	$a(2a-5)$	$2a^2-5a=a(2a-5)$
		$5a-2a^2$	$a(5-2a)$	$5a-2a^2=a(5-2a)$

Linking products to factors

Examples

1). $4(x+3)=4x+12$

- Therefore, $4x+12=4(x+3)$
- In this instance, there is a common factor in the algebraic expression.
- This algebraic expression has no special name, but when there is a common factor across the terms in an algebraic expression, we can factorise it, as we have done here.

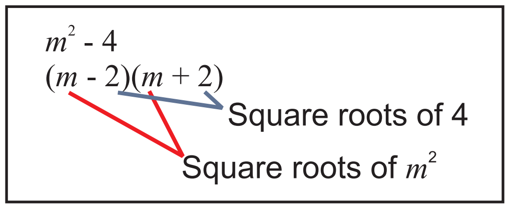
2). $p(k+3)+4(k+3)=(k+3)(p+4)$

- In this example, the expression has a repeated binomial factor which is a **common factor**. The expression can be factorised as we have shown.

3). $(x-5)(x+5)=x^2-25$

- Therefore, $x^2-25=(x-5)(x+5)$
- This type of expression is called a difference of squares. It can always be factorised in this way.
- In this instance we are able to make two binomial factors – each of them is made of the square root of the two terms in the algebraic expression.

- Draw learners attention to the pattern:
- They need to be able to apply this to any difference of squares expression.



Activity involving different types of factorising

1). Common factors

a). $-2x^3+6x^2-4x$

b). $25x^2-5x$

c). $abc-abd+ab$

d). $\frac{1}{2}ax-\frac{1}{2}bx$

2). Common binomial factors

a). $x(a+b)+y(a+b)$

b). $x(a-b)+y(a-b)$

c). $3x(2x-1)+(1-2x)$

d). $3m^4-5m^3+4m^2-8m$

3). Difference of squares

a). x^2-9y^4

b). a^2-16

c). x^2y^2-1

d). 100^2-99^2

Solutions

1). Common factors

a). $-2x^3+6x^2-4x=-2x(x^2-3x+2)$

b). $25x^2-5x=5x(5x-1)$

c). $abc-abd+ab=ab(c-d+1)$

d). $\frac{1}{2}ax-\frac{1}{2}bx=\frac{1}{2}x(a-b)$

2). Common binomial factors

a). $x(a+b)+y(a+b)=(a+b)(x+y)$

b). $x(a-b)+y(a-b)=(a-b)(x+y)$

c). $3x(2x-1)+(1-2x)=3x(2x-1)-(2x-1)=(2x-1)(3x-1)$

d). $3m^4-5m^3+4m^2-8m=m(3m^3-5m^2+4m-8)$

3). Difference of squares

a). $x^2-9y^4=(x-3y^2)(x+3y^2)$

b). $a^2-16=(a-4)(a+4)$

c). $x^2y^2-1=(xy-1)(xy+1)$

d). $100^2-99^2=(100-99)(100+99)=1(199)=199$

Factorising trinomials

- The following factorised and multiplied expressions are related: $(x+2)(x+3)=x^2+5x+6$
- Therefore, $x^2+5x+6=(x+2)(x+3)$

- This type of expression is called a trinomial. We can create two binomial factors for some trinomials when the terms work together as in the above example.

- One way of explaining trinomial factorisation is to show how you can re-write the expression to make it into a four-term expression with a common binomial factor, for example:

n^2+5n+6

$=n^2+2n+3n+6$ [Splitting 5n into terms with common factor with the constant term[

$=(n^2+2n)+(3n+6)$ [Grouping terms with a common factor]

$=n(n+2)+3(n+2)$ [$n+2$ is a common factor]

$=(n+2)(n+3)$ [Factorisation involving a common factor]

- Note that n^2+5n+6 could also be written as $n^2+3n+2n+6$
- This would lead to factors $(n+3)(n+2)$ which is the same as $(n+2)(n+3)$

- Learners should be made aware that factors of ax^2+bx+c will be of the form $(m+h)(n+k)$ where:
    - m and n are factors of ax^2
    - h and k are factors of c
    - $c>0$ implies that h and k are either both positive or negative
    - $h+k=b$ if $a=1$

- This type of factorising (trinomial) requires some thought and mental calculation, but lots of practice will help learners to become confident with the method of trinomial factorisation.

- The following examples show some different trinomials and their factors.
- Work through the examples with your learners – while you do so talk about the way in which the binomial factors are related to the trinomial expression.
- Remember that these are just a few examples to help you in the teaching of trinomial factorisation – you should find many more examples if you think your learners need more practice.

‘

Examples

a). $x^2+5x+6=(x+2)(x+3)$
2 and 3 are both positive and 2 + 3 = 5

b). $x^2-5x+6=(x-2)(x-3)$
2 and 3 are both negative and (-2) + (-3) = -5

c). $x^2-5x-6=(x+1)(x-6)$
1 and 6 have opposite signs, but their sum is -5

d). $x^2+5x-6=(x-1)(x+6)$
1 and 6 have opposite signs, but their sum is 5

e). $x^2-x-6=(x+2)(x-3)$
2 and 3 have opposite signs, but their sum is −1

f). $x^2+x-6=(x-2)(x+3)$
2 and 3 have opposite signs, but their sum is 1

Practice Exercise: Mixed examples

Factorise fully

1). $8p+4q$

2). $9a-45a^2b$

3). $4yz-6xy-2my$

4). $x(y-4)+3(y-4)$

5). $3p-7q-4t(3p-7q)$

6). $xy+yz+xw+zw$

7). $6mk+3mv-2nk-nv$

8). $25x^2-a^2$

9). $9abc^2-25abd^2$

10). $4x^2+2x-9y^2+3y$

11). $p^2+7a+12$

12). $p^2-4a-12$

13). $18a-12a^2+6a^3$

14). $28-7x^2$

15). $5a^2+20a-105$

Solutions

1). $8p+4q=4(2p+q)$

2). $9a-45a^2b=9a(1-5ab)$

3). $4yz-6xy-2my=2y(2z-3x-m)$

4). $x(y-4)+3(y-4)=(y-4)(x+3)$

5). $(3p-7q)-4t(3p-7q)=(3p-7q)(1-4t)$

6). $xy+yz+xw+zw=(x+z)(y+w)$

7). $6mk+3mv-2nk-nv$

$=3m(2k+v)-n(2k+v)$

$=(2k+v)(3m-n)$

8). $25x^2-a^2=(5x-a)(5x+a)$

9). $9abc^2-25abd^2$

$=ab(9c^2-25d^2)$

$=ab(3c-5d)(3c+5d)$

10). $4x^2+2x-9y^2+3y$

$=4x^2-9y^2+2x+3y$

$=(2x-3y)(2x+3y)+2x+3y$

$=(2x+3y)(2x-3y+1)$

11). $p^2+7a+12=(p+3)(p+4)$

12). $p^2-4a-12=(p+2)(p-6)$

13). $18a-12a^2+6a^3=6a(3+2a+a^2)$

14). $28 - 7x^2$

$= 7(4 - x^2)$

$= 7(2 - x)(2 + x)$

15). $5a^2 + 20a - 105$

$= 5(a^2 + 4a - 21)$

$= 5(a + 7)(a - 3)$

Other examples of how to test factorisation of algebraic expressions

ANA 2014 Grade 9 Mathematics Item 4.1

Factorise fully:

4.1 $3x^2y - 9xy^2 + 12x^3y^3$ [2]

ANA 2014 Grade 9 Mathematics Item 4.4

Factorise fully:

4.4 $x^2 - 11x + 18$ [2]

Notes:

Solving algebraic equations

ANA 2013 Grade 9 Mathematics Items 4.1, 4.2, 4.3 and 4.4

Solve for x:		
4.1	$3x-1=5$	[2]
4.2	$2(x-2)^2=(2x-1)(x-3)$	[4]
4.3	$\frac{2x-3}{2}+\frac{x+1}{3}=\frac{3x-1}{2}$	[4]
4.4	$x^3=64$	[2]

What should a learner know to answer these questions correctly?

Learners should be able to:

- Solve simple linear equations by inspection or using additive and multiplicative inverses;
- Solve equations that involve brackets;
- Solve equations that involve fractions;
- Solve equations that involve exponents.

Where is this topic located in the curriculum? Grade 9 Term 1 and 3

Content area: Patterns, Functions and Algebra.

Topic: Algebraic equations.

Concepts and skills:

- Solve equations by inspection;
- Solve equations using additive and multiplicative inverses;
- Solve equations using laws of exponents.

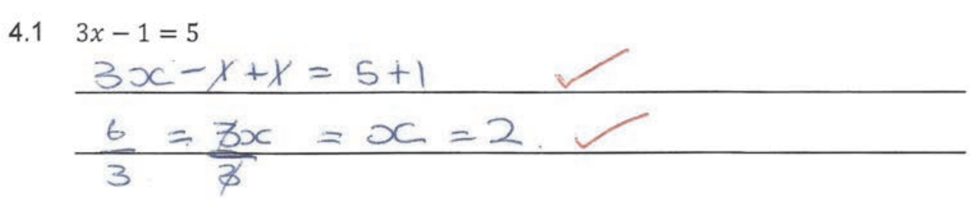
What would show evidence of full understanding?

If in all cases the learner applied the correct reasoning and processes and isolated the variable correctly to find the solution.

Item 4.1

If the learner obtained the correct answer through:

- Correctly adding 1 on both sides of the equal sign; and
- Correctly dividing both sides of the equal sign by 3.

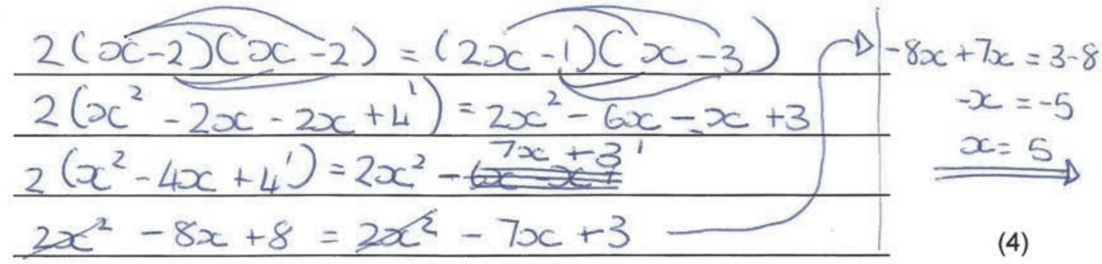


Item 4.2

If the learner obtained the correct answer through:

- Correctly removing brackets – correct multiplication of binomials and monomials;
- Correctly adding and subtracting like terms; and
- Correctly using additive and multiplicative inverses to isolate the variable and find the solution.

4.2 $2(x-2)^2 = (2x-1)(x-3)$

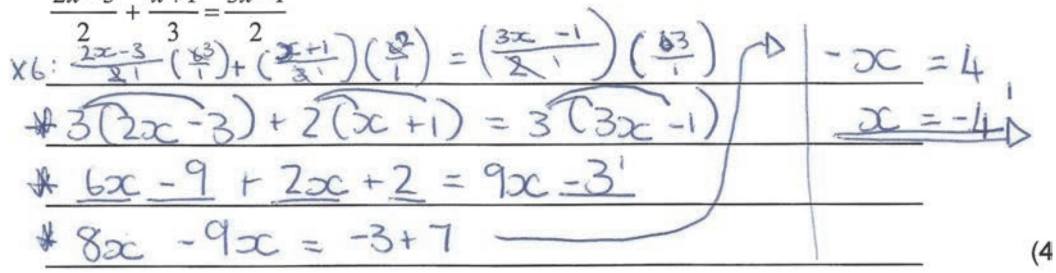


Item 4.3

If the learner obtained the correct answer through:

- Correctly reducing fractions to whole numbers by multiplying each term by the LCM of the denominators;
- Adding and subtracting like terms; and
- Correctly using additive and multiplicative inverses to isolate the variable and find the solution.

4.3 $\frac{2x-3}{2} + \frac{x+1}{3} = \frac{3x-1}{2}$

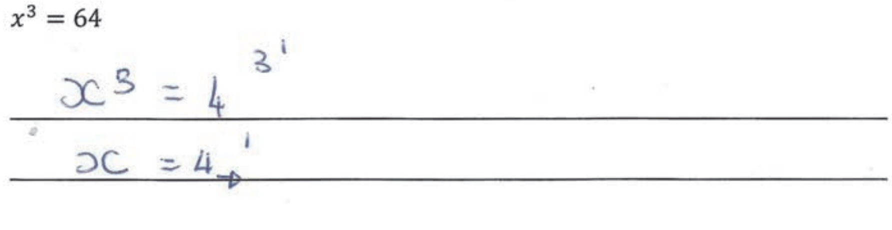


Item 4.4

If the learner obtained the correct answer through:

- Correctly expressing 64 as a power, with 4 as the base and 3 as the exponent; and
- Correctly equating the bases (taking cube roots on both sides of the equal sign).

4.4 $x^3 = 64$

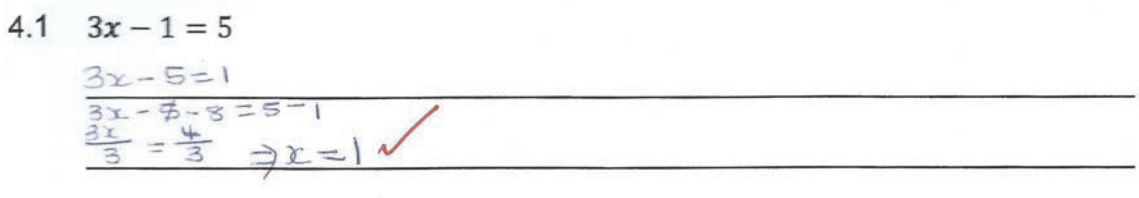


What would show evidence of partial understanding?

Item 4.1

If the learner obtained a partially correct answer because:

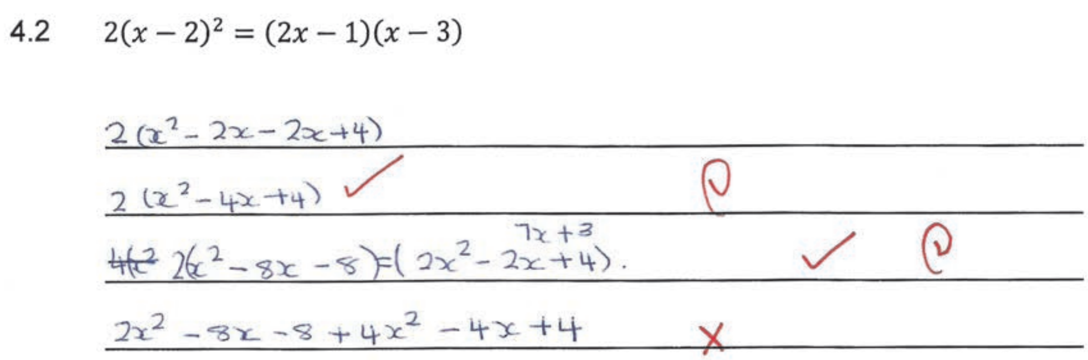
- The additive inverse was incorrectly used, but the multiplicative inverse rule was later correctly applied;
- Other variations of partially correct working were shown in the isolation process.



Item 4.2

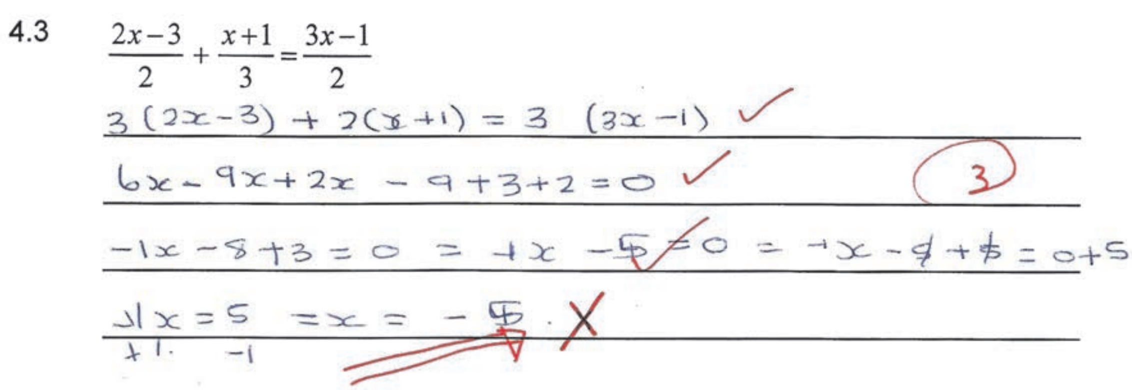
If the learner obtained a partially correct answer because:

- The binomial was correctly squared and correctly multiplied by 2;
- The binomials on the RHS of the equal sign were incorrectly multiplied;
- The equal sign was lost and like terms not added;
- Other variations of partially correct working were shown in the isolation process.



Item 4.3

- If the learner obtained a partially correct answer because:
- Fractions were correctly eliminated;
- The binomials on the RHS of the equal sign were incorrectly multiplied by 3.
- Other variations of partially correct working were shown in the isolation process.

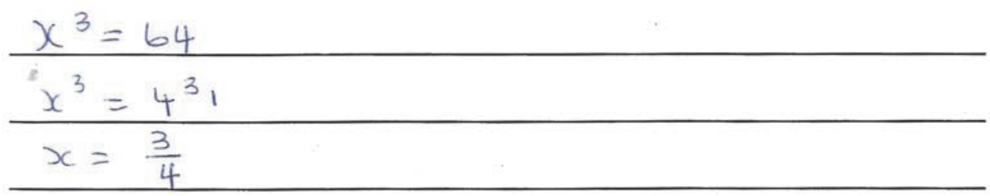


Item 4.4

If the learner obtained a partially correct answer because:

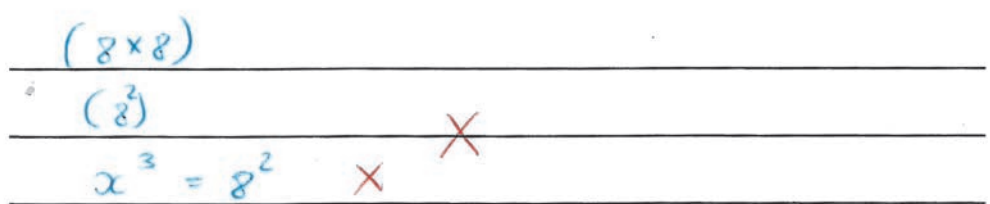
- The base and the power were correctly identified;
- The learner wrote the constant as an exponential expression (which was correct) but then simply wrote the exponential expression as a fraction.
- The learner should have equated the bases, since the exponents were the same.

4.4 $x^3 = 64$



- This learner wrote the constant as an exponential expression (using 8 as a base) which did not lead to the solution since although 64 = 8^2 this did not enable the learner to equate the bases as the exponents were not equal.

4.4 $x^3 = 64$



What would show evidence of no understanding?

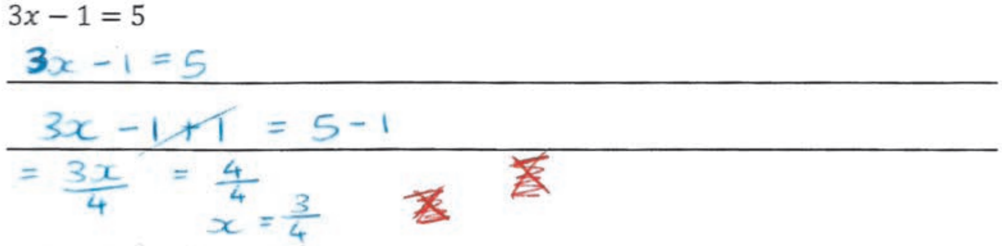
Item 4.1

If the learner obtained the incorrect answer because:

The inverses were incorrectly used (both additive and multiplicative). as in the example that follows;

Other working done showed no evidence of understanding of the isolation process.

4.1 $3x - 1 = 5$



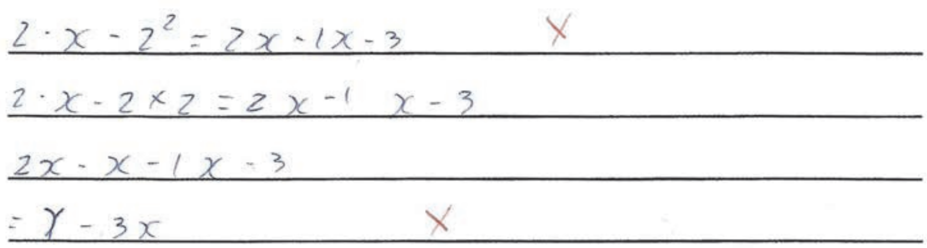
If the question was not answered.

Item 4.2

If the learner obtained the incorrect answer because:

- Brackets were incorrectly removed and unlike terms added as in the example shown.
- Other working done showed no evidence of understanding of the isolation process.

4.2 $2(x-2)^2 = (2x-1)(x-3)$


$2 \cdot x - 2^2 = 2x - 1x - 3$ ✗

$2 \cdot x - 2 \times 2 = 2x - 1 \quad x - 3$

$2x - x - 1x - 3$

$= Y - 3x$ ✗

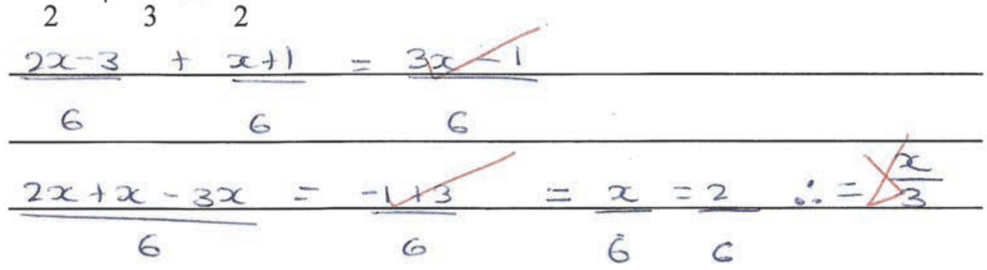
- If the question was not answered.

Item 4.3

If the learner obtained the incorrect answer because:

- Fractions were incorrectly eliminated and like terms incorrectly added as in the following example;
- Other working done showed no evidence of understanding of the isolation process.

4.3 $\frac{2x-3}{2} + \frac{x+1}{3} = \frac{3x-1}{2}$


$\frac{2x-3}{6} + \frac{x+1}{6} = \frac{3x-1}{6}$

$\frac{2x+x-3x}{6} = \frac{-1+3}{6} = \frac{x}{6} = \frac{2}{6} \therefore = \frac{x}{3}$

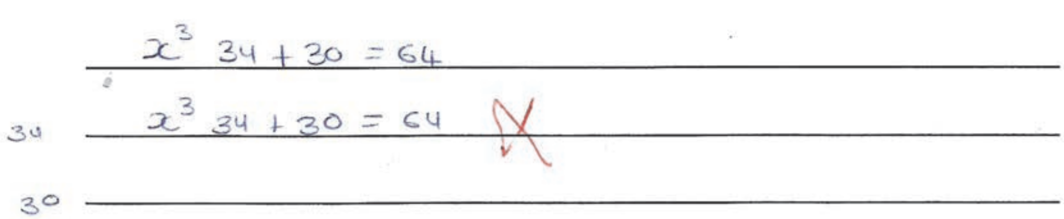
- If the question was not answered.

Item 4.4

If the learner obtained the incorrect answer because:

- As in the example shown the powers were not expressed as expressions with equal indices;
- Other working done showed no evidence of understanding of the isolation process.

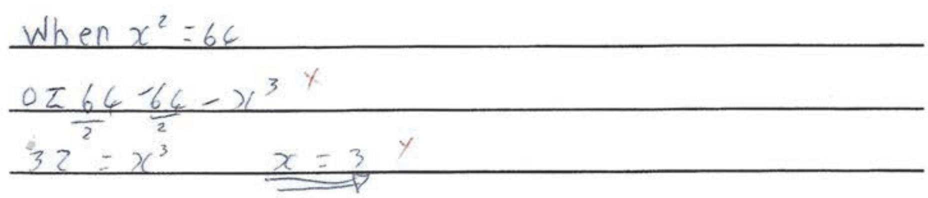
4.4 $x^3 = 64$


$x^3 \; 34 + 30 = 64$

34 $x^3 \; 34 + 30 = 64$ ✗

30

4.4 $x^3 = 64$



What do the item statistics tell us?

Item 4.1

22% of learners answered item 4.1 correctly.

Factors contributing to the difficulty of the item

- Learners are unable to meaningfully read (interpret) simple linear equations.
- Learners lack basic knowledge of how to use additive and multiplicative inverses to solve equations.

Item 4.2

3% of learners answered item 4.2 correctly.

Factors contributing to the difficulty of the item

- Learners cannot correctly multiply algebraic expressions (see item 2.2 and 2.4 above).
- They cannot correctly add and subtract like terms (see items 2.1 to 2.4 above).
- Learners lack the basic skills needed to solve algebraic equations.

Item 4.3

4% of learners answered item 4.3 correctly.

Factors contributing to the difficulty of the item

- Learners lack the skills necessary to solve equations with fractions.
- Learners lack the basic skills mentioned under Items 4.1 and 4.2 above.

Item 4.4

11% of learners answered item 4.1 correctly

Factors contributing to the difficulty of the item are:

- Learners are unable to interpret equations that involve exponents.
- Learner's lack understanding of the basic techniques for solving equations that involve exponents.

Teaching strategies

Algebraic language

- Ensure that learners are able to read equations meaningfully.
- Ensure that learners know and understand terminology used in algebra, for example: add, subtract, difference, double, quotient, product and dividend.
- You need to revisit algebraic language if necessary. You could use examples such as those shown in the table that follows.

Example of writing given expressions using algebra

Words	Symbolic representation
The sum of a certain number and 4 is 7	$x + 4 = 7$
The difference between a certain number and 12 is 3	$n - 12 = 3$
The product of 5 and a certain number is 10	$5m = 10$
A number is doubled and the product is increased by 7. The answer obtained is 15.	$2y + 7 = 15$

- Learners should use algebraic language to determine solutions of simple equations by inspection.
- This involves mental work during which learners must manipulate symbols and numbers in their heads. Such an activity will assist learners to use the correct logic when they isolate more complicated equations.

Examples

1). $x + 4 = 7$
What number is added to 4 to get 7?
[The expected response is 3. Thus, the solution is $x=3$]

2). $5m = 10$
What number when multiplied by 5 gives 10 as a product?
[The expected response is 2. Thus, solution is $m = 2$]

3). $3p - 5 = 16$
From which number do you subtract 5 and get 16?
[Learners should realise that $3p = 21$]
Which number do you multiply by 3 to get this number?
[The expected response is 7. Thus, solution is $p = 7$]

Activity

1). Write each of the following in symbolic notation:

a). The sum of a number and 7.

b). The product of 4 and a number.

c). The difference between a certain number and 3 is 5.

d). The product of 5 and a certain number is increased by 2. The answer is 17.

e). A number is increased by 7. The answer obtained is the same as when the same number is first multiplied by 3, and thereafter -5 is added to the product obtained.

2). Write each of the following in words

a). $d - 7$

b). $-3k$

c). $n + 12 = 21$

d). $3x = 24$

e). $m + 5 = 2m - 7$

.

3). Solve by inspection

a). $m + 37 = 30$

b). $27 - y = 15$

c). $3y = 45$

d). $\frac{p}{4} = 3$

e). $\frac{n}{3} + 5 = 15 = 3$

Solutions

1). Algebraic language: Variables may differ

a). $x + 7$

b). $4p$

c). $x - 3 = 5$

OR

$3 - x = 5$

d). $5y + 2 = 17$

e). $m + 7 = 3m - 5$

2). Accept any meaningful statement

a). 7 less than a certain number or a certain number that is decreased by 7.

b). The product of -3 and a certain number

c). A certain number is increased by 12. The answer obtained is 21.

d). he product of 3 and a certain number is 24.

e). A certain number is increased by 5. The answer obtained is the same as when the number is doubled, then decreased by 7.

3). Solve by inspection

a). $m+37=30$

$m=-7$

b). $27-y=15$

$y=12$

c). $3y=45$

$y=15$

d). $\frac{p}{4}=3$

$p=12$

e). $\frac{n}{3}+5=15$

$n=30$

Using trial and improvement or inverse rules to solve equations

- Learners should first solve linear equations with unknowns appearing on one side of the equal sign.
- Work through the isolation process with the learners. Make sure that learners are able to talk about the way they move the numbers around in the equation in order to find the solutions.
- Talking about the isolation process in relation to numbers (where learners can see the numeric calculation that confirms whether or not what they have done is correct) will help the learners apply these steps in the case of variables where the process might not immediately confirm that the moves were correct.

Examples

1). $x+4=7$

$x+4-4=7-4$

[Adding the additive inverse of 4 on both sides: the *additive* inverse in this case is – 4 since 4 – 4 = 0]

$=3$

2). $5m=10$

$\frac{1}{5}\times 5m=\frac{1}{5}\times 10$

[Multiplying both sides by the *multiplicative* inverse of 5: in this case this is $\frac{1}{5}$ since $\frac{1}{5}\times 5=1$]

$m=2$

3). $3p - 5 = 16$

$3p - 5 + 5 = 16 + 5$ [Use of *additive* inverse]

$3p = 21$

$\frac{1}{3} \times 3p = \frac{1}{3} \times 21$ [Use of *multiplicative* inverse]

$p = 7$

Extend activity to equations with unknown on both sides of the equal sign

Example

- A number is tripled. The answer obtained is the same as when the same number is increased by 8. Determine the number.
- Learners should express the statement in symbols.
- They may try to work out the solution by trial and improvement.
- They should ultimately use inverses to determine the solution.
- They should write the equation in symbols as: $3p = p + 8$

Solution by trial and improvement

p	$3p$	$p + 8$	Conclusion
1	3	9	$3p \neq p + 8$, *therefore* $p \neq 1$
2	6	10	$3p \neq p + 8$, *therefore* $p \neq 2$
3	9	11	$3p \neq p + 8$, *therefore* $p \neq 3$
4	12	12	$3p = p + 8$, *therefore* $p = 4$

Solution through the use of inverses

$3p = p + 8$

$3p - p = p + 8 - p$ [Adding the additive inverse of p on both sides]

$2p = 8$

$\frac{1}{2} \times 2p = \frac{1}{2} \times 8$ [Using the multiplicative inverse of 2]

$p = 4$

Extend equations to equations with brackets

- These may initially be done by inspection, but when the equations are more complex learners should work through the steps that will allow them to get to the correct answer.

Example

- Solve the equation $4(y - 5) = 12$

- Work through the example by asking questions such as the following:
- What number do you multiply by 4 and get 12? (The learners' response should be 3).

- This means that we can write $y - 5 = 3$.
- What inverse have we used to simplify the equation here? (The learners should respond "The multiplicative inverse").
- If $y - 5 = 3$, what is the value of y? (The response should be $y = 8$ to satisfy this equation).
- Check: $4(8 – 5) = 4(3) = 12$, as required.

- Here are some more examples that you could work through with your class to consolidate the use of inverse laws when solving equations:

More examples

1). $2(x - 2) = x - 3$

Solution

$2(x - 2) = x - 3$

$2x - 4 = x - 3$ [Removing brackets]

$2x - x = 4 - 3$ [Using *additive* inverses]

$x = 1$

2). $3(x - 2) = 2(x - 1)$

Solution

$3x - 6 = 2x - 2$ [Removing brackets]

$3x - 2x = 6 - 2$ [Using *additive* inverses]

$x = 4$

3). $2(x - 2)^2 = (2x - 1)(x - 3)$

Solution

$2(x - 2)(x - 2) = (2x - 1)(x - 3)$ [Expanding the square]

$2(x^2 - 4x + 4) = 2x^2 - 7x + 3$

OR

$(2x - 4)(x - 2) = 2x^2 - 7x + 3$

$2x^2 - 8x + 8 = 2x^2 - 7x + 3$ [Removing brackets]

$-x = -5$ [Using *additive* inverse]

$x = 5$ [Using *multiplicative* inverse]

- Learners should ultimately realise that if an equation with brackets is given, it is advisable to first remove the brackets and then use inverses.
- If there are no brackets (or if we have simplified the brackets) we then use inverses - we use the additive inverse first, then the multiplicative inverse.

Using trial and improvement or inverse rules to solve equations

Solving equations using inverse rules

1). $3m - 7 = m + 5$

2). $2x + 3 = -3x - 7$

3). $2x + 14 = 5 - x$

4). $5(p + 2) = p + 18$

5). $3(y + 2) = 2(y - 1)$

6). $2(x - 2) - (x - 1) = 2x - 2$

7). $(n + 3)(n - 2) = (n - 2)(n + 2)$

8). $(x + 3)(x - 2) = (x - 1)^2 - 2x + 3$

Solutions

1). $3m - 7 = m + 5$

$2m = 12$

$m = 6$

2). $2x + 3 = -3x - 7$

$5x = -10$

$x = -2$

3). $2x + 14 = 5 - x$

$3x = -9$

$x = -3$

4). $5(p + 2) = p + 18$

$5p + 10 = p + 18$

$4p = 8$

$p = 2$

5). $3(y + 2) = 2(y - 1)$

$3y + 6 = 2y - 2$

$y = -8$

6). $2(x - 2) - (x - 1) = 2x - 2$

$2x - 4 - x + 1 = 2x - 2$

$x = -1$

7). $(n + 3)(n - 2) = (n - 2)(n + 2)$

$n^2 + n - 6 = n^2 - 4$

$n = 2$

8). $(x+3)(x-2)=(x-1)^2-2x+3$

$x^2+x-6=x^2-2x+1-2x+3$

$5x=10$

$x=2$

Solving equations with fractions

Once learners can work with the above equations, they should be then introduced to equations with fractions. Learners do have greater difficulty working with fractions and so it is better to establish the isolation process used in the solution of equations in simpler numeric situations. Once the learners have mastered working with simpler numeric expressions, the activity can be extended to include fractions.

Examples

1). Half of a certain number is 4. Determine the number. Explain how you got the answer.

Expected solution

$\frac{n}{2}=4$ [Symbolic representation]

$2\times\frac{n}{2}=2\times4$ [Using inverse rule]

$n=8$

2). Half of a certain number is 2. Determine the number. Use the method you used above.

Expected solution

$\frac{k}{2}=2$ [Symbolic representation]

$k=4$ [Inspection]

3). Half of a certain number is one half. Determine the number. Explain how you got the answer.

Expected solution

$\frac{c}{2}=\frac{1}{2}$ [Symbolic representation]

$c=1$

4). Half of a certain number added to its quarter is 6. Determine the number.

Expected solution

$\frac{t}{2}+\frac{t}{4}=6$

$\frac{2t}{4}+\frac{t}{4}=6$ or $t\left(\frac{1}{2}+\frac{1}{4}\right)=6$ [Refer to Item 2.1 and Item 2.4]

$\frac{3t}{4}=6$

$t=8$

5). $\frac{2x-3}{2}+\frac{x+1}{3}=\frac{3x-1}{2}$

Expected solution

$\frac{2x-3}{2}+\frac{x+1}{3}=\frac{3x-1}{2}$

$\frac{6(2x-3)}{2}+\frac{6(x+1)}{3}=\frac{6(3x-1)}{2}$ [Using equivalence]

$3(2x-3)+2(x+1)=3(3x-1)$ [Simplifying fractions]

$6x-9+2x+2=9x-3$ [Removing brackets]

$x=-4$

Activity: Solving equations with fractions: Solve for the unknown

1). $\frac{x}{2}-\frac{x}{3}=2$
2). $3-\frac{x-2}{12}=4$
3). $\frac{y+2}{4}-\frac{y-6}{3}=\frac{1}{2}$
4). $\frac{x+1}{3}+\frac{2x-3}{2}=\frac{3x-1}{2}$
5). $p-2+\frac{1+p}{2}=\frac{3(p-1)}{2}$

Solutions

1). $\frac{x}{2}-\frac{x}{3}=2$

$6\left(\frac{x}{2}\right)-6\left(\frac{x}{3}\right)=6(2)$

$3x-2x=12$

$x=12$

2). $3-\frac{x-2}{12}=4$

$36-(x-2)=48$ [LCD = 12]

$-x+2=12$

$x=-10$

3). $\frac{y+2}{4}-\frac{y-6}{3}=\frac{1}{2}$

$3(y+2)-4(y-6)=6$ [LCD = 12]

$3y+6-4y+24=6$

$-y=-24$

$y=24$

4). $\frac{x+1}{3}+\frac{2x-3}{2}=\frac{3x-1}{2}$

$2(x+1)+3(2x-3)=3(3x-1)$ [LCD = 6]

$2x+2+6x-9=9x-3$

$8x-7=9x-3$

$x=-4$

5). $p-2+\frac{1+p}{2}=\frac{3(p-1)}{2}$

$2(p-2)+1+p=3(p-1)$ [LCD = 2]

$2p-4+1+p=3p-3$

$3p-3=3p-3$

$0=0$

- This equation is true for all values of p. It is always true, since it does not depend on p.

Solving equations with exponents

- Lastly, learners should be taught how to solve simple equations with exponents.
- To work with exponential equations, learners need to be able to apply their knowledge of exponents and the exponential laws.
- You should revise the laws of exponents with your class if necessary. (See Item 2.3 for other work in relation to manipulation of exponential expressions).

Examples

1). A number is squared. The answer is 9. Determine the number.

Expected solution

$a^2 = 9$ [Symbolic representation]

$a^2 = 3^2$ [Expressing the number as a power of the same index/exponent]

$a = \pm 3$

2). A number is cubed. The answer is 1 000. Determine the number.

Expected solution

$v^3 = 1\,000$

$v^3 = 10^3$ [Powers of the same base]

$v = 10$

3). A number is cubed. The answer is 64. Determine the number.

Expected solution

$x^3 = 64$

$x^3 = 4^3$

$x = 4$

Activity: Solving equations with exponents

Solve for the unknown:

1). $m^2 = 225$

2). $x^3 = 729$

3). $3n^2 = 27$

4). $2^m = 32$

5). $3^x = 81$

Solutions

1). $m^2 = 225$

$m^2 = 15^2$

$m = \pm 15$

2). $x^3 = 729$

$x^3 = 9^3$

$x = 9$

3). $3n^2 = 27$

$n^2 = 9$

$n = \pm 3$

4). $2^m = 32$

$2^m = 2^5$

$m = 5$

5). $3^x = 81$

$3^x = 3^4$

$x = 4$

Other examples of how to test solving algebraic equations

ANA 2014 Grade 9 Mathematics Item 5.1

Solve for x:

5.1 $(x - 2)^2 + 3x - 2 = (x + 3)^2$ [4]

ANA 2014 Grade 9 Mathematics Item 5.3

5.3 $\frac{x+2}{3} - \frac{x-3}{4} = 0$ [3]

Notes:

Algebraic equations: word problems

ANA 2013 Grade 9 Mathematics Item 14

14. The 200 Grade 9 boys in a school play soccer, hockey or both.
If 150 boys play soccer and 130 play hockey, calculate how many of them play BOTH soccer and hockey. [3]

What should a learner know to answer this question correctly?

Learners should be able to:

- Set up equations to describe problem situations;
- Analyse and interpret equations that describe a given situation.

Where is this topic located in the curriculum? Grade 9 Terms 1 & 3

Content area: Patterns, Functions and Algebra.

Topic: Algebraic equations.

Concepts and skills:

- Set up equations to describe problem situations;
- Analyse and interpret equations that describe a given situation;
- Solve equations by inspection using additive and multiplicative inverses and laws of exponents;
- Determine the numerical value of an expression by substitution;
- Use substitution in equations to generate tables of ordered pairs;
- Extend solving equations to include using factorisation and equations of the form: a product of factors = 0.

What would show evidence of full understanding?

If the learner obtained the correct solution by using an appropriate mathematical strategy.

Let x boys play soccer and hockey

$150 + (130 - x) = 200$ ✓M
$280 - x = 200$ ✓M
$x = 80$ ✓A

or $130 + (150 - x) = 200$ ✓M
$280 - x = 200$ ✓M
$x = 80$ ✓A

or

Total number of boys who play hockey and soccer
$= 150 + 130 = 280$ ✓M
But this is 80 more than the number of boys in grade 9 which means 80 boys must play both soccer and hockey ✓✓M

What would show evidence of partial understanding?

- If the learner worked out the problem, but did not write the initial equation correctly using the given total number of learners at the school (200);

The 200 Grade 9 boys in a school play soccer, hockey or both. If 150 boys play soccer and 130 play hockey, calculate how many of them play BOTH soccer and hockey.

$$130 + (150 - x) = 280$$
$$150 - x = 280 - 130$$
$$x = 0$$

The 200 Grade 9 boys in a school play soccer, hockey or both. If 150 boys play soccer and 130 play hockey, calculate how many of them play BOTH soccer and hockey.

$$150 + (130 - x) = 280$$
$$\cancel{150}\ 130 - x = 280 - 150$$
$$x = 0$$

- If the learner worked out the problem, but used the incorrect operation.

The 200 Grade 9 boys in a school play soccer, hockey or both. If 150 boys play soccer and 130 play hockey, calculate how many of them play BOTH soccer and hockey.

$$150 + (130 + x) = 200$$
$$130 + x = 50$$
$$x = 50 - 130$$
$$= -80$$

What would show evidence of no understanding?

- If the learner did not attempt the question;
- If the learner solved the problem incorrectly with no mathematical reasoning or logic.

The 200 Grade 9 boys in a school play soccer, hockey or both. If 150 boys play soccer and 130 play hockey, calculate how many of them play BOTH soccer and hockey.

~~130×150~~ 130 + 150 x 200

= R30130

The 200 Grade 9 boys in a school play soccer, hockey or both. If 150 boys play soccer and 130 play hockey, calculate how many of them play BOTH soccer and hockey.

20 of them play both soccer and hockey.

The 200 Grade 9 boys in a school play soccer, hockey or both. If 150 boys play soccer and 130 play hockey, calculate how many of them play BOTH soccer and hockey.

150 boys play Soccer and 130 play hockey they will be 280 boys.

What do the item statistics tell us?

23 % of learners answered the question correctly.

Factors contributing to the difficulty of the item

- Learners may have poor understanding of the concepts and skills tested in these items.
- Learners may be unable to set up equations to describe problem situations.
- Learners may be unable to analyse and interpret equations that describe a given situation.

Teaching strategies

Setting up equations to describe problem situations

- Learners must understand that equations are used to solve problems.
- Revise key words that learners are required to use when setting up equations. Use the following list of key words:
  - Addition: plus, sum, added to, increased by;
  - Subtraction: minus, difference, less than;
  - Multiplication: times, product, multiplied by;
  - Division: quotient, divided by, share.

- Explain to your learners that they need to read the question carefully and then highlight, underline or circle key information.
- Explain to the learners that the next steps are to:
  - Identify what has to be found;
  - Introduce a variable x or y to represent the unknown item/number that needs to be found by calculation; and
  - Write down the equation.
- Lastly help learners solve for the unknown variable and write down the solution in words.
- Allow learners to practice by using examples in class. The following examples may be used:

Examples

1. The sum of two times a number and five equals 75. Find the number.
2. The sum of four consecutive integers is 230. Find the integers.
3. The 400 Grade 9 boys in a school play soccer, cricket or both. If 250 boys play soccer and 220 play cricket, calculate how many of them play BOTH soccer and cricket.

Solutions

1). Underline key information:

The <u>sum of two times</u> a number <u>and five</u> <u>equals</u> 75. Find the number.

Let the number $=x$

Write the equation:

$2x+5=75$

Solve the equation:

$2x=75-5$

$2x=70$

$x=35$

Write the solution in words:

The number is 35.

2). Underline key information:

The <u>sum of four consecutive integers is 230</u>. Find the integers.

Let the first integer $= x$

The second integer is $x + 1$

The third integer is $x + 2$

The fourth integer is $x + 3$

Write the equation:

$x + x + 1 + x + 2 + x + 3 = 230$

Solve the equation by gathering like terms:

$x + x + x + x = 230 - 1 - 2 - 3$

$4x = 224$

$x = 56$

Write the solution in words:

The first integer $= x = 56$

The second integer is $x + 1 = 56 + 1 = 57$

The third integer is $x + 2 = 56 + 2 = 58$

The fourth integer is $x + 3 = 56 + 3 = 59$

3). Underline the key information:

The <u>400</u> Grade 9 boys in a school play soccer, cricket or both.

If <u>250 boys play soccer and 220 play cricket</u>, calculate how many of them play <u>BOTH</u> soccer and cricket.

Let the number $= x$

Write the equation:

$250 + (220 - x) = 400$

Solve the equation:

$220 - x = 400 - 250$

$x = 220 - 150$

$x = 70$

OR

$250 + 220 = 470$

470 is 70 more than the total number of boys at the school

$470 - 400 = 70$

Write the solution in words:

The number of boys who play both soccer and cricket is 70.

Once learners are confident with working on examples allow them to engage in an activity. The following activities may be used.

Activities

1). The sum of three times a number and 6 equals 69. Find the number.

2). Three times a number decreased by 5 equals 79. Find the number.

3). The sum of three consecutive integers is 126. Find the integers.

4). Thabi is 6 years older than Angela. In 6 years' time Thabi will be one and a half times as old as Angela, How old is Thabi now?

5). In a school there are twice as many boys than there are girls. If the total number of learners is 1 236, how many girls are there in the school?

6). The 200 Grade 9 girls in a school play netball, hockey or both. If 130 girls play netball and 170 play hockey, calculate how many of them play BOTH netball and hockey.

Solutions

1). Underline the key information:

The sum of three times a number and 6 equals 69. Find the number.

Let the number $=x$

Write the equation:

$3x+6=69$

Solve the equation:

$3x=69-6$

$3x=63$

$x=21$

Write the solution in words:

The number is 21

2). Underline the key information:

Three times a number decreased by 5 equals 79. Find the number.

Let the number $=x$

Write the equation:

$3x-5=79$

Solve the equation:

$3x = 79 + 5$

$3x = 84$

$x = 28$

Write the solution in words:

The number is 28.

3). Underline the key information:

The <u>sum of three consecutive integers is 126</u>. Find the integers.

Let the first integer $= x$

The second integer is $x + 1$

The third integer is $x + 2$

Write the equation:

$x + x + 1 + x + 2 = 126$

Solve the equation by gathering like terms:

$x + x + x = 126 - 1 - 2$

$3x = 123$

$x = 41$

Write the solution in words:

The first integer is $x = 41$

The second integer is $x + 1 = 41 + 1 = 42$

The third integer is $x + 2 = 41 + 2 = 43$

4). Underline the key information:

Thabi is <u>6 years older</u> than Angela. In <u>6 years' time</u> Thabi will be <u>one and a half times as old</u> as Angela. How old is Thabi now?

Write the equation:

Now: Let Angela's age $= x$ and Thabi's age $= x + 6$

In 6 years' time:

Angela's age $= x + 6$ and Thabi'as age $= 6 + (x + 6)$

$1{,}5(x + 6) = 6 + (x + 6)$

Solve the equation:

$1{,}5x + 9 = x + 12$

$1{,}5x - x = 12 - 9$

$0{,}5x = 3$

$x = 6$

Write the solution in words:

Angela's age is 6, thus Thabi is $6 + 6 = 12$ years old.

5). Underline the key information:

In a school there are twice as many boys than there are girls. If the total number of learners is 1 236, how many girls and how many boys are there in the school?

Let the number $= x$

Girls $= x$ and Boys $= 2x$

Write the equation:

$2x + x = 1\,236$

Solve the equation:

$3x = 1\,236$

$x = 412$

Write the solution in words:

The number of girls in the school is 412.

The number of boys in the school is $2 \times 412 = 824$.

6). Underline the key information:

The <u>200 Grade 9 girls</u> in a school play netball, hockey or both. If <u>130 play netball</u> and <u>170 play hockey</u>, calculate how many girls play BOTH netball and hockey.

Let the number playing both netball and hockey $= x$

Write the equation:

$130 + (170 - x) = 200$

Solve the equation:

$170 - x = 200 - 130$

$170 - x = 70$

$x = 170 - 70$

$x = 100$

OR

$130 + 170 = 300$

300 is 100 more than the total number of girls at the school.

$300 - 200 = 100$

Write the solution in words:

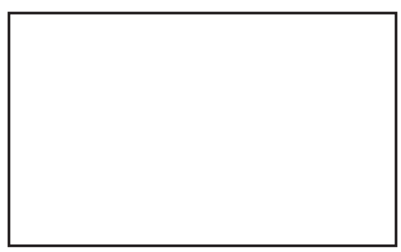
The number of girls who play both netball and hockey is 100.

Analyse and interpret equations that describe a given situation

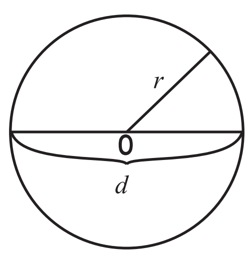
- Once learners are confident setting up equations explain to them the importance of analysing and interpreting a given situation.
- Equations are generally used to describe relationships.
- Remember that diagrams or sketches may assist learners to solve problems in geometry.
- Allow learners to practice by using examples in class. The following examples may be used.

Examples

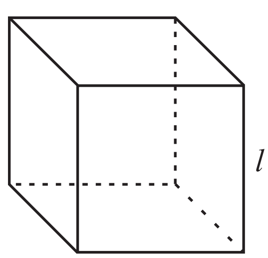
1). Provide an equation for the breadth of a rectangle if the area of the rectangle is $64\ \text{cm}^2$ and the length is x.



2). Provide an equation for the radius and the area of a circle given that the diameter is $36x$.



3). Provide an equation for the length of a cube given that the volume is $512s^3$



Solutions

1). Area of rectangle = Breadth x length

Breadth = $\frac{Area\ of\ rectangle}{length}$

Breadth = $\frac{64\ cm^2}{x}$

2). Area of a circle $= \pi r^2$, with radius r.

But diameter $= 2r = 36x$ [given]

$\therefore r = 18x$

$\therefore A = \pi r^2 = \pi(18x)^2$

3). Volume of a cube $= l^3$

$512s^3 = l^3$

$l^3 = 512s^3$

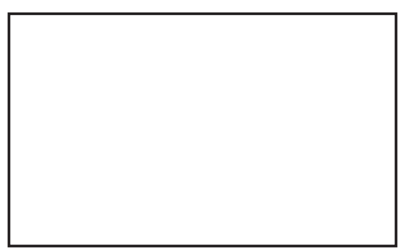
$\therefore l = \sqrt[3]{512s^3}$

$l = 8s$

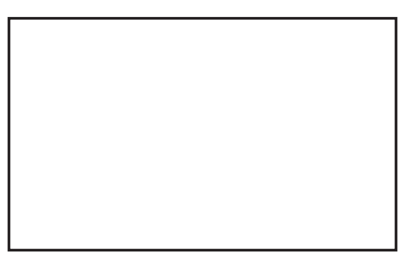
Once learners are confident with working with examples, allow them to engage in an activity. The following activities may be used.

Activities

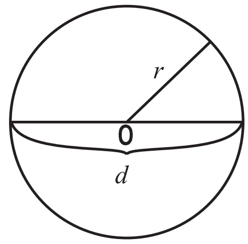
1). Provide an equation for the breadth of a rectangle if the area of the rectangle is $121\ \text{cm}^2$ and the length is y.



2). Provide an equation for the length of a rectangle given that the area is $169\ \text{cm}^2$ and the breadth is $(x-4)$ cm.



3). Provide an equation for the radius and the area of a circle given that the diameter is $64x$.



4). Provide an equation for the length of a cube given that the volume is $1\,331x^3$.

5). Provide an equation for the base of a triangle given that the area is $50x^2$ and the perpendicular height is $10x$.

6). We use the formula $F=\frac{9}{5}C+32$ to convert degrees Celsius to degrees Fahrenheit.

a). What is the temperature in $°F$ when it is $86°C$?

b). Convert $280°F$ to °C.

Solutions

1). Area of rectangle $= length \times breadth$

$121\ cm^2 = y \times breadth$

$breadth = \frac{121cm^2}{y}$

2). Area of rectangle $= length \times breadth$

$169\ cm^2 = length \times (x-4)\ cm$

$length = \frac{169\ cm^2}{(x-4)cm}$

3). Area of a circle $= \pi r^2$, with radius r.

but $2r = diameter = 64x$ [given]

$\therefore\ r = 32x$

$\therefore A = \pi r^2 = \pi(64x)^2$

4). Volume of a cube $= l \times l \times l = l^3$

$V = 1\,331\,x^3 = l^3$

$l = \sqrt[3]{1\,331x^3}$

$\therefore l = 11x$

5). Area of triangle $= \frac{1}{2} \times base \times perpendicular\ height$

$50x^2 = \frac{1}{2} \times base \times 10x$

$base = \frac{2 \times 50x^2}{10x} = \frac{100x^2}{10x} = 10x$

6. a). $F = \frac{9}{5}(86) + 32$

Temperature $= 154{,}8 + 32 = 186{,}8°F$

6). $F = \frac{9}{5}C + 32$

$102 = \frac{9}{5}x + 32$

$102 - 32 = \frac{9}{5}x$

$\frac{9}{5}x = 70$

$x = \frac{5}{9} \times 70$

$x = 38{,}9°C$

Notes:

Numeric and geometric sequences

ANA 2013 Grade 9 Mathematics Items 5.1, 5.2 and 5.3

5.1	Write down the next TWO terms in the number sequence 7; 11; 15;	[2]
5.2	Write down the general term Tn of the above number sequence. $Tn =$	[2]
5.3	Calculate the value of 50th term.	[2]

What should a learner know to answer these questions correctly?

Learners should be able to:

- Extend a number pattern;
- Write a general rule for a number sequence;
- Use the general rule to determine any term in a number sequence.

Where is this topic located in the curriculum? Grade 9 Term 1

Content area: Patterns, Functions and Algebra.

Topic: Numeric patterns.

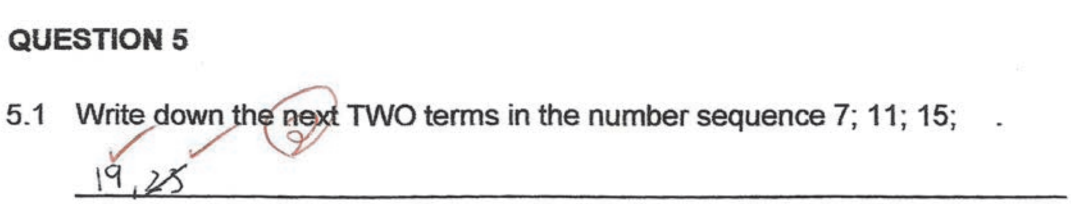
Concepts and skills:

- Investigate and extend numeric and geometric patterns looking for relationships between numbers;
- Describe and justify the general rules for observed relationships between numbers in own words or in algebraic language.

What would show evidence of full understanding?

If the learner obtained the correct answers by:

- Correctly identifying the common difference and using it to extend the sequence;
- Correctly used the observed pattern to determine the general term (Tn);
- Correctly used the general term to determine the 50th term.


QUESTION 5

5.1 Write down the next TWO terms in the number sequence 7; 11; 15;

19, 23

5.2 Write down the general term Tn of the above number sequence.

$Tn =$ 4n + 3 ✓ ✓ (2)

5.3 Calculate the value of the 50th term.

= 4(50) + 3 ✓

= 203 ✓ (2)

What would show evidence of partial understanding?

If the learner answered some but not all parts of the item correctly, for instance:

- If the learner extended the sequence correctly, but not by the number of terms as per the instruction;
- If the general term the learner obtained works for the first term only;
- If the leaner calculated the 50th term incorrectly.

QUESTION 5

5.1 Write down the next TWO terms in the number sequence 7; 11; 15; .

19 ✓ (1)

5.2 Write down the general term Tn of the above number sequence.

$Tn =$ Tn = 3n + 4 ✓ (1)

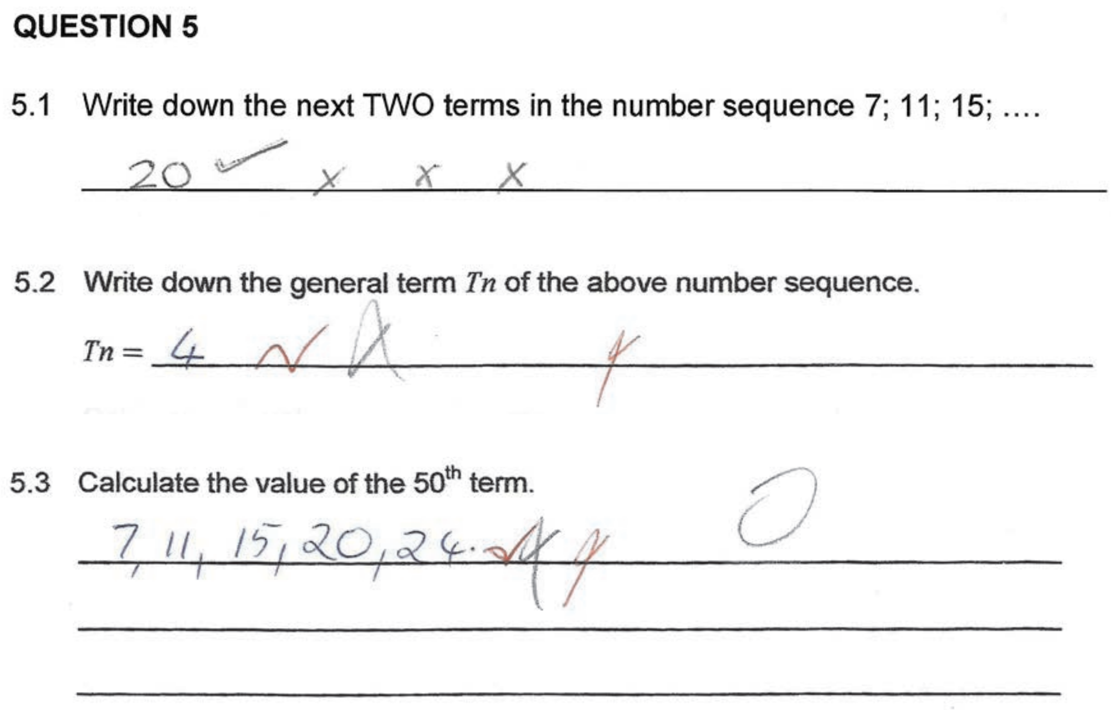
5.3 Calculate the value of the 50th term.

~~7; 11; 15; 19; 23~~

50th term = 123 ✗

What would show evidence of no understanding?

- If the learner extended the sequence incorrectly;
- If the general term the learner obtained is incorrect;
- If the learner could not calculate the 50th term at all.



QUESTION 5

5.1 Write down the next TWO terms in the number sequence 7; 11; 15; ….

20

5.2 Write down the general term Tn of the above number sequence.

$Tn =$ 4

5.3 Calculate the value of the 50^{th} term.

7, 11, 15, 20, 24.

What do the item statistics tell us?

Item 5.1

76% of learners answered the question correctly.

Factors contributing to the difficulty of the item

- Learners may be unable to count in 4s starting at 7.

Item 5.2

8% of learners answered the question correctly.

Factors contributing to the difficulty of the item

- Learners may confuse the general term with the common difference or constant difference between the terms;
- Learners may be unable to establish the relationship between each term and its position.

Item 5.3

9% of learners answered the question correctly.

Factors contributing to the difficulty of the item

- Learners may resort to listing numbers to find the required terms;
- Learners may rely on being given a formula to calculate the general term.

Teaching strategies

Relating numbers using patterns

Learners should do exercises to practice how one number can be expressed in terms of other numbers in different ways. Doing this will help learners to become used to manipulating numbers when they are trying to find the rules of number patterns.

Example

$12 = 3 \times 4;$

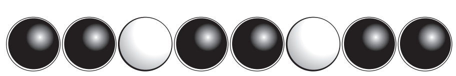
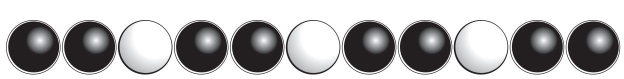
$12 = 2 \times 5 + 2;$

$12 = \frac{7 \times 5 + 1}{3}$; etc.

- Learners should assign positions to terms of a sequence and use the positions to investigate relationships between the terms. Tables and flow diagrams may be used for this activity.
- Allow learners to develop numeric pattern rules for counts based on geometric patterns. They should use sketches and objects to generate their own patterns and establish relationships

Example

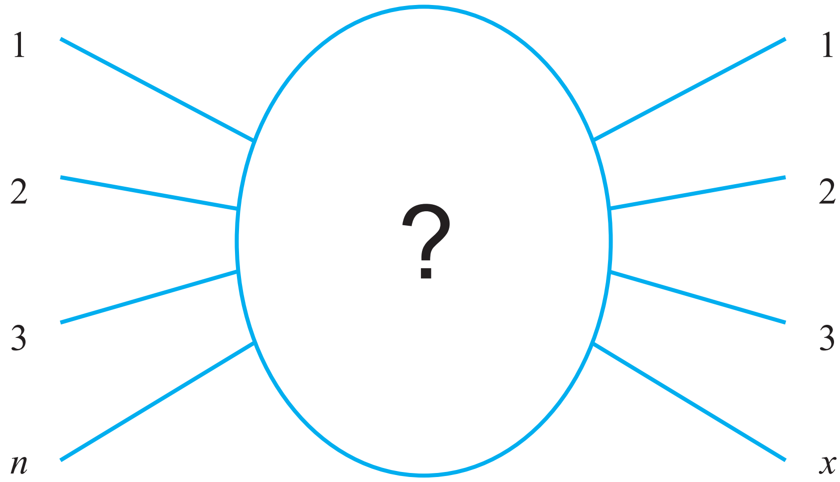
What is the n^{th} term of the sequence of counters shown in the following table?

Position	Counters/Beads	White	Black	Pattern (No. of beads)
1		1	4	White = 1 Black = 4 = 2 + 2 = 2 × 1 + 2
2		2	6	White = 2 Black = 6 = 4 + 2 = 2 × 2 + 2
3		3	8	White = 3 Black = 8 = 6 + 2 = 2 × 3 + 2
n		x	y	

To generate the rule you could use a flow diagram.

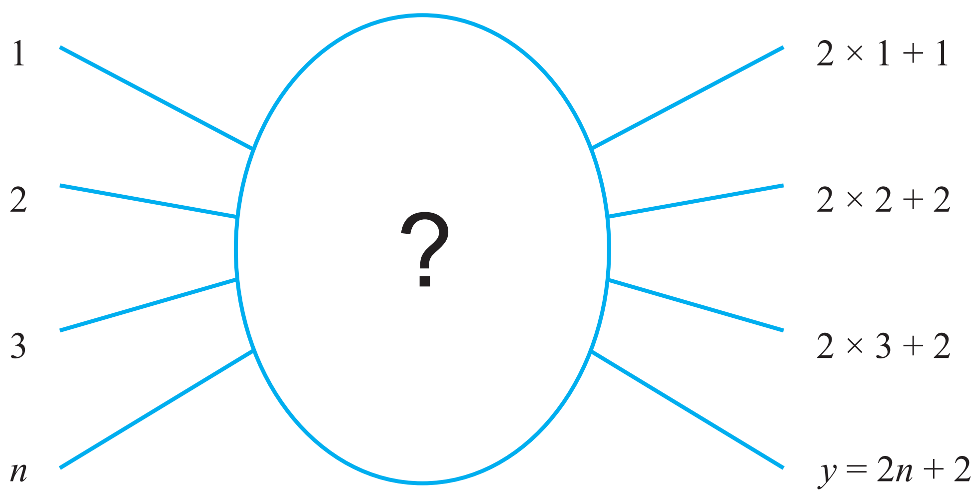
- Work through this with your learners explaining how to do this for both the white and the black beads.

- The sequence of white beads can be presented using a flow diagram as follows:



- For the white beads: $x = n$ or $T_n = n$, starting with $n = 1$

- The sequence of black beads can be presented using a flow diagram as follows:



For the black beads: $y = 2n + 2$, or $T_n = 2n + 2$, starting with $n = 1$

- Learners need to be able to use the n^{th} term expression to calculate other terms in the sequence. For example, in the sequence of black and white beads above, the 50^{th} term would be the sum of the 50^{th} terms for the black and the white sequences:

For the white beads: $T_n = n$

$\therefore T_{50} = 50$

For the black beads: $T_n = 2n + 2$

$\therefore T_{50} = 2(50) + 2$

This means that for the black and white beads (together):

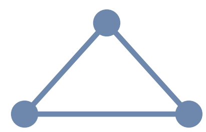
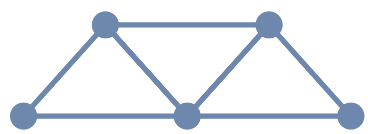
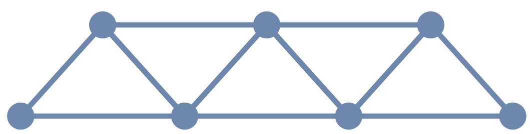
$T_{50} = 50 + 2(50) + 2 = 152$

- This shows that patterns which are made of shapes can be reduced to numeric patterns by using counting.

Activity: Relating numbers found in geometric patterns using numeric patterns

Examples

1). Nina makes a geometric pattern using triangles.

Stage	Geometric Shapes	Number of triangles	Number of sticks
1		1	3
2		3	7
3		5	11

a). Draw Nina's geometric shape for the 4^{th} stage.

b). How many triangles are in the 4^{th} stage?

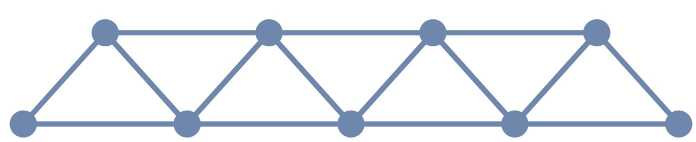
c). How many sticks does she need to make the pattern for the 4^{th} stage?

d). Write a formula for the number of triangles in the nth stage.

e). Write a formula for the number of sticks in the nth stage.

Solution

1). a). The 4^{th} stage will look like this:



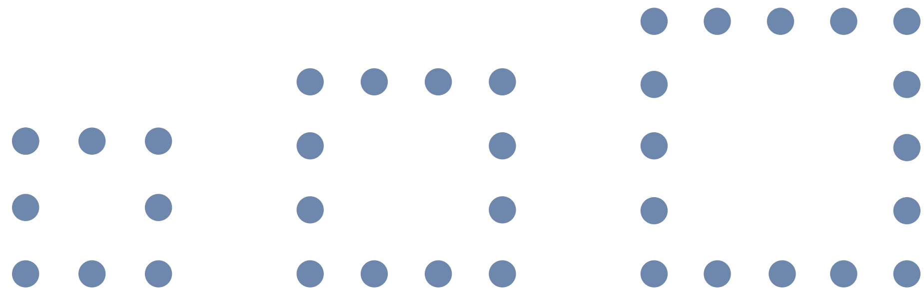
b). The number of triangles in the 4^{th} stage is 7.

c). The number of sticks needed to make the pattern in the 4^{th} stage is 15.

d). The formula is: Number of triangles $= 2n - 1$ where n is the stage number.

e). The formula is: Number of sticks $= 4n - 1$.

2). Tau puts fence around plots of different sizes. The dots show the number of poles used for each plot.



Plot 1 Plot 2 Plot 3

a). In your own words describe the pattern.

b). How many poles will Tau needs for the 4^{th} plot?

c). Determine the formula for the number of poles in the n^{th} plot.

d). Use the formula to determine the number of poles needed for the 25^{th} plot.

Solution

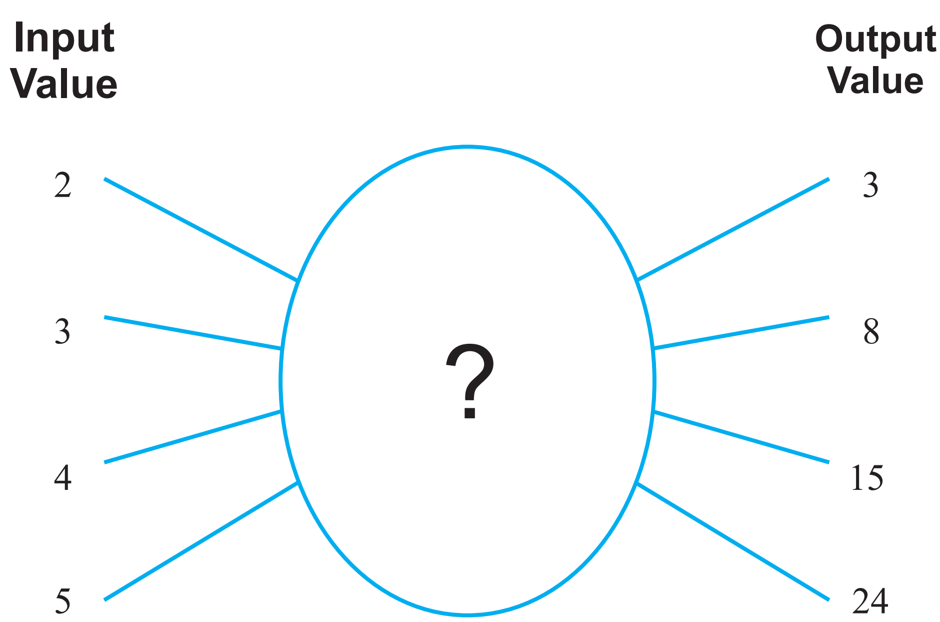
a). The shapes/figure/plots become bigger; more poles will be needed as we move from Plot 1 to Plot 2 (or some similar explanation).

b). Tau needs 20 poles.

c). Number of poles $=4(n+1)$ or $4n+4$

d). No of poles $=4(25+1)=104$

3). The flow diagram shows some input values and corresponding output values.



a). Determine the rule used to get the output value for each input value.

b). Use the rule to determine the output value of the input value 12.

c). Use the rule to determine the input value if the output value is 80.

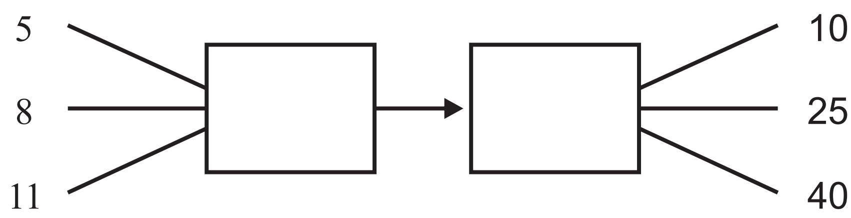
Solution

a). Square the input value and then subtract 1

b). Output value = $(12)^2 - 1 = 143$

c). Input value = $\sqrt{80 + 1} = 9$

4). Determine the rule for the next flow diagram:



Solution

Subtract 3 from the input value and then multiply the answer (difference) by 5.

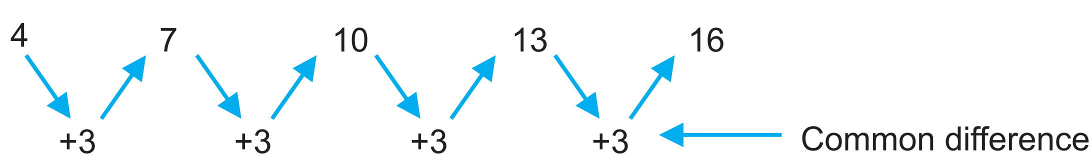
Patterns with a common difference

- Learners should know that a sequence has a common difference if the difference between consecutive terms is constant.
- This can be written in terms of nth terms in the following way: $T_{(n+1)} - T_n = T_n - T_{(n-1)}$ where $n = 1; 2; 3; \ldots$

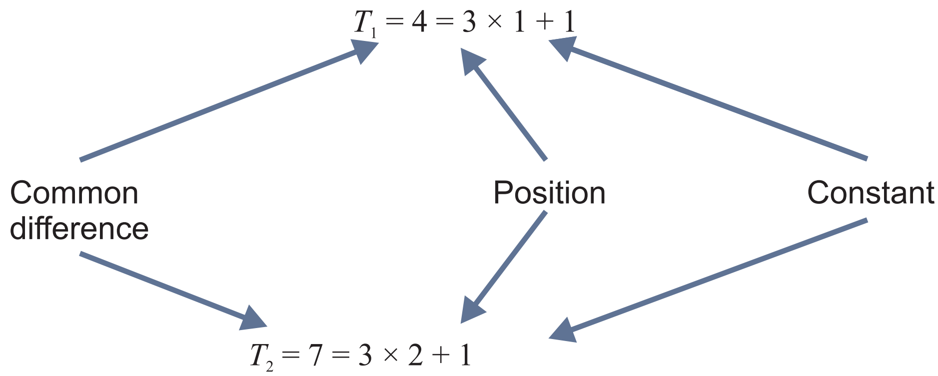
Example

4, 7, 10, 13, 16, … has a common difference

- If $T_n = 12, T_{(n+1)} = 14$ and $T_{(n-1)} = 10$
- $T_{(n+1)} - T_n = 14 - 12 = 2$
- $T_n - T_{(n-1)} = 12 - 10 = 2$
- Therefore $T_{(n+1)} - T_n = T_n - T_{(n-1)}$



- The sequence can be presented as follows:
- Each term can then be related to its position and the common difference as follows:



- The general term $T_n = 3n + 1$
- Therefore, $T_{(50)} = 3 \times 50 + 1 = 151$

Activity: Patterns with a common difference

- Determine the next term in each of the following sequences.
- Write down the formula for the nth term (T_n).
- Use you formula to determine the 50th term (T_{50}).

a). 3, 5, 7, 9, …
b). 7, 12, 17, 22, …
c). 8, 11, 14, 17, …
d). 48, 41, 34, 27, …
e). 1, 5, 9, 13, …

Solutions

a). Next term is 11
$T_n = 2n + 1$
$T_{50} = 2(50) + 1 = 101$

b). Next term is 27
$T_n = 5n + 2$
$T_{50} = 5(50) + 2 = 252$

c). Next term is 20
$T_n = 3n + 5$
$T_{50} = 3(50) + 5 = 155$

d). Next term is 20
$T_n = -7n + 55$
$T_{50} = -7(50) + 55 = -295$

e). Next term is 17

$T_n = 4n - 3$

$T_{50} = 4(50) - 3 = 197$

Other patterns

- Learners should be made aware that some patterns do not have a common difference.

Example

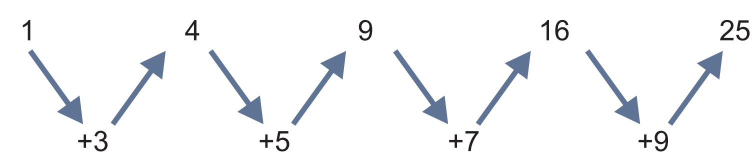
1, 4, 9, 16, …

- The pattern could be written as follows:
- These are square numbers and their nth term formulae reflect this. Discuss the link between square

n	T_n
1	$1 = 1 \times 1 = 1^2$
4	$4 = 2 \times 2 = 2^2$
3	$9 = 3 \times 3 = 3^2$
4	$16 = 4 \times 4 = 4^2$
n	n^2

numbers and the formulae with your class.

- Draw the learners' attention to the differences between consecutive terms.



- Patterns that have differences that are not constant, but have a common difference between the differences are quadratic patterns.
- This difference between the differences is called the second difference.
- It suffices for this grade to say you use a square term for these patterns (square the position) and then check what must be added or whether anything else should be done to generate the nth term.

Example

4, 7, 12, 19, 28, …

- The pattern has no common difference.
- First term $= 1 \times 1 + 3$;
- Second term $= 2 \times 2 + 3$;
- Third term $= 3 \times 3 + 3$, etc.
- Therefore $T_n = n^2 + 3$

Activity: Other patterns

- Determine the next term in each of the sequences that follow.
- Write down the formula for the nth term (T_n).
- Use you formula to determine the 8th term (T_8).

a). 2, 5, 10, 17, …
b). 6, 9, 14, 21, …
c). 0, 3, 8, 15, …
d). 2, 8, 18, 32, …
e). 3, 9, 19, 33, …

Solutions

a). Next term is 26
$T_n = n^2 + 1$
$T_8 = 8^2 + 1 = 65$

b). Next term is 30
$T_n = n^2 + 5$
$T_8 = 8^2 + 5 = 69$

c). Next term is 24
$T_n = n^2 - 1$
$T_8 = 8^2 - 1 = 63$

d). Next term is 50
$T_n = 2n^2$
$T_8 = 2 \times 8^2 = 128$

e). Next term is 51
$T_n = 2n^2 + 1$
$T_8 = 2 \times 8^2 + 1 = 129$

Other examples of how to test for numeric patterns

ANA 2014 Grade 9 Mathematics Items 6.1, 6.2 and 6.3

6.1 Complete the table below:

Position in pattern	1	2	3	4	5
Term	1	8	27		

[2]

6.2 Write down the general term T_n of the above number pattern. [1]

6.3 If $T_n = 512$, determine the value of n. [2]

Notes:

Ratio, rate, proportion and financial mathematics

ANA 2013 Grade 9 Mathematics Items 6.1, 6.2 and 6.3

6.1	How long will it take to travel 432 kilometres at an average speed of 96 kilometres per hour?	[2]
6.2	Calculate the simple interest on R3 500 invested at 6% per annum for 3 years.	[5]
6.3	Calculate how much money you will owe the bank after 3 years if you borrow R7 500 from the bank at 13% per annum compound interest.	[4]

What should a learner know to answer these questions correctly?

Learners should be able to:

- Solve problems that involve rate;
- Solve financial problems that involve simple interest;
- Solve financial problems that involve compound interest.

Where is this topic located in the curriculum? Grade 9 Term 1

Content area: Numbers, Operations and Relationships.

Topic: Whole numbers.

Concepts and skills:

- Solve problems in contexts involving ratio and rate and direct and indirect proportion;
- Solve problems that involve whole numbers, percentages and decimal fractions in financial contexts such as profit, loss, discount and vat, calculating budgets, accounts and loans, calculating simple interest and hire purchase, exchange rates and commission, rentals and compound interest.

What would show evidence of full understanding?

If the learner obtained the correct answer by:

- Correctly identifying the appropriate formula; and
- Correctly applying the identified formula.

6.1 How long will it take to travel 432 kilometres at an average speed of 96 kilometres per hour?

$T = \frac{s}{d} = \frac{432\,km}{96\,km/h}$ ✓

$T = 4{,}5\,h = T = 5$ hours (2)

6.2 Calculate the simple interest on R3 500 invested at 6% per annum for 3 years.

$Si = \frac{Prt}{100}$ ✓

$Si = \frac{3500 \times 6 \times 3}{100}$ ✓✓✓ (5)

$Si = R630$ ✓

6.3 Calculate how much money you will owe the bank after 3 years if you borrow R7 500 from the bank at 13% per annum compound interest.

$A = P(1+i)^n$ ✓

$A = 7500\left(1 + \frac{13}{100}\right)^3$ ✓ (4)

$A = 10\,821{,}73$ ✓

What would show evidence of partial understanding?

- If the learner chose an inappropriate formula, but applied it correctly; or
- If the leaner chose the correct formula but applied it incorrectly.

6.1 How long will it take to travel 432 kilometres at an average speed of 96 kilometres per hour?

$\frac{432}{96} \times 60$

$= 270$ Km

6.2 Calculate the simple interest on R3 500 invested at 6% per annum for 3 years.

$A = P(1 \times i \times n)$

$= 3500(1 + 0{,}06 \times 3)$

$= R\,4\,130$

What would show evidence of no understanding?

- If the learner used an inappropriate formula incorrectly.

6.1 How long will it take to travel 432 kilometres at an average speed of 96 kilometres per hour?

$T = \frac{s}{d}$

= 6km Per hour ✗

6.2 Calculate the simple interest on R3 500 invested at 6% per annum for 3 years.

~~3500~~ $A = \frac{N}{10} \times 1/6$

3500 + 6% - 36

= R45 R. ✗

6.3 Calculate how much money you will owe the bank after 3 years if you borrow R7 500 from the bank at 13% per annum compound interest.

7500 - 13% - 3

= 130 ✗

What do the item statistics tell us?

- 33% of learners answered item 6.1 correctly;
- 22% of learners answered item 6.2 correctly;
- 16% of learners answered item 6.3 correctly.

Factors contributing to the difficulty of the items

- Learners may not conceptualise rate as a comparison of quantities.
- Learners may not be able to calculate the percentage of a quantity.
- Learners may not be able to differentiate between simple and compound interest.
- Learners my not know the formulae for calculating the required rates.

Teaching strategies

Provide definitions of key terms that learners will be using with examples.

You can use the table of key terms given here to assist you:

Key terms	Explanation	Examples
Ratio	A **ratio** is a comparison of two quantities of the same units.	For example if we want to compare the number 4 to the number 5, we would write it one of the following ways: 4 to 5 or 4 : 5 or $\frac{4}{5}$
Rate	A **rate** is a ratio that compares two different kinds of numbers.	Kilometres per hour (km/hr) metres per second (m/s) or (ms^{-1}).
Proportion	A proportion is composed of two ratios or fractions that are equal.	4 is to 5 as 8 is to 10 or $\frac{4}{5} = \frac{8}{10}$

Using proportional reasoning

- Learners should initially be given problems on rate that they can solve by inspection.

Example

Sipho reads 180 pages in 2 days. Let us assume that he reads at a constant rate.

Number of pages	Number of days
180	2
x	1
y	4

1). How many pages does he read a day?

2). How many pages will he read in 4 days?

Solution

1). If in 2 days he reads 180 pages, then in a day he should read half the number of pages.

Number of pages	Number of days	Calculation
180	2	
x	1	$2x = 180$ $x = \frac{180}{2}$ $x = 90$

OR

Half of 180 is 90
Therefore Sipho reads 90 pages in 1 day.

2). If in 2 days he reads 180 pages, then in 4 days he should read double the number of pages.

Number of pages	Number of days	Calculation
180	2	
y	4	$2y = 180 \times 4$ $y = \frac{270}{2}$ $y = 360$

OR

Double 180 is 360
Therefore Sipho reads 360 pages in 4 days.

- The learners should be allowed to solve problems by trial and improvement.

Example

576 sweets are packed into small boxes. 24 sweets should be put in each box.

1). How many boxes will be needed?
2). How many sweets can be packed in 120 boxes?

Solution

1). A table similar to the one shown may be used:

No. of sweets	No. of boxes
24	1
48	2
96	4
192	8
384	16
768	32

- The number of boxes required is more than 16 but less than 32.
- If for 192 sweets I need 8 boxes and for 384 sweets I need 16 boxes, then for 192 + 384 sweets I need 8 + 16 boxes.
- 192 + 384 = 576
- 8 + 16 = 24
- Therefore for 576 sweets I need 24 boxes.

2). A table similar to the one that follows may be used:

No. of boxes	No. of sweets
24	576
48	1 152
96	2 304
192	4 608

- 120 boxes will hold more than 2 304 sweets, but less than 4 608 sweets.
- 96 + 24 boxes will hold 2 304 + 576 sweets.
- 96 + 24 = 120
- Therefore120 boxes will hold 2 304 sweets + 576 sweets
- = 2 880 sweets.

Ultimately encourage the learners to use the unitary strategy. This entails comparing one quantity to a unit of another quantity.

Example

576 sweets are packed into small boxes. 24 sweets should be put in each box.

1). How many boxes will be needed?

2). How many sweets can be packed in 120 boxes?

Solution

1). If 1 box takes 24 sweets, then the number of boxes needed

$\frac{576}{24}$ =

= 24 boxes

2). If 1 box takes 24 sweets, then the number of sweets in 120 boxes

= 120 × 24

= 2 880 sweets

Activity: Using proportional reasoning

1). Solve by inspection:

a). If Peter was driving at a constant speed of 80 km/h, how far did he travel in 2 hours?

b). If 45 tiles cover an area of $9\ m^2$, how many tiles are needed to cover a surface of $36\ m^2$?

c). If water flows from a tap at 7 litres per minute, it takes 5 minutes to fill a container. If the flow of water is increased to 14 litres per minute, how long will it take to fill the same container?

2). Solve by trial and improvement:

a). A box of sweets is filled with 15 sweets. How many sweets will fill 327 boxes?

b). In a factory 5 tyres are allocated to each car. How many cars will get a full allocation of tyres if there are 3 257 tyres available?

c). After $1\frac{1}{2}$ hours, Peter discovered that he had covered 198 kilometres. What distance (km) did he cover every minute?

3). Solve using the unitary strategy:

a). 3 450 litres of water is required to prepare meals for 25 people. How much water will be required to prepare the same meal for 15 people?

b). If there are 300 calories in 100 g of a certain food, how many calories are there in a 30 g portion of this food?

c). If a train can travel 126 kilometres in 3 hours, how long will it take to travel 21 kilometres if the speed remains constant?

d). Petrus takes a bus to school. The bus travels at an average speed of 40 km/h. The school is 9 kilometres from his house. How long does it take him to get to school?

Solutions

1). Solve by inspection:

a). 160 km

b). 180 tiles

c). 2,5 minutes

2). Solve by trial and improvement:

a). 1 box → 15 sweets
2 boxes → 30 sweets
5 boxes → 75 sweets
7 boxes → 105 sweets
20 boxes → 300 sweets
100 boxes → 1 500 sweets
200 sweets → 3 000 sweets
Therefore 320 boxes will contain 4 800 sweets.
Therefore 327 boxes will contain 4 905 sweets.

b). 5 tyres → 1 car
50 tyres → 10 cars
500 tyres → 100 cars
1 000 tyres → 200 cars
3 000 tyres → 600 cars
200 tyres → 40 cars
3 250 tyres → 650 cars
Therefore 651 cars will get a full allocation of tyres.

c). 1,5 hours = 90 minutes → 198 km

45 minutes → 99 km

15 minutes → 33 km

5 minutes → 11 km

1 minutes → 2,2 km

Therefore Peter covers 2,2 km every minute.

3). Solve using the unitary strategy:

a). For 25 people we require 3 450 litres

1 person requires 3 450 ÷ 25 litres = 138 litres

Therefore 15 people require 138 × 15 litres = 2 070 litres.

b). 100 g has 300 calories

10 g has 300 ÷ 10 calories = 30 calories

Therefore 30 g has 30 × 3 calories = 90 calories.

c). 126 km takes 3 hours = 180 minutes

1 km will take 180 ÷ 126 minutes = 1,43 minutes

Therefore 21 km will take 21 ×1,43 = 30 minutes.

OR

126 ÷ 21 = 6. Therefore 21 km will take 180 ÷ 6 = 30 min.

d). 40 km take 1 hour = 60 minutes

1 km will take 60 ÷ 40 minutes = 1,5 minutes

Therefore 9 km will take 1,5 × 9 = 13,5 minutes

It takes Petrus 13,5 minutes to get to school.

Using tables (spread sheets) and formulae

- Leaners should revise calculating the percentage of a given quantity.

Example

Calculate 6% of R3 500

- Learners may express 6% as 6 ÷ 100 = 0,06.
- They should then multiply the given quantity by 0,06.
- Thus 6% of R3 500 = R3 500 × 0,06 = R210

- Learners should then use tables/spread sheets to solve simple and compound angle problems.

Example

1). Calculate the simple interest on R3 500 invested at 6% per annum for 3 years.

2). Calculate how much money you will owe the bank after 3 years if you borrow R7 500 from the bank at 13% per annum compound interest

Solution using tables (spread sheets)

1). Opening balance: R3 500
Interest rate: 6% p.a.

Year	Principal amount	Calculation	Interest per annum	Total interest
1	R3 500	$\frac{6}{100}$ × R3 500 = R210	R210	R210
2	R3 500	$\frac{6}{100}$ × R3 500 = R210	R210	R210 + R210 = R420
3	R3 500	$\frac{6}{100}$ × R3 500 = R210	R210	R210 + R210 + R210 = R630

- Learners should note that the principal amount is constant.
- Interest accrued each year is also constant.

2). Opening balance: R7 500
Interest rate: 13% p.a.

Year	Principal amount	Calculation	Interest per annum	Amount owed
1	R7 500	$\frac{13}{100}$ × R7 500 = R975	R975	R7 500 + R975 = R8 475
2	R8 475	$\frac{13}{100}$ × R8 475 = R1 101,75	R1 101,75	R8 475 + R1 101,75 = R9 576,75
3	R9 576,75	$\frac{13}{100}$ × R9 576,75 = R1 244,98	R1 244,98	R9 576,75 + R1 244,98 = R10 821,73

- Learners should note that the principal amount changes each year.
- Interest per annum also changes.

Solution using formulae

Learners should ultimately use formulae to calculate interest. However, it is recommended that learners should not be rushed into the use of formulae.

1). Simple interest $(SI) = Pni$
where P is the invested amount, n is the investment period and i is the interest rate expressed as a decimal.
Thus $SI = R3\ 500 \times 3 \times 0{,}06 = R630$

2). Compound interest $(CI) = P(1 + i)^n$
where P is the invested amount, n is the investment period and i is the interest rate expressed as a decimal.
Thus $CI = R7\ 500(1 + 0{,}13)^3$
$= R10\ 821{,}73$

Activity: Using tables (spread sheets) and formulae

1). Philani bought a scooter for R15 000. He paid 15% of the amount in cash. How much did he pay in cash?
2). Joan bought a new watch that cost R1 250 from the jewellers. She was offered a discount of 7,5%. How much did she pay for the watch?
3). Calculate the amount that Zinhle will receive at the end of 2 years if she invests R25 000 at 12% per year simple interest.
4). Bongiwe invests R12 000 in a savings account at 6,5% per annum compound interest. Calculate how much there will be in the account after 5 years.
5). Patrick invests R18 000 at 7% compound interest per year for 5 years. How much money will he have at the end of the period?

Solutions

1). Philani paid R15 000 × 0,15 = R2 250.

2). Discount = R1 250 × 0,075 = R93,75
Therefore Joan paid R1 250 – R93,75 = R1 156,25.

3). Interest = R25 000 × 0,12 × 2 = R6 000
Therefore Zinhle will receive R25 000 + R6 000 = R31 000.

4). Amount in the account = R12 000$(1 + 0{,}065)^5$ = R16 441,04.

5). Patrick will have R18 000$(1 + 0{,}07)^5$ = R25 245,93.

Other examples of how to test ratio, rate, proportion and financial mathematics

ANA 2014 Grade 9 Mathematics Item 8.2

8.2 Nthabi's car uses 1 litre of fuel to travel 12 km.
How much fuel will be needed to travel 420 km? [2]

ANA 2014 Grade 9 Mathematics Items 8.5 and 8.6

8.5 Calculate how long it will take for an investment of R4 000 at 3% per annum simple interest to earn an interest of R840. [6]

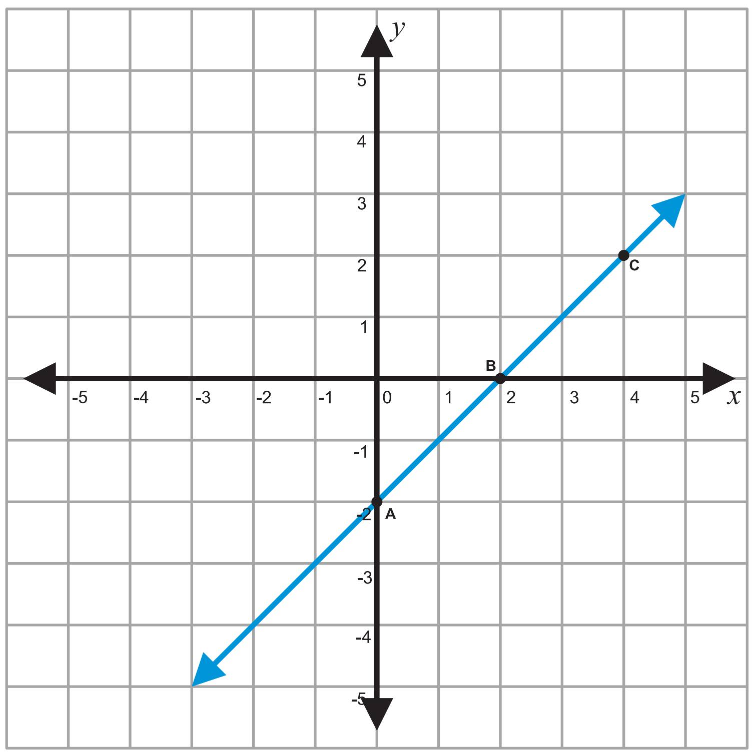
8.6 Calculate the final amount that I will have in my savings account if I invest R600 for 2 years at a rate of 6% per annum compound interest. [4]

Notes:

Linear Graphs: Equations and interpretation

ANA 2013 Grade 9 Mathematics Items 7.1.1 and 7.1.2

7.1 Use the graph below to answer the questions that follow.



7.1.1 Write down the coordinates of points A, B and C in the table. [3]

	A	B	C
x-coordinate			
y-coordinate			

7.1.2 Use the table in question 7.1.1 or any other method to determine the equation of the line ABC. [2]

What should a learner know to answer these questions correctly?

Learners should:

- Know that the equation of a straight line is y=mx+c ;
- Be able to Identify coordinates of a point on a Cartesian plane;
- Be able to use coordinates or a sketch to determine the equation of a graph.

Where is this topic located in the curriculum? Grade 9 Term 3

Content area: Algebra, patterns and functions.

Topic: Graphs.

Concepts and skills:

- Determine equations from given linear graphs.

What would show evidence of full understanding?

- If the learner correctly identified the coordinates of the given points; and
- If the learner correctly used the coordinates or the sketch to determine the equation of the graph.

7.1.1 Write down the coordinates of points A, B and C in the table.

	A	B	C
x-coordinate	0	2	4
y-coordinate	-2	0	2

7.1.2 Use the table in question 7.1.1 or any other method to determine the equation of line ABC.

neg line = y
so m = -

$y = mx + c$ $\qquad \frac{y_2 - y_1}{x_2 - x_1}$

$y = -x - 2$ $\qquad \frac{2-(-2)}{4-0} = \frac{4}{4} = 1 = -1$

What would show evidence of partial understanding?

- If the learner correctly read all or some of the coordinates, but gave the incorrect equation;
- If the learner gave a straight line graph equation, but the equation given was not completely correct.

7.1.1 Write down the coordinates of points A, B and C in the table.

	A	B	C
x-coordinate	0	2	4
y-coordinate	-2	0	2

7.1.2 Use the table in question 7.1.1 or any other method to determine the equation of line ABC.

$y = 4x + 2$

What would show evidence of no understanding?

- If the learner gave the coordinates of the points incorrectly;
- If, instead of giving a coordinate (x or y), the learner gave a coordinate pair (x; y).
- If the type of equation the learner provided was incorrect (e.g. a parabola rather than a linear equation).
- If the answer given was not an equation at all.

7.1.1 Write down the coordinates of points A, B and C in the table.

	A	B	C
x-coordinate	(-2 ; -1)	(2 ; 0)	(4 ; 2)
y-coordinate	(2 ; 1)	(-2 ; 0)	(-4 ; -2)

7.1.2 Use the table in question 7.1.1 or any other method to determine the equation of line ABC.

(-y -2) (2,0)(4,2) X

What do the item statistics tell us?

Item 7.1.1

32% of learners answered the question correctly.

Factors contributing to the difficulty of the item

- Learners are unable to correctly read the coordinates of the points on a graph;
- Some learners do not know what a coordinate is.

Item 7.1.2

4% of learners answered the question correctly.

Factors contributing to the difficulty of the item

- Learners cannot identify the coordinates of the points A, B and C correctly.
- Learners cannot use a sketch to determine the gradient and the y-intercept of a graph.

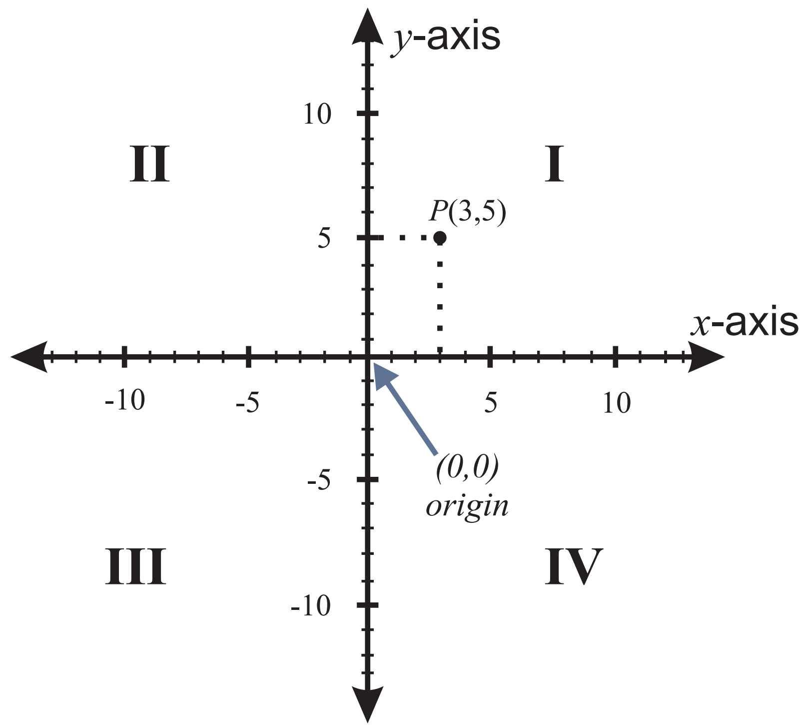
Teaching strategies

Determining coordinates of points in the Cartesian plane

- The Cartesian plane should be discussed with the learners. The discussion should ensure that learners can correctly read coordinates of points on a Cartesian plane.

Example

- A Cartesian coordinate plane has two intersecting number lines that form axes (see diagram).



  - The horizontal axis is called the x-axis.
  - The vertical axis is called the y-axis.
  - The axes intersect at the point called the origin.
  - The axes divide the coordinate plane into four quadrants (indicated by I, II, III and IV).
  - A point on the plane can be described by its x- and y-coordinates. These coordinates are written as an ordered pair: $(x; y)$.
  - The coordinates of the origin are (0; 0)
  - The coordinates of point P are (3; 5), where 3 is the x-coordinate, 5 is the y-coordinate and (3; 5) is the ordered pair.

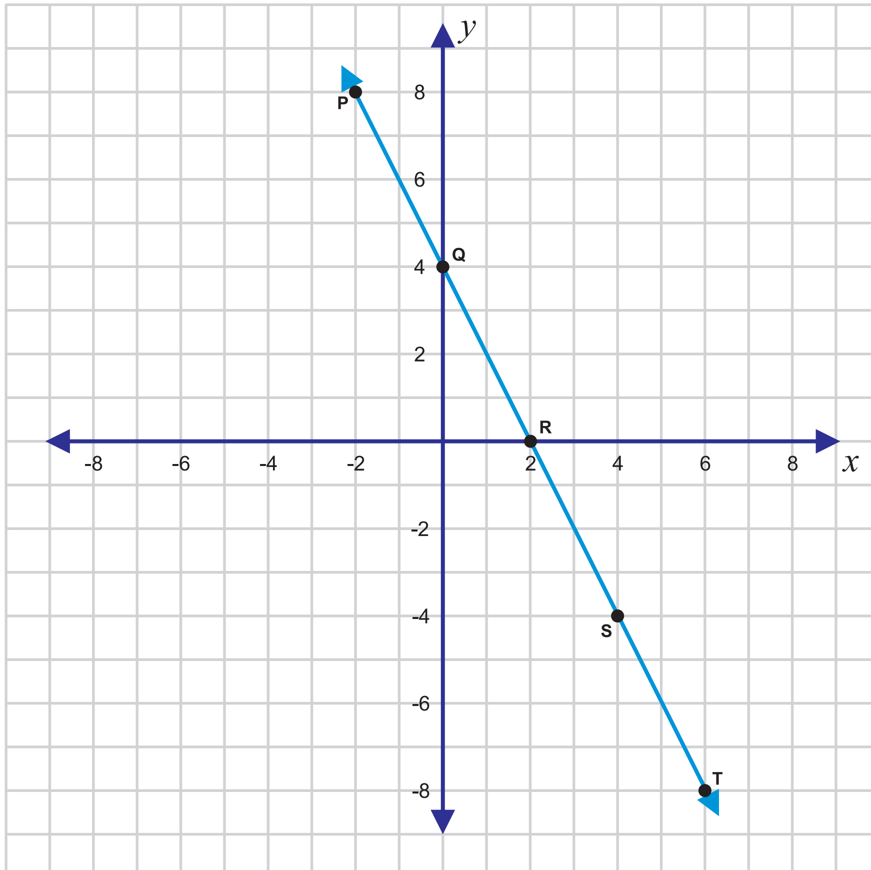
Determining coordinates using a line graph

- Next discuss a line graph with your learners. This should assist learners to correctly read coordinates of points on a given line.
  - Remind the learners that they need to find the x-intercept and y-intercept of the point in order to write the ordered pair for each of the intercepts on the axes.
  - The x-intercept is the x-value at which the line/point would cut the x-axis and the y-intercept is the y-value at which the line/point would cut the y-axis.

‘

Example

PT is a straight line that contains points Q, R and S.



Complete the following:

1). The y-intercept of the line is point ____ with coordinates ____.

2). The x-intercept is point ____ with coordinates ____.

3). Complete the table

- Discuss the solutions with your class making sure the learners can read the information from the graph and that they can speak confidently about points and lines in the Cartesian plane.

	P	**S**	**T**
x-coordinate			
y-coordinate			

Solution

1). The y-intercept of the line is Q which has coordinates (0; 4)

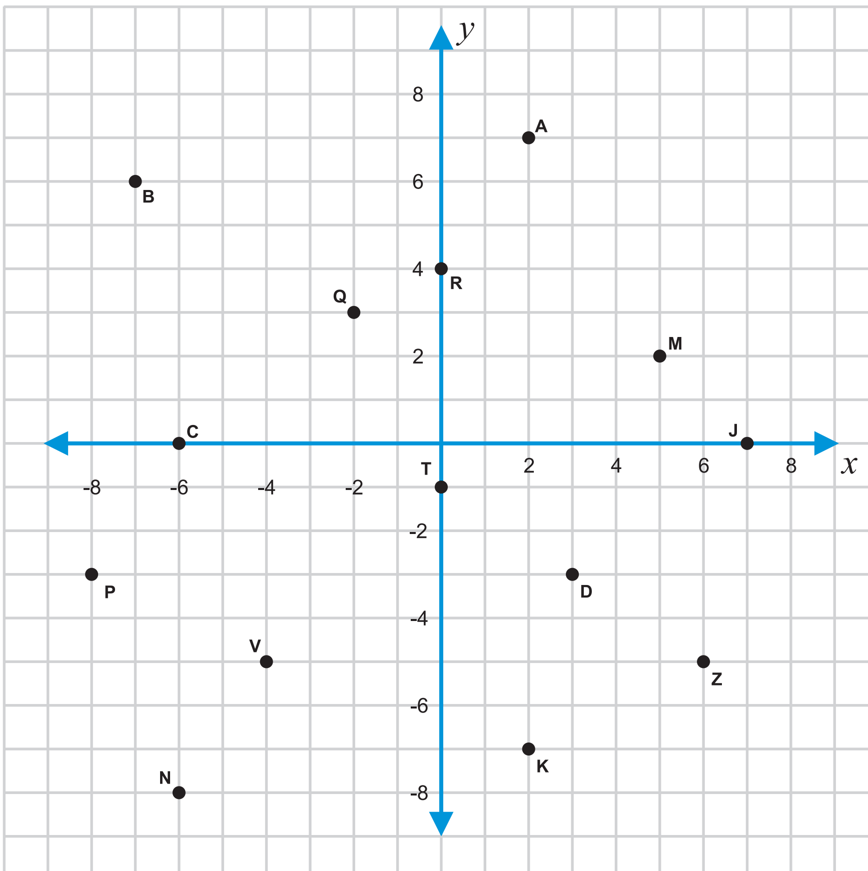
2). The x-intercept is R which has coordinates (2; 0)

3). The coordinates of the points P, S and T can be tabulated as shown:

	P	**S**	**T**
x-coordinate	-2	4	6
y-coordinate	8	-4	8

Activity: Determining coordinates of points in the Cartesian plane

1). Use this Cartesian plane to complete the table that follows.



Point	x-coordinate	y-coordinate	Ordered pair
A	2	7	(2;7)
B			
C			
D			
J			
K			
M			
N			
P			
Q			
R			
T			
Z			

2). Which points lie in the first quadrant?

3). How would you describe their coordinates?

4). Which points lie in the second quadrant?

5). How would you describe their coordinates?

6). Describe the coordinates of:

a). Points in the third quadrant.

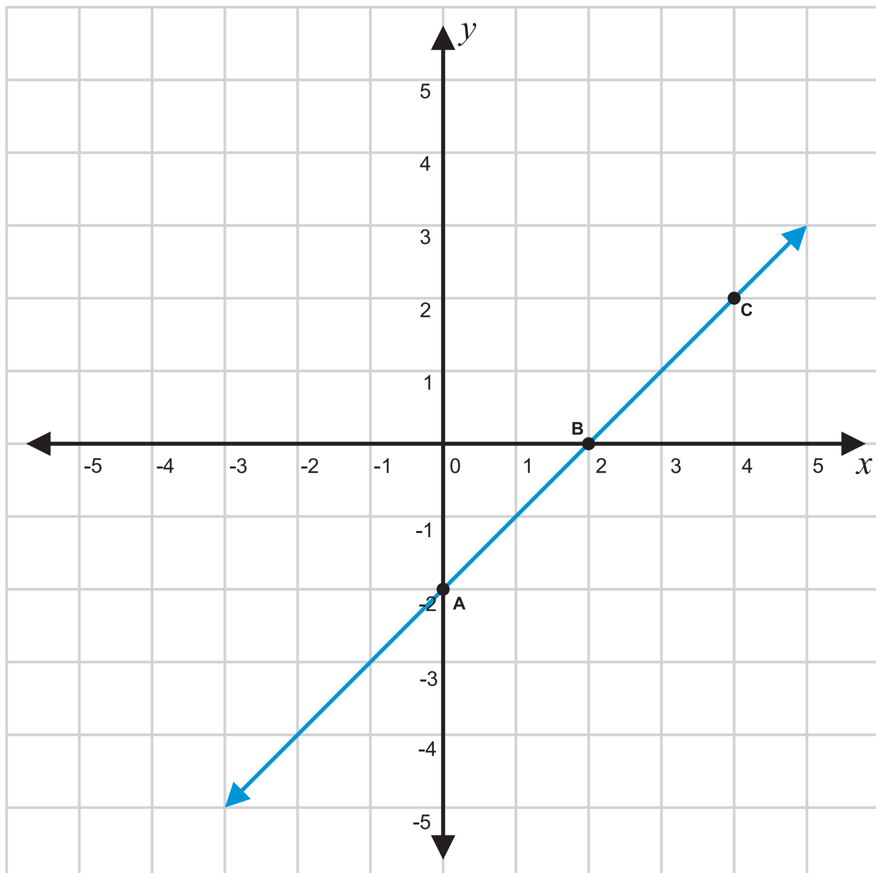
b). Points in the fourth quadrant.

c). Points on the x-axis.

d). Points on the y-axis.

7). Use the graph to answer the questions that follow.

a). Write down the coordinates of points A, B and C in the table that follows:



	A	B	C
x-coordinate			
y-coordinate			

b). Write down the coordinates of the y-intercept.

c). Write down the coordinates of the x-intercept.

Solution

1).

Point	x-coordinate	y-coordinate	Ordered pair
A	2	7	(2; 7)
B	-7	6	(-7; 6)
C	-6	0	(-6; 0)
D	3	-3	(3; -3)
J	7	0	(7; 0)
K	2	-7	(2; -7)
M	5	2	(5; 2)
N	-5	-8	(-5; -8)
P	-8	-3	(-8; -3)
Q	-2	3	(-2; 3)
R	0	4	(0; 4)
T	0	-1	(0; -1)
Z	6	-5	(6; -5)

2). Points A and M

3). The coordinates are both positive ($x > 0$ and $y > 0$)

4). Points B and Q

5). The x-coordinate is negative and the y-coordinate is positive ($x < 0$ and $y > 0$)

6). a). Both coordinates are negative ($x < 0$ and $y < 0$)

b). The x-coordinate is positive and the y-coordinate is negative ($x > 0$ and $y > 0$)

c). The y-coordinate is 0

d). The x-coordinate is 0

7). a).

	A	B	C
x-coordinate	0	2	4
y-coordinate	-2	0	2

b). (0; -2)

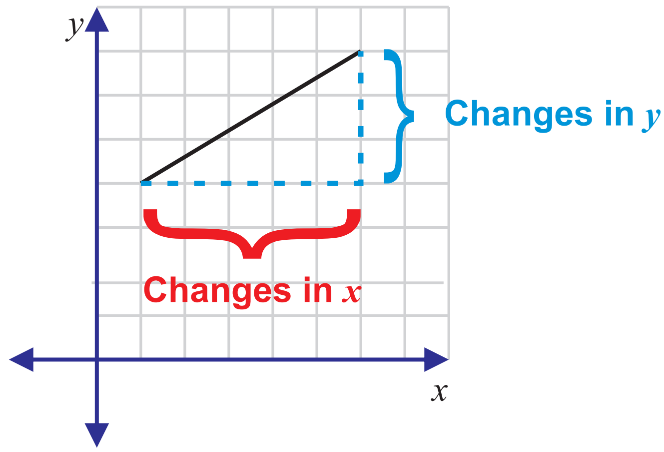
c). (2; 0)

Determining the equation of a line

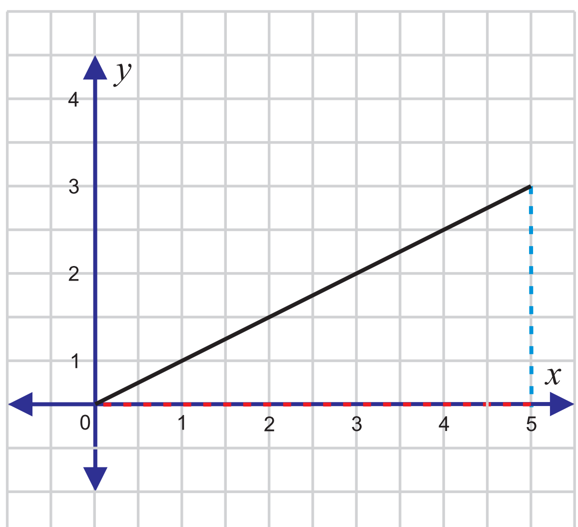
- The teacher should then demonstrate for learners how the gradient of a line passing through given points is calculated.
- The gradient of a line is calculated by finding the rise (change in y) over the run (change in x) for a section of the line:

$$\text{Gradient} = \frac{rise}{run} = \frac{Change\ in\ y}{Change\ in\ x} = \frac{\Delta y}{\Delta x}$$

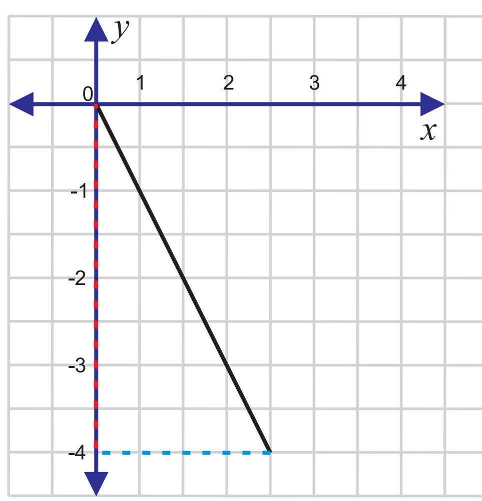
- You can use the diagram shown to assist with your explanation on gradient.



- Learners should be given an example of a positive gradient (see the diagram that follows).
  - Discuss why the gradient is positive (it is going up).

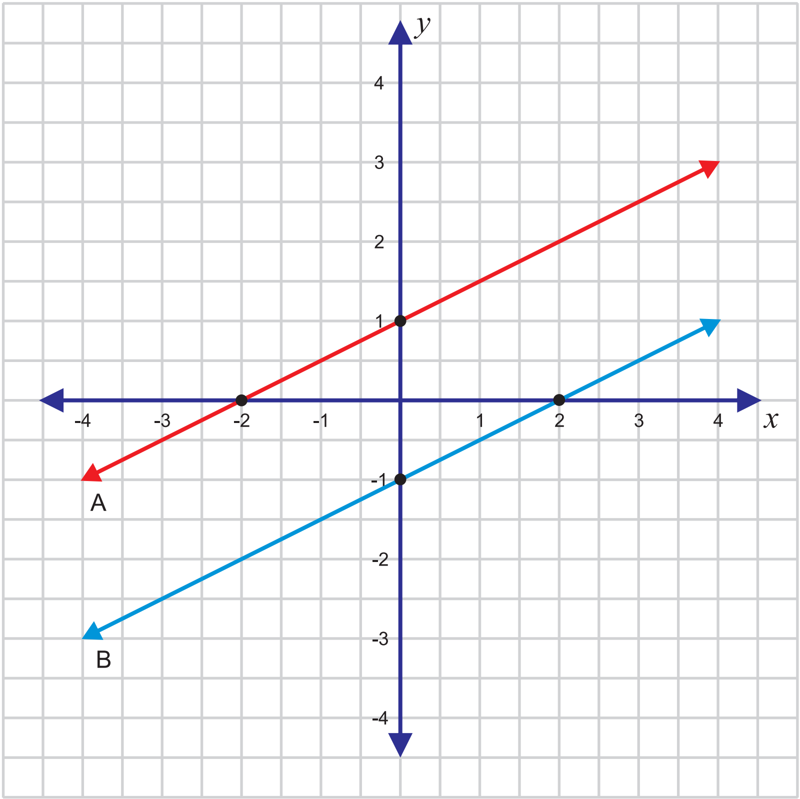


- Also show learners an example of a negative gradient (see the next diagram).
  - Discuss why the gradient is negative (it is going down).

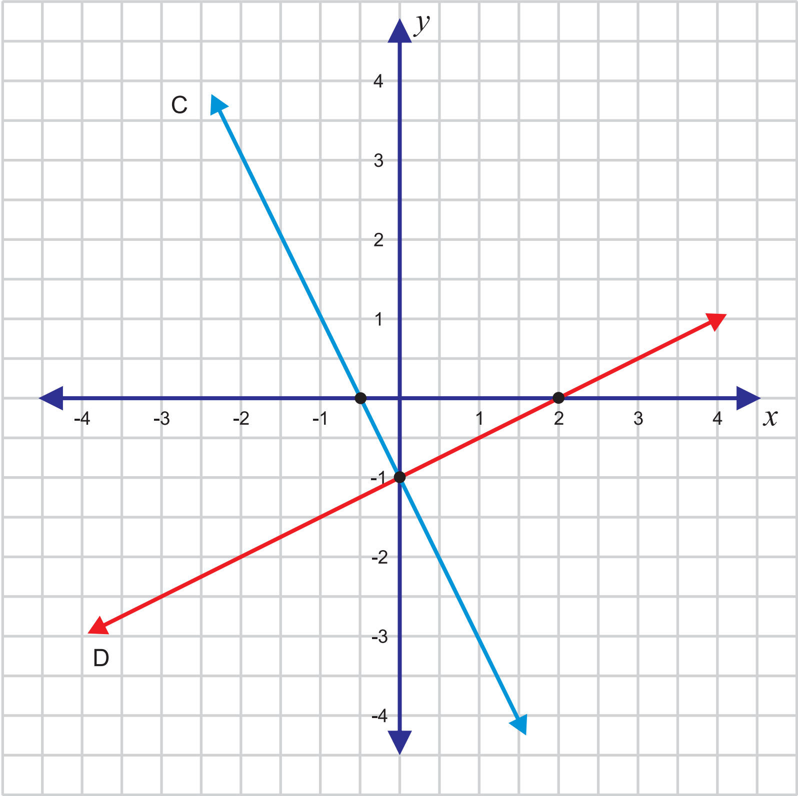


- Explain that for gradient up is positive, and down is negative.

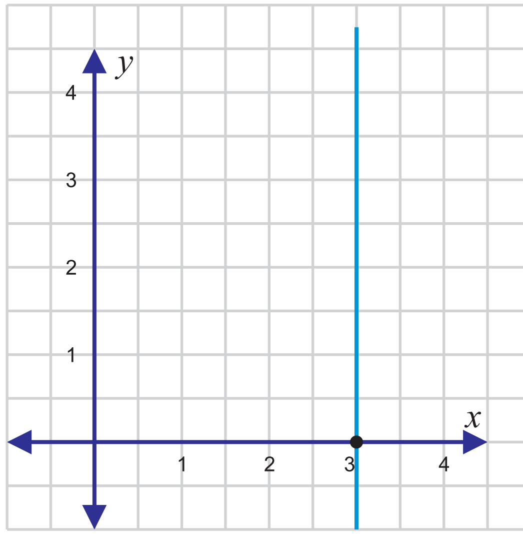
- You could also discuss the gradients of lines that are perpendicular and lines that are parallel.
  - Parallel lines have equal gradients, $m_A = m_B$



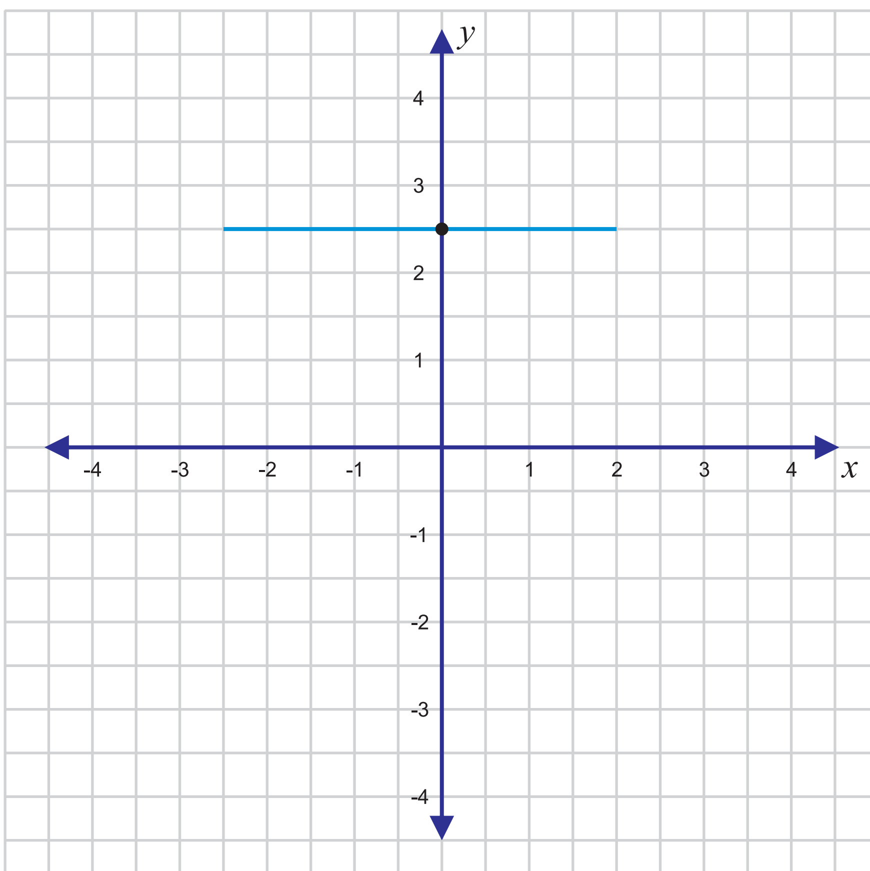
  - When you multiply the gradients of two perpendicular lines you would get an answer of -1, $m_C \times m_D = -1$



- Lines that are parallel to the y-axis have a gradient which is undefined (see the diagram that follows).



- Lines that are parallel to the x-axis have a 0 gradient m=0 (see the next diagram follows)

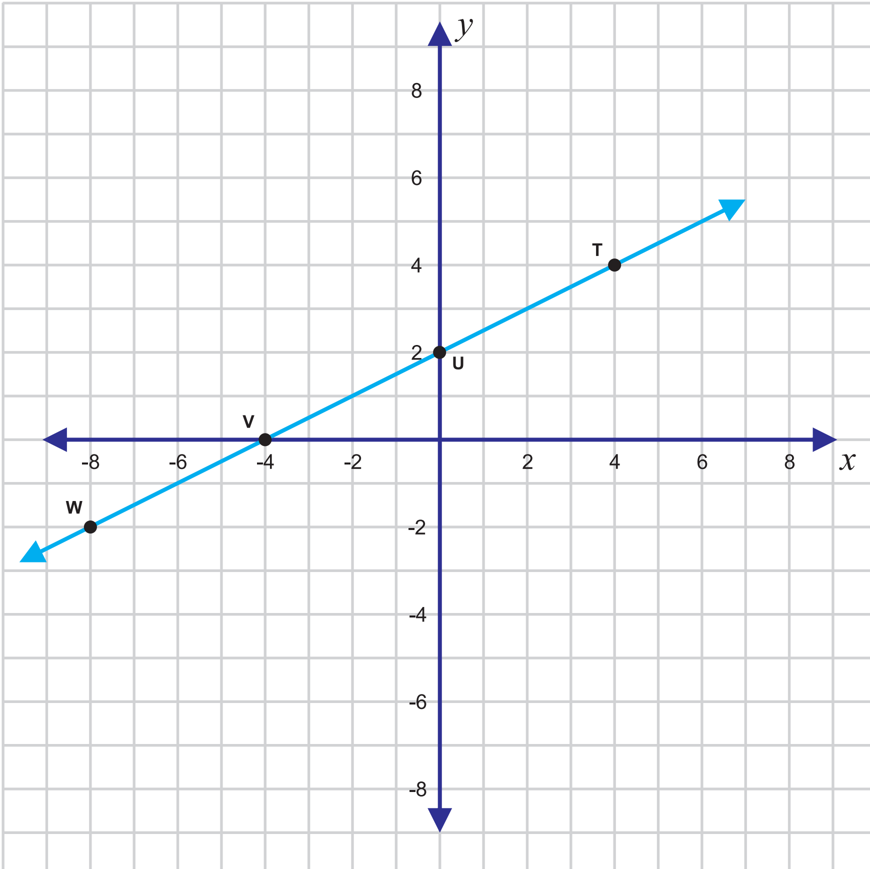


- You could also demonstrate how the gradient and the y-intercept are used to determine the equation of a line.

- Remind the learners that the general form of the equation of a straight line is given by $y = mx + c$, where m is the gradient of the line and c is the y-intercept of the line.

Example

Consider the following sketch:



- The gradient of the line WT could be calculated using any two points on the line WT.
- For example, the gradient could be calculated as follows using points T (4; 4) and U (0; 2):

$$\text{Gradient} = \frac{\text{Change in } y}{\text{Change in } x} = \frac{\Delta y}{\Delta x}$$

$$\frac{\Delta y}{\Delta x} = \frac{4-2}{4-0} = \frac{2}{4} = \frac{1}{2}$$

or

$$\frac{\Delta y}{\Delta x} = \frac{2-4}{0-4} = \frac{-2}{-4} = \frac{1}{2}$$

- Learners' attention should be drawn to the importance of the order in which the change is calculated.
- You can calculate the change in y in any direction, but then you must calculate the change in x in the SAME direction.
- Points W and T could also be used for consolidation, since they give us other pairs of points on the same line: if we use W and T to find the gradient we should get the same gradient as we did when we used T and U.

- Using points W and T: T (4; 4) and W (- 8; - 2):

$$\text{Gradient} = \frac{\text{Change in } y}{\text{Change in } x} = \frac{\Delta y}{\Delta x}$$

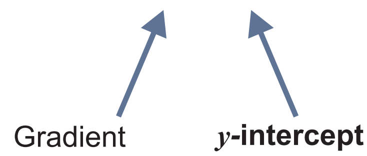
$$\frac{\Delta y}{\Delta x} = \frac{4-(-2)}{4-(-8)} = \frac{6}{12} = \frac{1}{2}$$

or

$$\frac{\Delta y}{\Delta x} = \frac{-2-4}{-8-4} = \frac{-6}{-12} = \frac{1}{2}$$

- The ***y*-intercept** of the line WT is Q (0; 2).

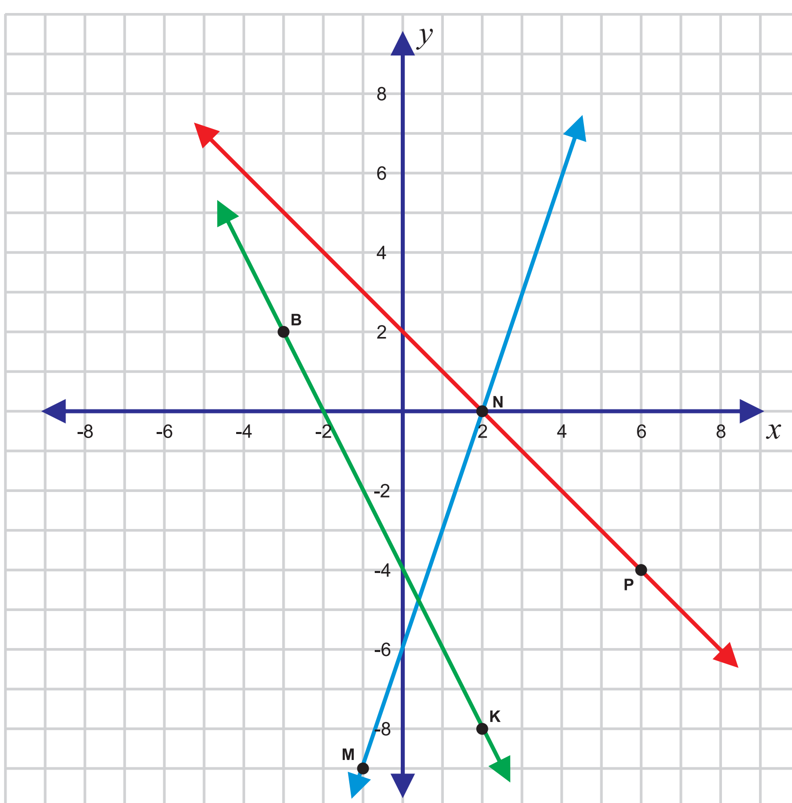
Equation: $y = \quad x + \frac{1}{2}2$



- Discuss the solution with the learners. Use other pairs of points on the same line (or draw another line in the plane with other coordinates labelled) to be sure that your learners feel confident about how to find equations of straight lines before you give them the consolidation exercise.

Activity: Determining the equation of a line

Use the following graph to answer the questions that follow.



1). Calculate the gradient and y-intercept of each of the following lines:

a). BK

b). MN

c). PN

2). Use the solutions obtained in 1 above to determine the equation of:

a). BK

b). MN

c). PN

3). Write down the coordinates of the point of intersection of MN and NP:

Solution

1). a).Gradient of BK = $\frac{2-(-8)}{-3-2} = \frac{-8-2}{2-(-3)} = -2$

y-intercept of BK is (0; -4)

b).Gradient of MN = $\frac{-6-0}{0-2} = \frac{0-(-6)}{2-0} = 3$

y-intercept of MN is (0; -6)

c).Gradient of PN = $\frac{0-(-4)}{2-6} = \frac{-4-0}{6-2} = -1$

y-intercept of PN is (0; 2)

2). a).Equation of BK is: $y = -2x - 4$

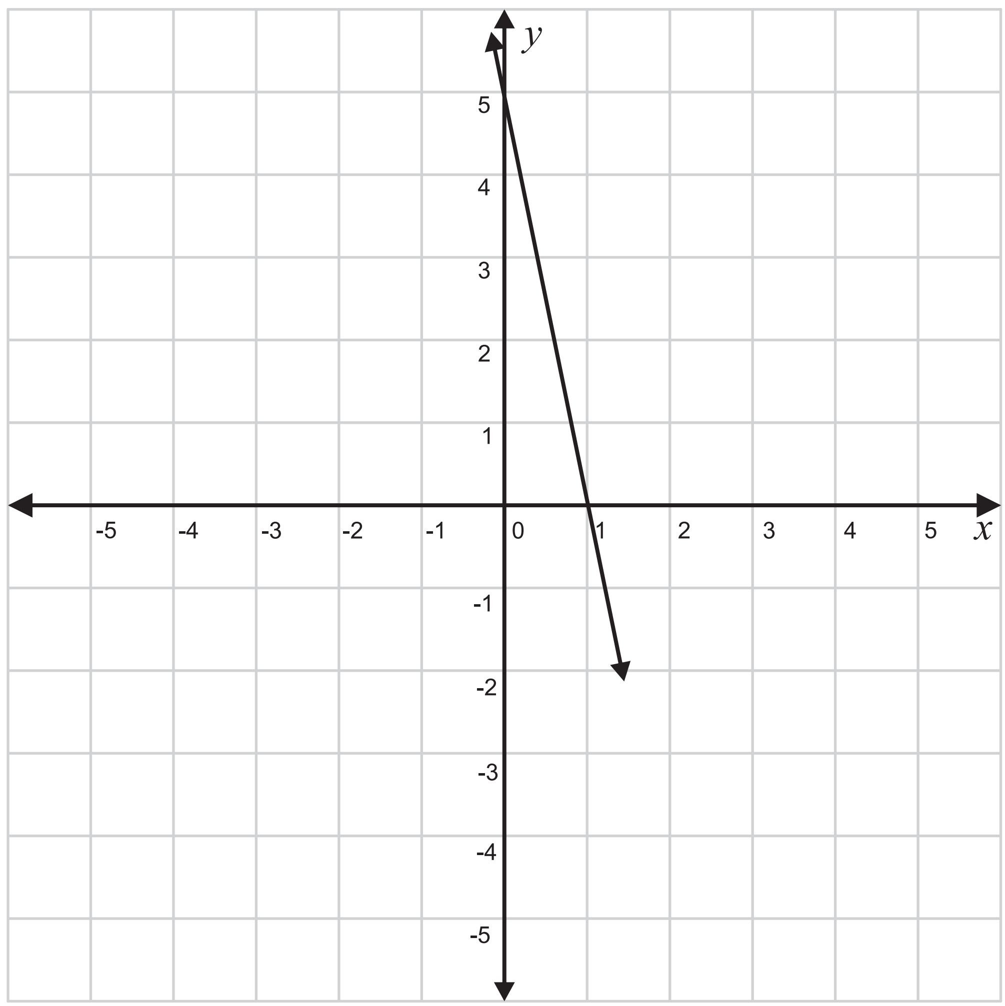
b).Equation of MN is: $y = 3x + 6$

c).Equation of NP is: $y = -x + 2$

3). (2; 0)

Notes:

Other examples of how to test linear graphs



ANA 2014 Grade 9 Mathematics Items 7.1 and 7.2

7.1 Use the graph to calculate the gradient of the straight line. [3]

7.2 Determine the equation of the straight line. [2]

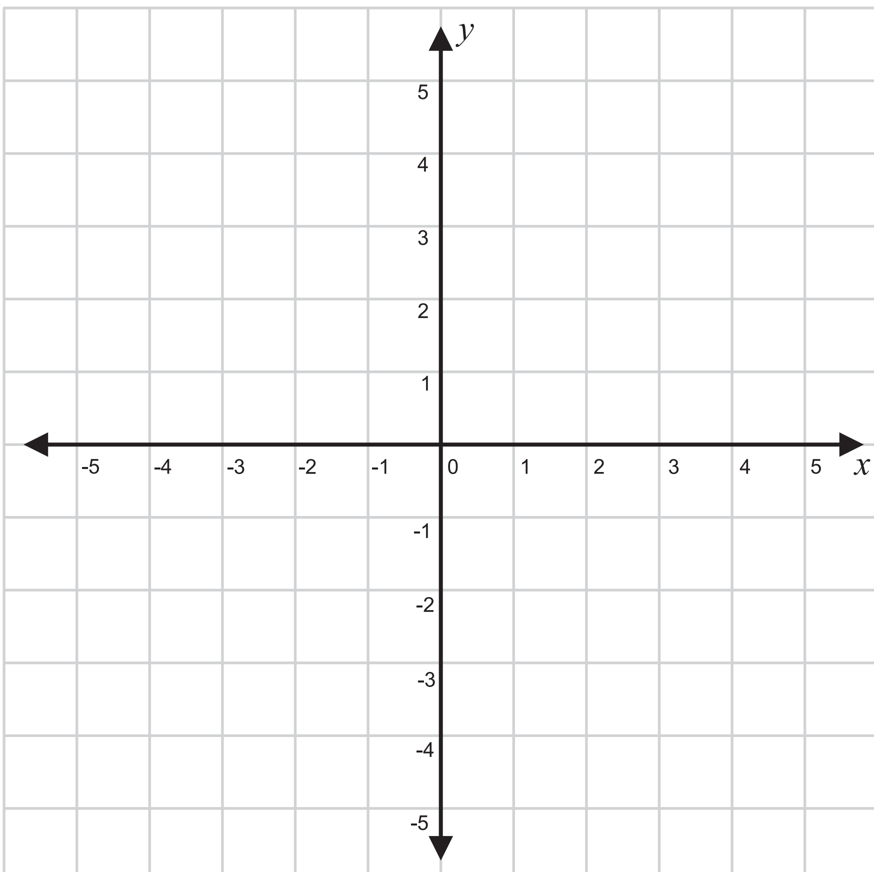
Notes:

Linear Graphs: Sketching and interpretation

ANA 2013 Grade 9 Mathematics Items 7.2.1 and 7.2.2

7.2 Use the grid below to answer the questions that follow.

7.2.1 Draw the graph defined by $y=-2x+4$ and $x=1$ on the given set of axes. Label each graph and clearly mark the points where the lines cut the axes. [5]



7.2.2 Write down the coordinates of the point where the two lines cut one another. [2]

What should a learner know to answer these questions correctly?

Learners should be able to:

- Sketch the graph of a given equation;
- Identify coordinates of the point of intersection of two linear graphs.

Where is this topic located in the curriculum? Grade 9, Term 3

Content area: Patterns, Algebra and Functions.

Topic: Graphs.

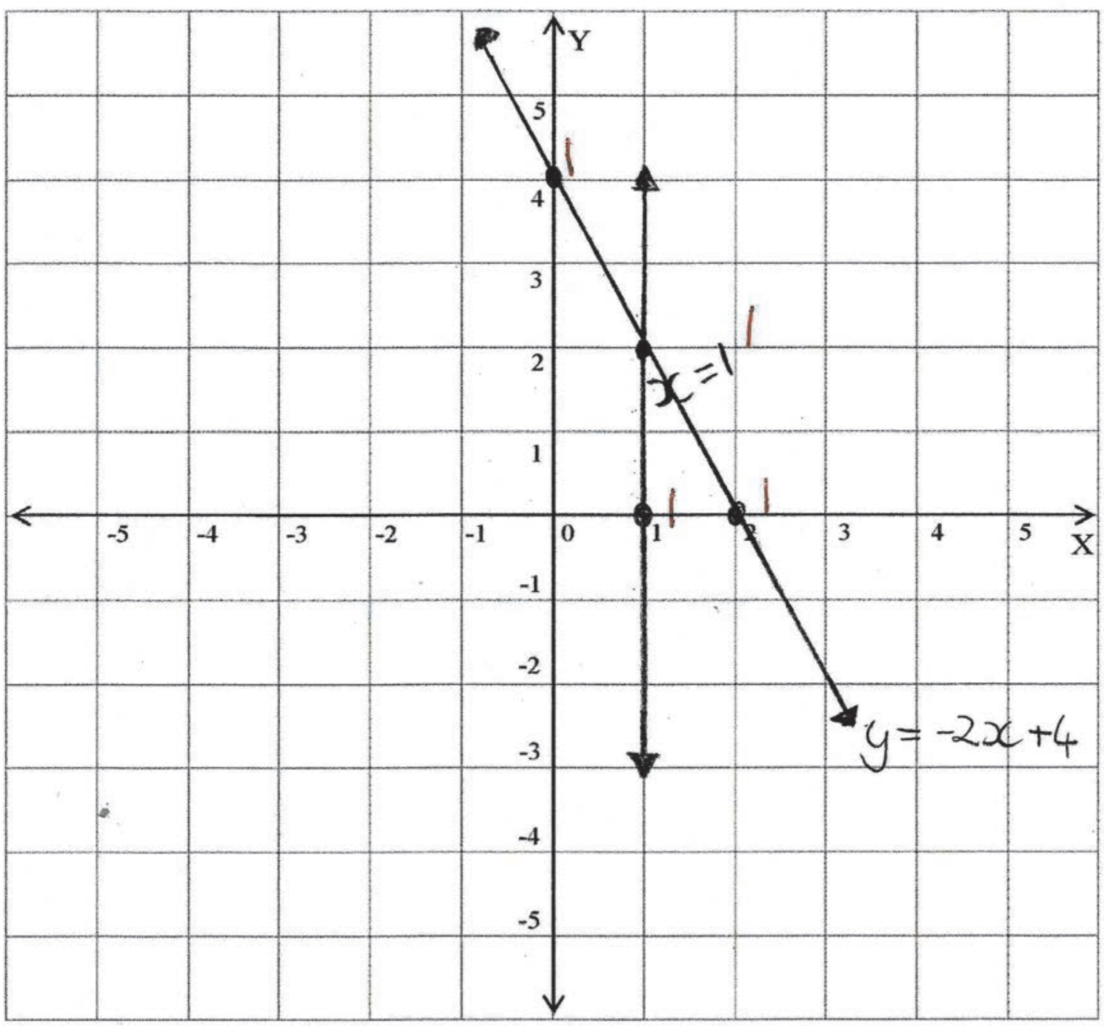
- Concepts and Skills:
- Drawing linear graphs from given equations;
- Graph interpretation.

What would show evidence of full understanding?

- If the learner drew the graphs correctly; and
- If the learner correctly identified the coordinates of the point of intersection.

7.2 Use the grid below to answer the questions that follow.

7.2.1 Draw the graphs defined by $y = -2x + 4$ and $x = 1$ on the given set of axes. Label each graph and clearly mark the points where the lines cut the axes.



7.2.2 Write down the coordinates of the point where the two lines cut one another.

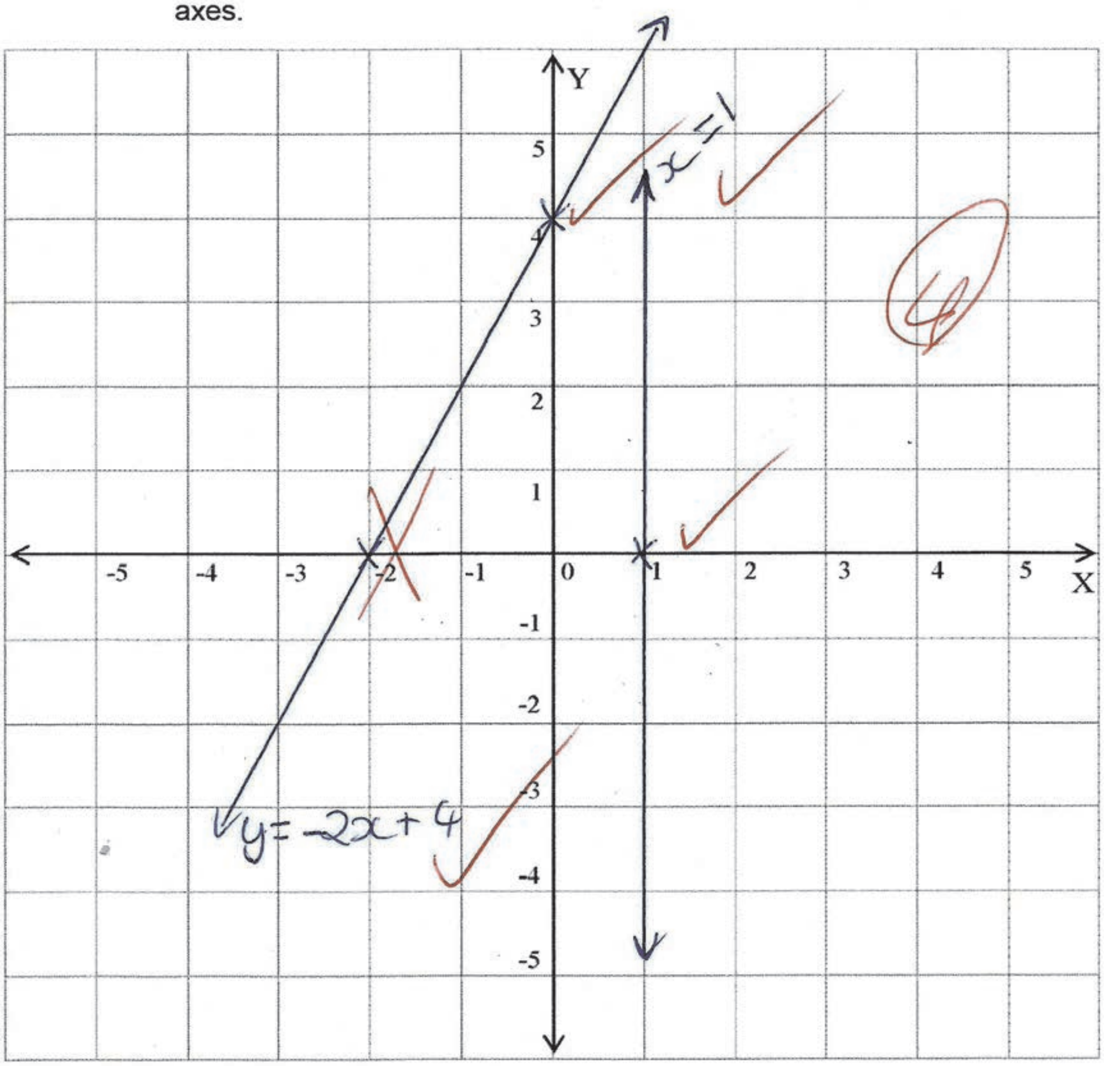
(1; 2)

What would show evidence of partial understanding?

- If the learner drew one or both graphs correctly; but did not identify the point of intersection correctly;
- If the learner drew both graphs incorrectly, but correctly identified a point of intersection.

7.2 Use the grid below to answer the questions that follow.

7.2.1 Draw the graphs defined by $y = -2x + 4$ and $x = 1$ on the given set of axes. Label each graph and clearly mark the points where the lines cut the axes.



7.2.2 Write down the coordinates of the point where the two lines cut one another.

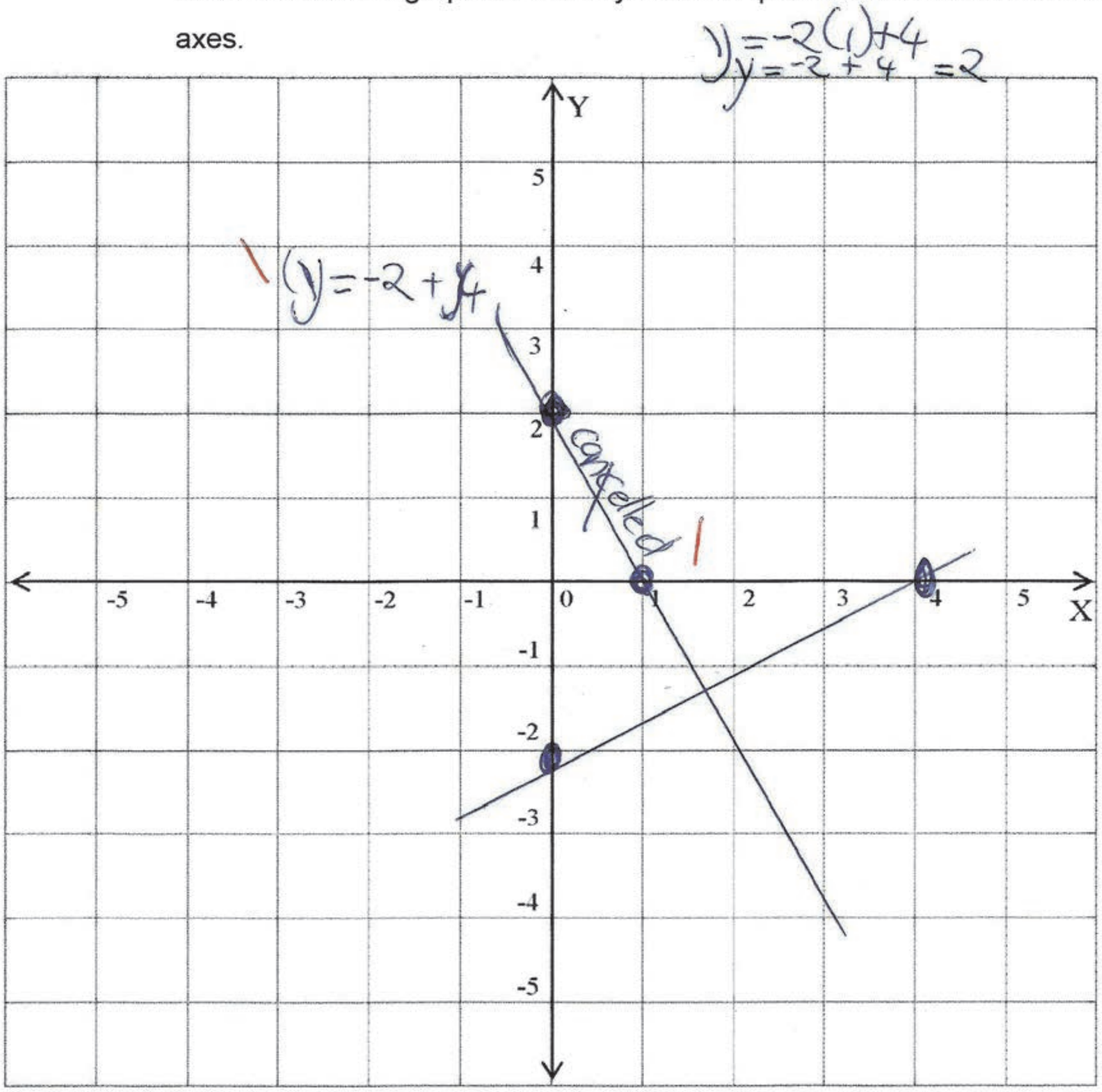
(1 + 4) ✗

What would show evidence of no understanding?

- If the learner sketched the graphs incorrectly; and
- If the learner identified the coordinates of the point of intersection incorrectly.

7.2 Use the grid below to answer the questions that follow.

7.2.1 Draw the graphs defined by $y = -2x + 4$ and $x = 1$ on the given set of axes. Label each graph and clearly mark the points where the lines cut the axes.



7.2.2 Write down the coordinates of the point where the two lines cut one another.

(-2 , 4)

What do the item statistics tell us?

Item 7.2.1

9% of learners answered the question correctly.

Factors contributing to the difficulty of the item

- Learners are unable to sketch graphs using equations.

Item 7.2.2

7% of learners answered the question correctly.

Factors contributing to the difficulty of the item

- Learners are unable to read the coordinates of a point on the Cartesian plane.

Teaching strategies

Point-by-point sketching

- It is advisable to start with constant graphs as they do not require any calculations.
- These are graphs of the form $x = a$ or $y = a$, where a is any constant value.

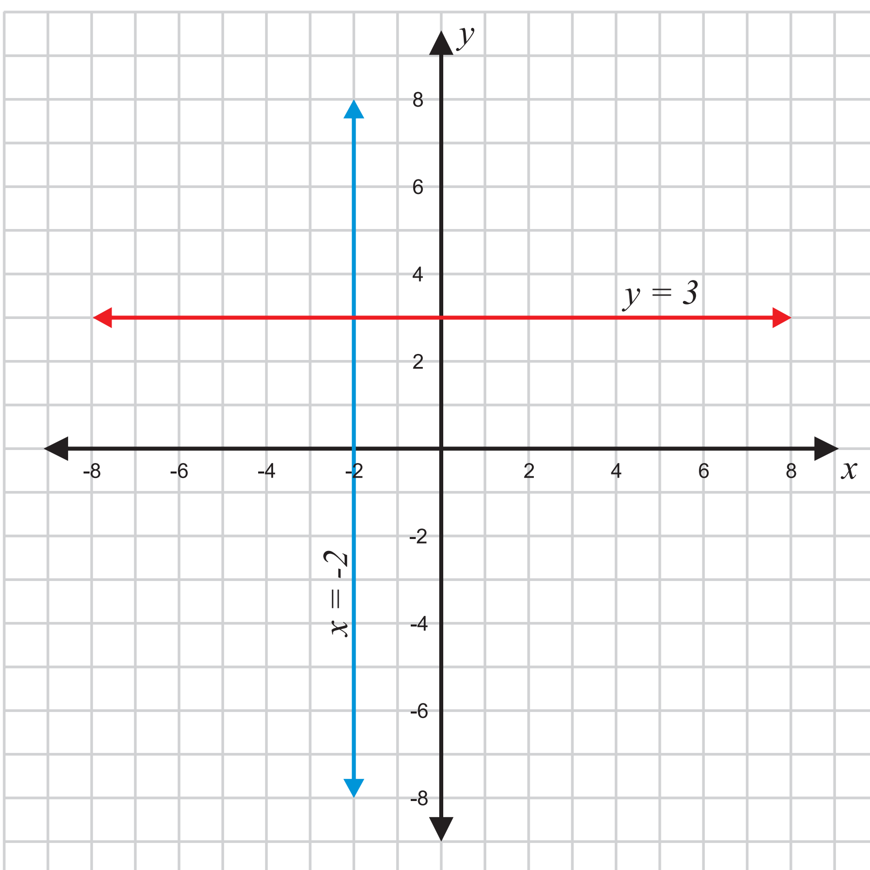
Example

Sketch graphs for the following equations:

$y = 3$

$x = -2$

Solution



- Discuss the two forms of constant graphs based on this sketch:
  - $x = 2$ (the x-value is constant, it does not change);
  - $y = 3$ (the y-value is constant, it does not change);
  - The x-value constant graph is a vertical line which goes through the given x-value: this means

that the graph cuts the x-axis at the point 2.
  - The y-value constant graph is a horizontal line which goes through the given y-value: this means that the graph cuts the y-axis at the point 3.

- Next you could move onto a discussion of oblique line graphs. In these graphs the x- and corresponding y-values change and so point plotting can be used to determine the slope of the line.
- Learners should be reminded that the point-by-point method entails assigning an arbitrary value to x and then substituting that value in the equation to determine the corresponding value of y.
  - To find a few points to use to draw the sketch, a table of points of x-values and corresponding y-values should be drawn up.
  - Make learners aware of the fact that while calculating coordinates of 2 points only is sufficient, it advisable that coordinates of 3 or more points be determined so that careless errors can be identified and the correct plots confirmed.
  - Suggest that learners select values that will make calculations easy.

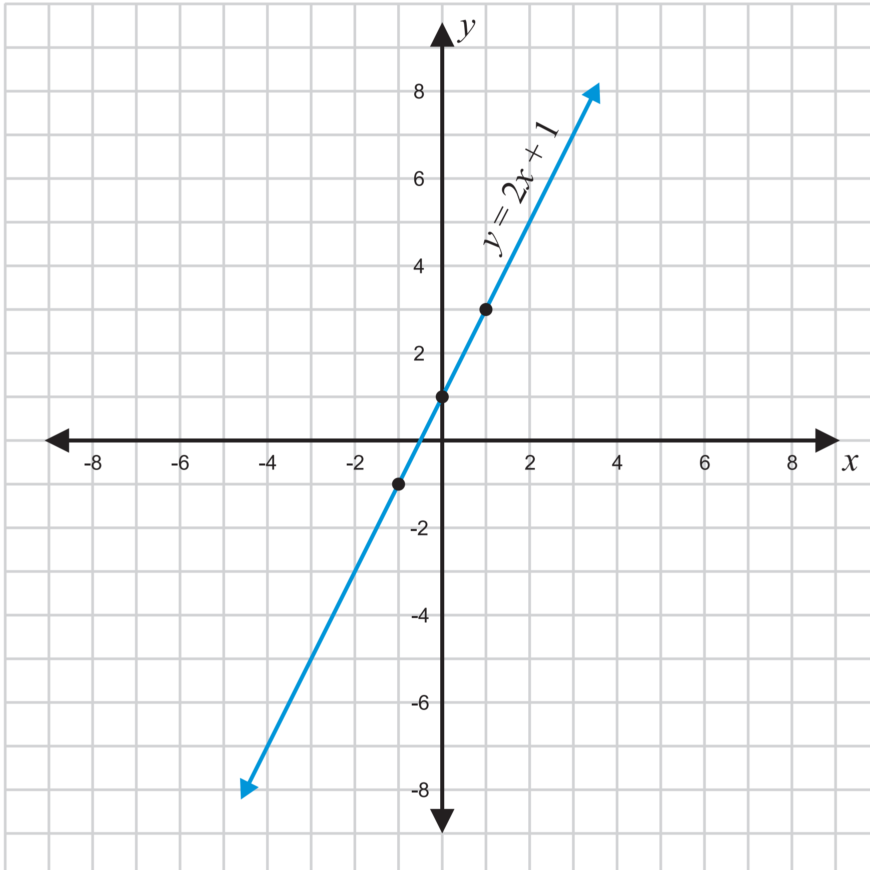
Example

Sketch the graph defined by the equation $y = 2x + 1$

- Select values of x for which you will calculate the corresponding values of y.
- -1, 0 and 1 have been selected as arbitrary values of x.
- For each value of x, a corresponding y-coordinate has been calculated.
  - $[= -1 \Longrightarrow y = 2x + 1 = 2(-1) +1 = -2 + 1 = -1]$
  - $[= 0 \Longrightarrow y = 2x + 1 = 2(0) + 1 = 0 + 1 = 1]$
  - $[= 1 \Longrightarrow y = 2x + 1 = 2(1) + 1 = 2 + 1 = 3]$

x	-1	0	1
$y = 2x + 1$	-1	1	3

- The coordinates can then been presented in a table as shown:
- The coordinates can be plotted on a Cartesian plane and the plotted points joined to sketch the graph:



Activity: Point-by-point sketching

Where necessary, use the point-by-point method to sketch, on the same system of axes, the graphs defined by the following equations:

1). $y = -3$

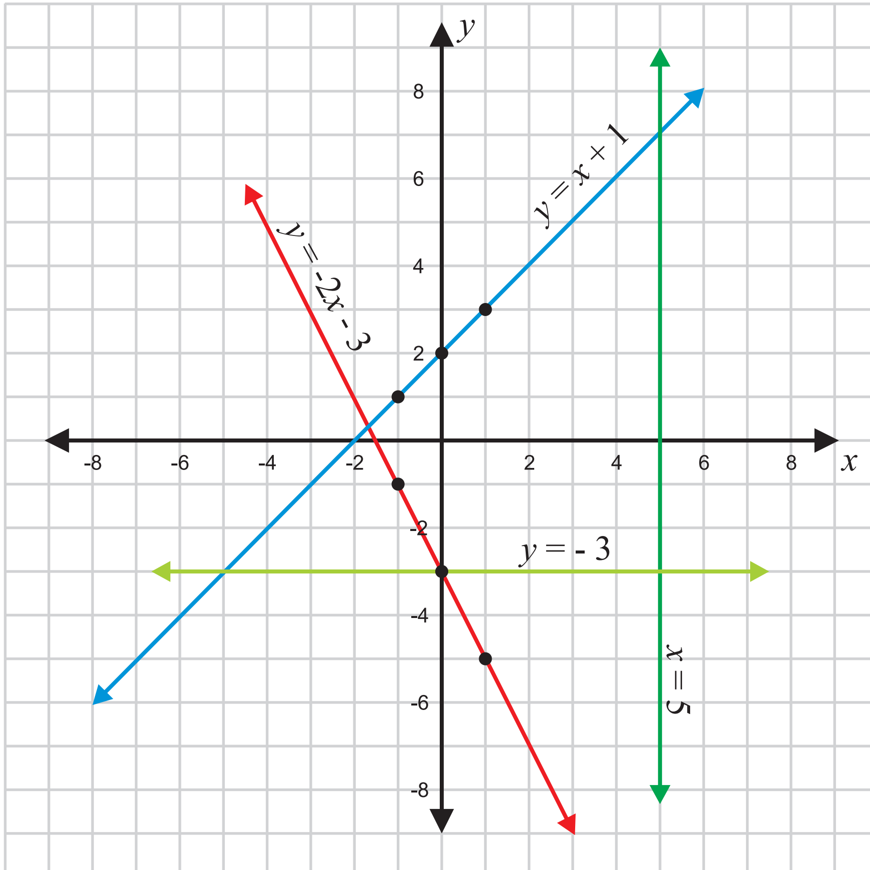
2). $x = 5$

3). $y = x + 2$

4). $y = -2x - 3$

Solution

x	-1	0	1
$y = x + 2$	1	2	3
$y = -2x - 3$	-1	-3	-5



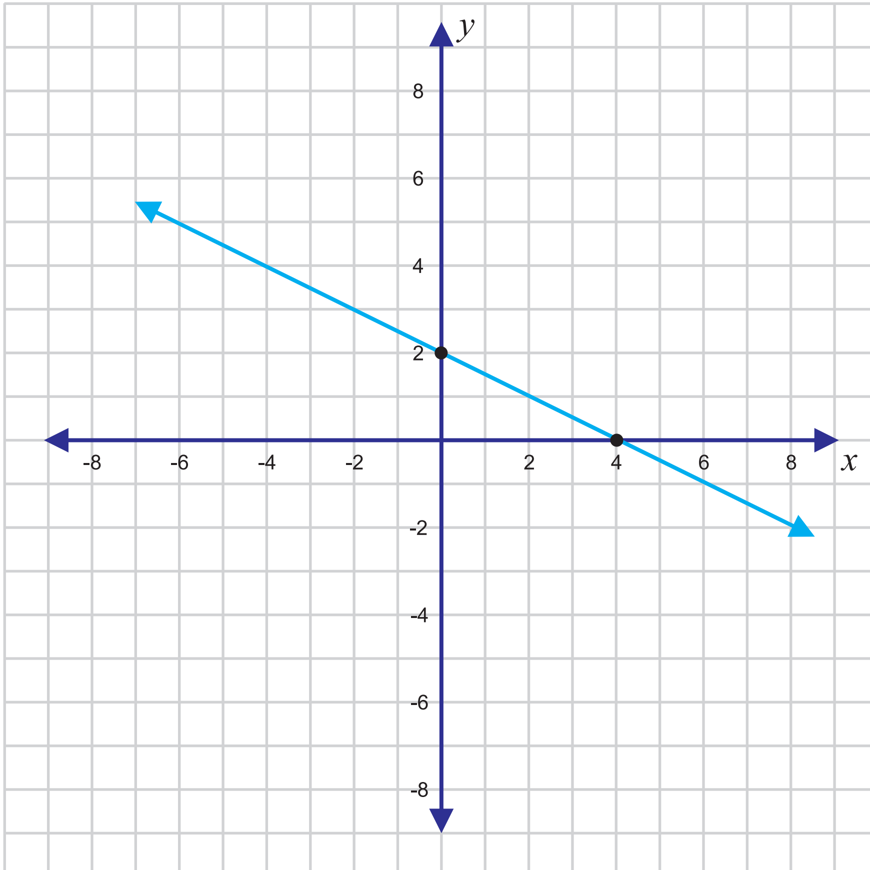
The dual intercept method for straight line graph sketching

- The dual intercept method entails calculating coordinates of the x-intercept and the y-intercept in order to sketch the graph.
- The coordinates are then plotted on the Cartesian plane and joined by a straight line.

Example

Sketch the graph defined by the equation $y = \frac{-x}{2} + 2$

- The y-intercept is calculated by making the value of $x = 0$. This is because when $x = 0$, the graph will cross the y-axis.
    - $y = \frac{-0}{2} + 2$
    - $= 2$
    - Therefore the y-intercept is at (0; 2)
- We could also read the y-intercept value directly from the graph if we remember that $y = mx + c$, where $c = y$-intercept. In this case, $c = 2$.
- The x-intercept is calculated by making the value of $y = 0$. This is because when $y = 0$, the graph will cross the x-axis.
    - That is: $\frac{-x}{2} + 2 = 0$
    - Thus $x = 4$
    - x-intercept is at (4; 0)
- The two points (0; 2) and (4; 0) are plotted as follows and joined by a straight line:



Activity: The dual intercept method for straight line graph sketching.

Use the dual intercept method to sketch the graphs defined by the following equations:

1). $y = x + 2$

2). $y = \frac{1}{2}x - 3$

3). $y = -x + 4$

4). $y = -2x - 3$

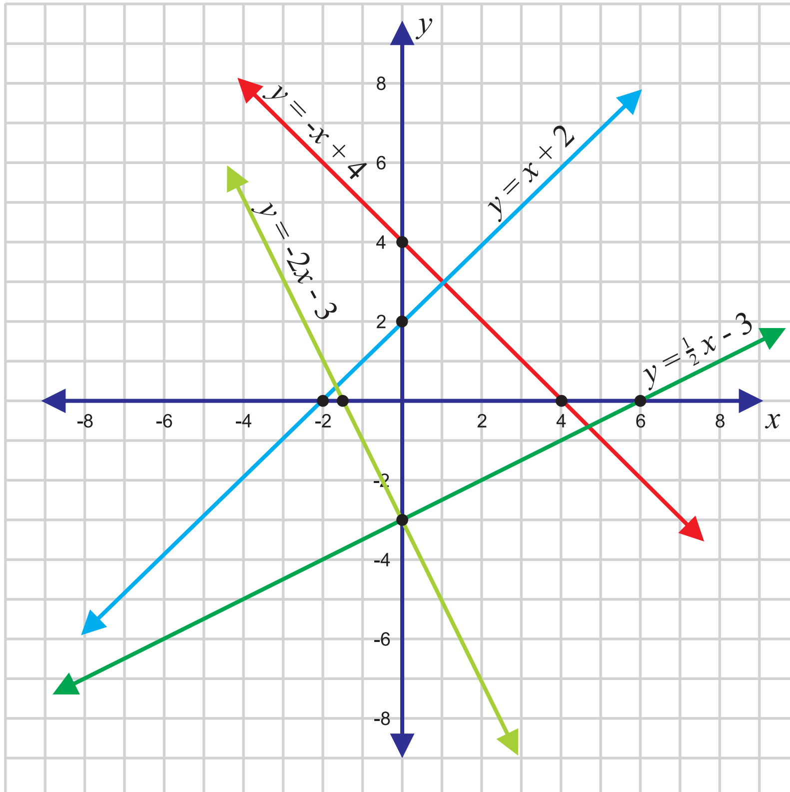
Solution

1). y-intercept is (0; 2) and x-intercept is (-2; 0)

2). y-intercept is (0; -3) and x-intercept is (6; 0)

3). y-intercept is (0; 4) and x-intercept is (4; 0)

4). y-intercept is (0; -3) and x-intercept is $(-\frac{3}{2}; 0)$

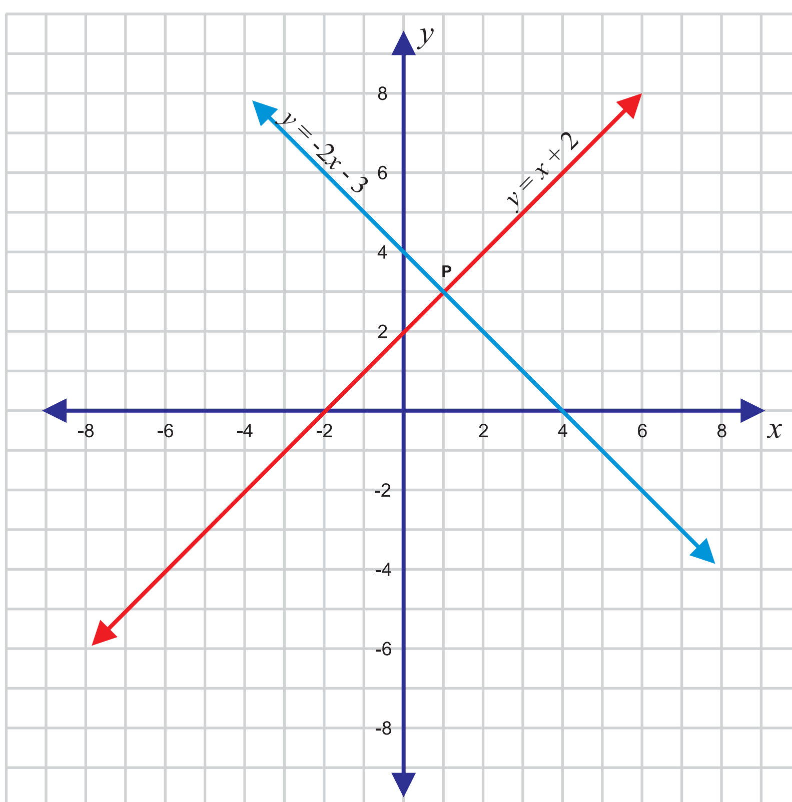


Determining points of intersection of graphs

- Learners should also be shown how to determine the point of intersection of two graphs.
- The point of intersection is the point where two (or more) graphs cross.

Example

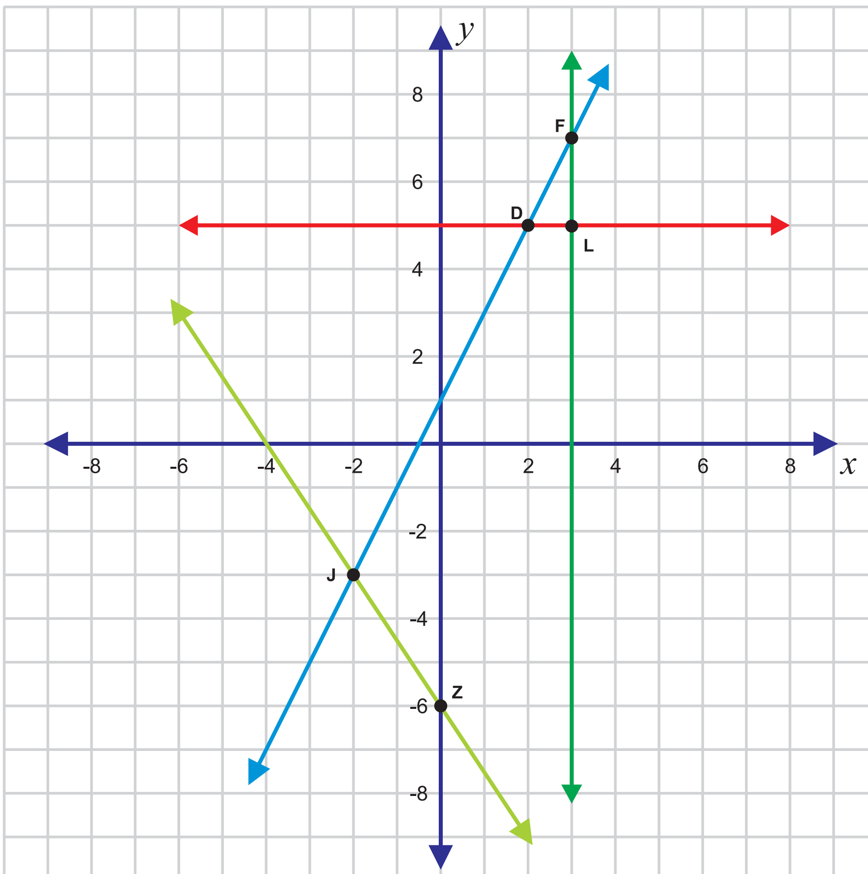
Consider the graphs of $y = -x + 4$ and $y = x + 2$ drawn as follows:



- The lines intersect at P
- The coordinates of P are $x = 1$ and $y = 3$.
- The point of intersection of the two line graphs is P (1; 3)

Activity: Linear graphs – mixed questions

Use the sketched graphs to answer the following questions.



1). Write down the equations of the following lines:

a). DL

b). FL

c). JF

d). JZ

2). Write down the coordinates of the y-intercepts of:

a). JF

b). JZ

3). Write down the coordinates of the x-intercepts of:

a). JF

b). JZ

4). Write down the coordinates of the point of intersection of:

a). DL and JF

b). DL and FL

c). JF and FL

d). JF and JZ

Solutions

1). Equations

a). $y = 5$

b). $x = 3$

c). $x = 2x + 1$

d). $y = \frac{-3x}{2} - 6$

2). y-intercepts

a). (0; 1)

b). (0; -6)

3). x-intercepts of:

a). $(-\frac{1}{2}; 0)$

b). (-4; 0)

4). Coordinates of the points of intersection

a). (2; 5)

b). (3; 5)

c). (3; 7)

d). (-2; 3)

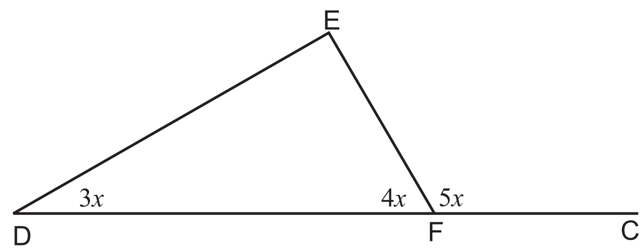
Notes:

Geometry of 2-D shapes: Properties of triangles

ANA 2013 Grade 9 Mathematics Items 1.9, 8.1, 8.2, 8.3 and 8.4

1.9 In the figure below, side DF of ΔEDF is produced to C.

Cacluate the size of $\hat{E}$ in terms of x.



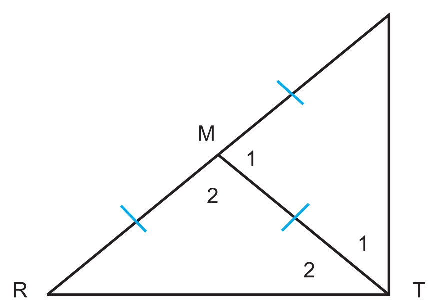
A. $2x$

B. $12x$

C. $7x$

D. $9x$ [1]

8.1 In ΔPRT below, M is the midpoint of PR and $MR = MT$



If $\hat{P} = 25°$, calculate with reasons:

8.1.1 The size of $\hat{T}_1$

[1]

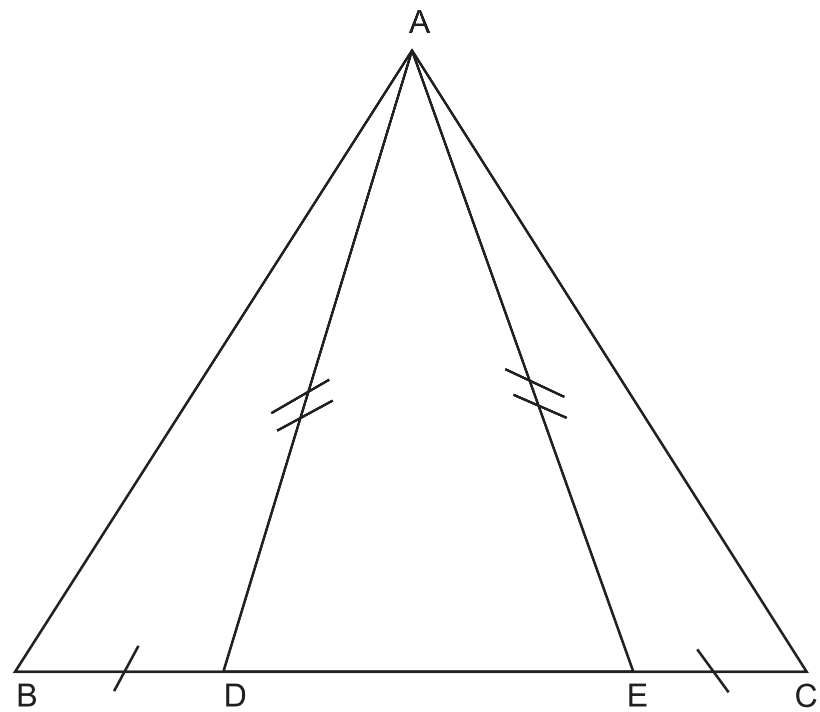
8.1.2 The size of $\widehat{M}_2$

[1]

8.1.3 The size of $\hat{R}$

[3]

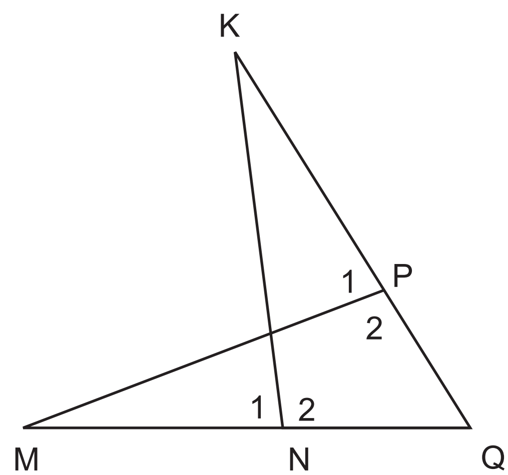
8.2 In ΔABC, D and E are points on BC such that $BD = CE$ and $AD = AC$.



8.2.1 Why is $BE = DC$? [1]

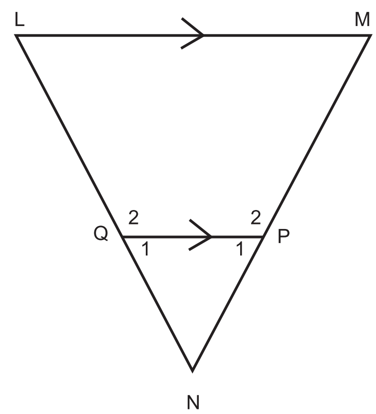
8.2.2 Which triangle is congruent to ΔABE? [1]

8.3 In the figure below ΔKNQ and ΔMPQ have a common vertex Q.
P is a point on KQ and N is a point on MQ.
$\text{KQ} = \text{MQ}$ and $PQ = QN$.



Prove with reasons that $\Delta KNQ \equiv \Delta MPQ$ [4]

8.4 In $\Delta\, NML$ below P and Q are points on the sides MN and LN respectively such that $QP \parallel LM$ $MN = 16\ cm$, $QP = 3\ cm$ and $LM = 8\ cm$.



8.4.1 Complete the following (Give reasons for the statements).

Prove with reasons that $\Delta\, QPN \,|||\, \Delta\, LMN$.

In $\Delta\, QPN \,|||\, \Delta\, LMN$

$\hat{N} =$ …………………… ……………………

$\hat{P}_1 =$ …………………… ……………………

$\hat{Q}_1 =$ …………………… ……………………

$\therefore \Delta\, QPN \,|||\, \Delta$ …………….. …………………… [4]

8.4.2 Hence, calculate the length of PN. [3]

What should a learner know to answer these questions correctly?

Learners should be able to:

Item 1.9

- Solve simple geometric problems involving unknown angles in triangles using known properties and definitions;
- Use the property that the sum of the angles of a triangle adds up to 180°.

Item 8.1 and item 8.2

- Use their knowledge regarding the sum of the angles of a triangle;
- Use their knowledge of the properties of an isosceles triangles;
- Use their knowledge of the exterior angle of a triangle theorem.

Item 8.3 and 8.4

- Prove that triangles are similar;
- Prove that triangles are congruent;
- Show how similarity and congruency may be applied to solve problems.

Where are these topics located in the curriculum? Grade 9 Term 2

Content area: Space and shape (Geometry).

Topic: Construction of geometric figures;

Geometry of 2-D shapes.

Concepts and skills:

- By construction, investigate the angles in a triangle, focusing on the relationship between the exterior angle of a triangle and its interior angles;
- Properties and definitions of triangles in terms of their sides and angles, distinguishing between equilateral triangles, isosceles triangles and right-angled triangles;
- Through investigation, establish the minimum conditions for triangles to be congruent;
- Through investigation, establish the minimum conditions for triangles to be similar;
- Solve simple geometric problems involving unknown sides and angles in triangles and quadrilaterals using known properties and definitions as well as properties of congruent and similar triangles.

What would show evidence of full understanding?

Item 1.9

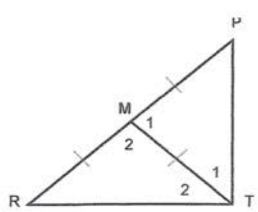
If the learner chose option A as the answer.

Item 8.1

If the learner calculated the angles correctly and provided acceptable reasons for all statements:

QUESTION 8

8.1 In ΔPRT below, M is the midpoint of PR and $MR = MT$.



If $\hat{P} = 25°$, calculate with reasons:

8.1.1 The size of $\hat{T}_1$

$\hat{T}_1 = 25°$ (base ∠'s of an Isosceles Δ)

(1)

8.1.2 The size of $\hat{M}_2$

$\hat{M}_2 = 180° - \hat{M}_1$ (∠'s on a straight line)

$\hat{M}_2 = 180° - 130°$ $\hat{M}_2 = 50°$

(1)

8.1.3 The size of $\hat{R}$

$\hat{R} = \hat{T}_2$ (base ∠'s of a isosceles Δ)

$\hat{R} = 65°$

(3)

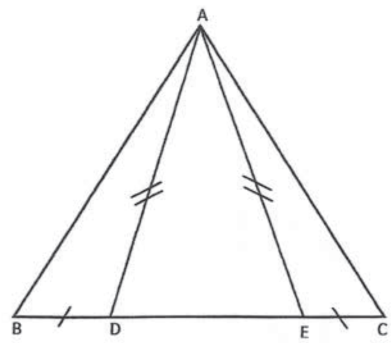
⊛ $T_2 = 90° - 25°$ (complementary ∠'s = 90°)

$T_2 = 65°$

Item 8.2

If the learner provided an acceptable reason for BE to be equal to CD and correctly identified a triangle that is congruent to ΔABE:

8.2 In ΔABC, D and E are points on BC such that $BD = EC$ and $AD = AE$.



8.2.1 Why is $BE = CD$? Because DB + DE = CE + DE ✓ (1) (1)

or

8.2.1 Why is $BE = CD$? Δ ABE = ΔADC (SSS)

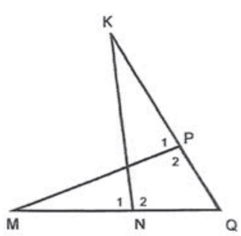
8.2.2 Which triangle is congruent to ΔABE ?

ΔADC (SSS)

Item 8.3

If the learner correctly proved that triangle KNQ is congruent to triangle:

8.3 In the figure below ΔKNQ and ΔMPQ have a common vertex Q.
P is a point on KQ and N is a point on MQ.
$KQ = MQ$ and $PQ = QN$.



Prove with reasons that $\Delta KNQ \equiv \Delta MPQ$.

KQ = MQ (Given)

PQ = QN (Given)

$\hat{Q} = \hat{Q}$ (Common ∠)

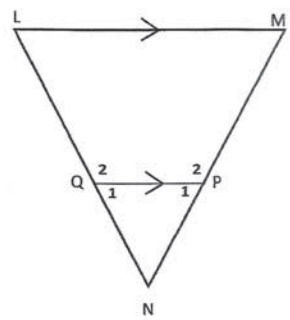
Δ KNQ ≡ Δ MPQ (SAS)

Item 8.4

If the learner correctly proved that the two given triangles are similar and correctly used proportionality to calculate the length of PN:

8.4 In ΔNML below, P and Q are points on the sides MN and LN respectively such that $QP \parallel LM$.

$MN = 16$ cm, $QP = 3$ cm and $LM = 8$ cm.



8.4.1 Complete the following (give reasons for the statements):

Prove with reasons that $\Delta QPN \parallel\!| \Delta LMN$.

In ΔQPN and ΔLMN

1. $\hat{N} = \hat{N}$ Common
2. $\hat{P}_1 = \hat{M}$ Corresponding angles
3. $\hat{Q}_1 = \hat{L}$ Corresponding angles

$\therefore$ $\Delta QPN \parallel\!| \Delta LMN$ (A.A.A)

8.4.2 Hence, calculate the length of PN.

$\frac{QP}{LM} = \frac{PN}{MN} = \frac{QN}{LN}$

$\frac{3cm}{8cm} = \frac{PN}{16cm}$

$= \frac{8cm}{48cm^2}$

$PN = 6cm$

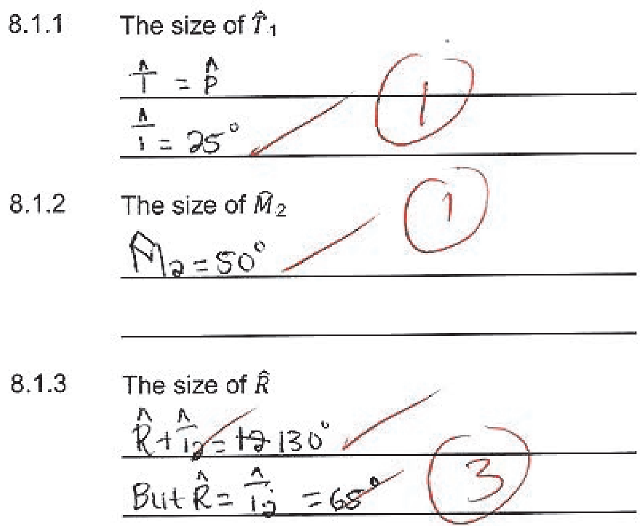
What would show evidence of partial understanding?

Item 1.9

- If the learner selected option B: this indicates the learner added up all the x values instead of using knowledge of the properties of the exterior angle of a triangle, i. e. that the exterior angle of a triangle is equal to the sum of the interior opposite angles;
- If the learner selected option C: this indicates the learner added up all the interior x values instead of using knowledge of the properties of the exterior angle of a triangle, .i.e. that the exterior angle of a triangle is equal to the sum of the interior opposite angles;
- If the learner selected option D: this shows the learner added up the adjacent x values instead of using knowledge of the properties of the exterior angle of a triangle, .i.e. that the exterior angle of a triangle is equal to the sum of the interior opposite angles.

Item 8.1

If the learner calculated the angles correctly without providing reasons:



8.1.1 The size of $\hat{T}_1$

$\hat{T} = \hat{P}$

$\hat{1} = 25°$ ✓ (1)

8.1.2 The size of $\hat{M}_2$

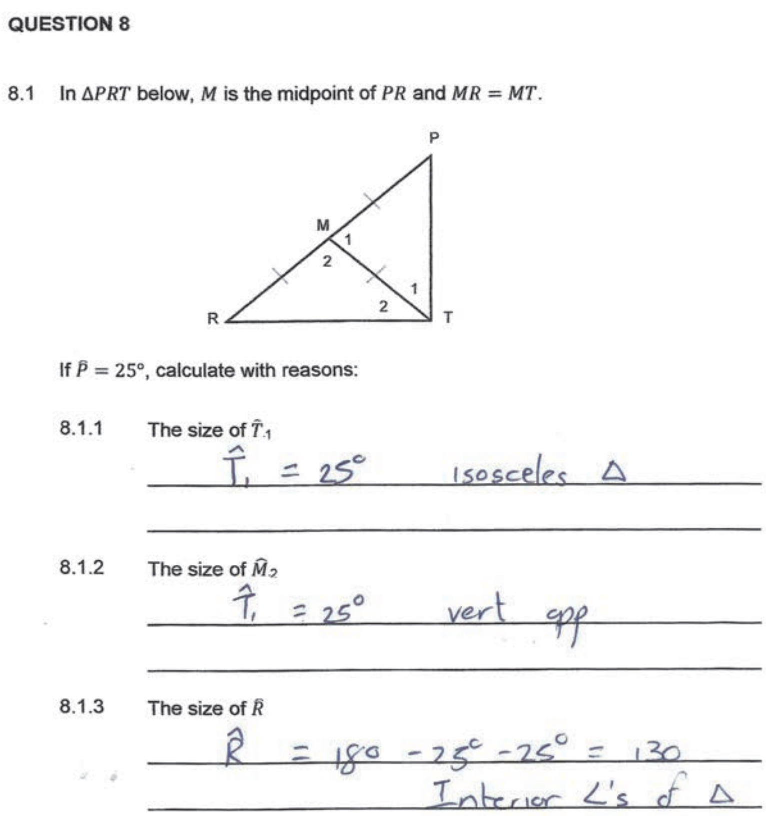
$\hat{M}_2 = 50°$ ✓ (1)

8.1.3 The size of $\hat{R}$

$\hat{R} + \hat{T}_2 = 130°$ ✓

But $\hat{R} = \hat{T}_2 = 65°$ ✓ (3)

- If the learner incorrectly calculated the measure of some angles;
- If the learner provided some reasons which were incorrect:



QUESTION 8

8.1 In ΔPRT below, M is the midpoint of PR and $MR = MT$.

If $\hat{P} = 25°$, calculate with reasons:

8.1.1 The size of $\hat{T}_1$

$\hat{T}_1 = 25°$ isosceles Δ

8.1.2 The size of $\hat{M}_2$

$\hat{T}_1 = 25°$ vert opp

8.1.3 The size of $\hat{R}$

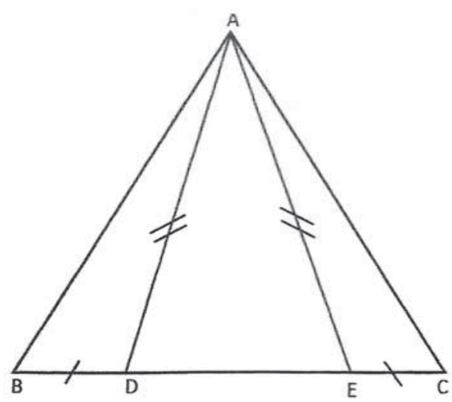
$\hat{R} = 180 - 25° - 25° = 130$

Interior ∠'s of Δ

Item 8.2

If the learner could not provide the reason BE is equal to CD, but correctly identified a triangle that is congruent to ΔABE:

8.2 In ΔABC, D and E are points on BC such that $BD = EC$ and $AD = AE$.



8.2.1 Why is $BE = CD$? Given

8.2.2 Which triangle is congruent to ΔABE ?

$\Delta ABE \equiv ADC$

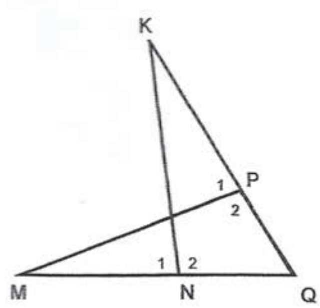
Item 8.3

If the learner provided correct statements, but did not provide reasons for the statements:

8.3 In the figure below ΔKNQ and ΔMPQ have a common vertex Q.

P is a point on KQ and N is a point on MQ.

$KQ = MQ$ and $PQ = QN$.



Prove with reasons that $\Delta KNQ \equiv \Delta MPQ$.

$\hat{Q} = \hat{Q}$ ✓

$MQ = KQ$ ✓

$QN = QP$ ✓

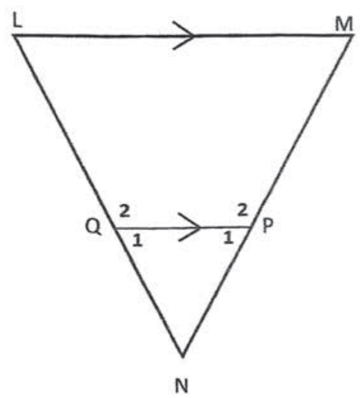
$\therefore \Delta KNQ \equiv \Delta MPQ$ ✓ (4)

Item 8.4

- If the learner correctly identified some of the equal parts, but failed to provide acceptable reasons for the equality;
- If the learner equated angles that are not alternate angles of parallel lines and so they are not equal sized angles:

8.4 In ΔNML below, P and Q are points on the sides MN and LN respectively such that $QP \parallel LM$.

$MN = 16$ cm, $QP = 3$ cm and $LM = 8$ cm.



8.4.1 Complete the following (give reasons for the statements):

Prove with reasons that $\Delta QPN \parallel\!\mid \Delta LMN$.

In ΔQPN and ΔLMN

1. $\hat{N} = \hat{N}$ ✓ Common vertex
2. $\hat{P}_1 = \hat{O}_2$ alternate ∠'s
3. $\hat{Q}_1 = \hat{P}_2$ alternate ∠'s

$\therefore$ $\Delta QPN \parallel\!\mid \Delta LMN$ ✓ (A, A, A)

- If the learner did not divide by the length of the side that corresponds to PN:

8.4.2 Hence, calculate the length of PN.

PN = PN × QP × LM

= 16 cm × 3 cm × 8 cm

PN = 384 cm² ✗

What would show evidence of no understanding?

Item 1.9

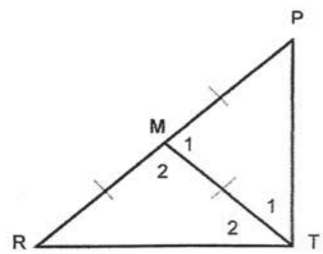
If the learner did not attempt the question.

Item 8.1

- If the learner calculated the angles incorrectly; or
- If the learner provided incorrect reasons:

QUESTION 8

8.1 In ΔPRT below, M is the midpoint of PR and $MR = MT$.



If $\hat{P} = 25°$, calculate with reasons:

8.1.1 The size of $\hat{T}_1$

$\hat{T}_1 = M_2$ (altanate angles) ✗

8.1.2 The size of $\hat{M}_2$

$M_2 = M_1$ (opposit to eact other) ✗

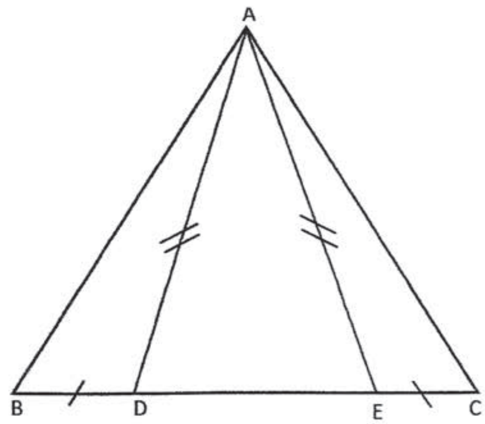
8.1.3 The size of $\hat{R}$

$\hat{R} = \hat{T}$, $\hat{R} = \hat{M}$, $\hat{R} = \hat{R}$ (s.s.s) ✗

Item 8.2

- If the learner could not provide a reason why $BE = CD$;
- If the learner could not identify a triangle that is congruent to ΔABE.

8.2 In ΔABC, D and E are points on BC such that $BD = EC$ and $AD = AE$.



8.2.1 Why is $BE = CD$? ~~Ge~~ Given ✗

8.2.2 Which triangle is congruent to ΔABE ?

~~AB~~ ADC ✗ ACD

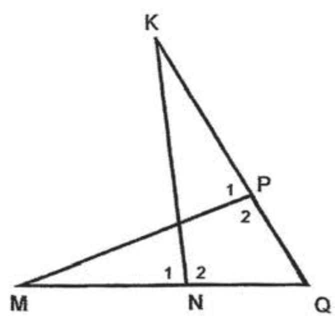
Item 8.3

If the learner did not show that equal parts should be identified to prove congruency:

8.3 In the figure below ΔKNQ and ΔMPQ have a common vertex Q.

P is a point on KQ and N is a point on MQ.

$KQ = MQ$ and $PQ = QN$.



Prove with reasons that $\Delta KNQ \equiv \Delta MPQ$.

(kq) is given ✗

(mq) given ✗

(bq) given ✗

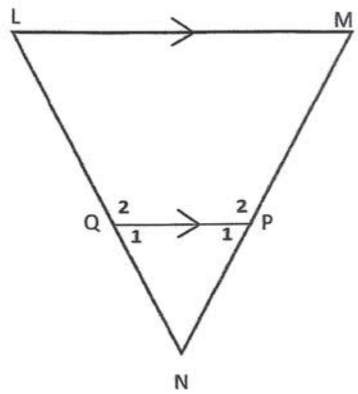
(qN) given ✗

Item 8.4

- If the learner was unable to write a proof for the similarity of triangles;
- If the learner was unable to use proportionality to determine the length of a side of a triangle:

8.4 In ΔNML below, P and Q are points on the sides MN and LN respectively such that $QP \parallel LM$.

$MN = 16$ cm, $QP = 3$ cm and $LM = 8$ cm.



8.4.1 Complete the following (give reasons for the statements):

Prove with reasons that $\Delta QPN \parallel\!| \Delta LMN$.

In ΔQPN and ΔLMN

1. $\hat{N} =$ 180° − 16 = 164 Alternate ∠'s
2. $\hat{P}_1 =$ 180° − 3 = 177 Corresponding ∠'s
3. $\hat{Q}_1 =$ 180 − 3 = 171 Corresponding ∠'s

∴ $\Delta QPN \parallel\!| \Delta$ LMN RHS

8.4.2 Hence, calculate the length of PN.

PN // QP

PN = 3 cm

180° − 3 cm

= 177°

8.4.2 Hence, calculate the length of PN.

PN = 16 cm − 5

= 11 cm

What do the item statistics tell us?

Item 1.9

21% of learners answered 1.9 correctly.

Factors contributing to the difficulty of the item

- Poor understanding of the concepts and skills tested in this item;
- Learners are unable to solve simple geometric problems involving unknown angles in triangles using known properties and definitions.

Item 8.1

8.1.1

11% of learners answered 8.1.1 correctly.

Factors contributing to the difficulty of the item

- Learners do not know the properties of isosceles triangles.

8.1.2

4% of learners answered 8.1.2 correctly.

Factors contributing to the difficulty of the item

- Learners cannot relate the exterior angle of a triangle to the interior angles.

8.1.3

2% of learners answered 8.1.3 correctly.

Factors contributing to the difficulty of the item

- Learners do not know the properties of isosceles triangles.
- They also do not know that the sum of the interior angles of a triangle is 180º.

Item 8.2

8.2.1

2% of learners answered 8.2.1 correctly.

Factors contributing to the difficulty of the item:

- Learners do not know how to prove that two lines are equal.

8.2.2

9% of learners answered 8.2.2 correctly.

Factors contributing to the difficulty of the item

- Learners cannot identify overlapping congruent triangles.

Item 8.3

11% of learners answered the question correctly.

Factors contributing to the difficulty of the item

- Learners do not know how to prove that two triangles are congruent.

Item 8.4

8.4.1

10% of learners answered 8.4.1 correctly.

Factors contributing to the difficulty of the item

- Learners do not know how to prove that two triangles are similar.

8.4.2

4% of learners answered 8.4.2 correctly.

Factors contributing to the difficulty of the item

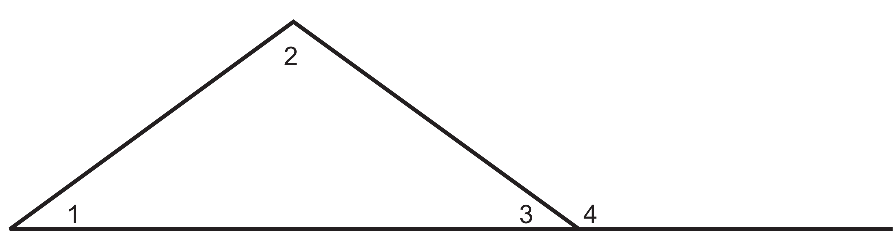
- Learners do not know that corresponding sides of similar triangles are proportional.

Teaching strategies

Using experiments and constructions to discover properties of triangles

Exploring exterior angle relationship to interior angles

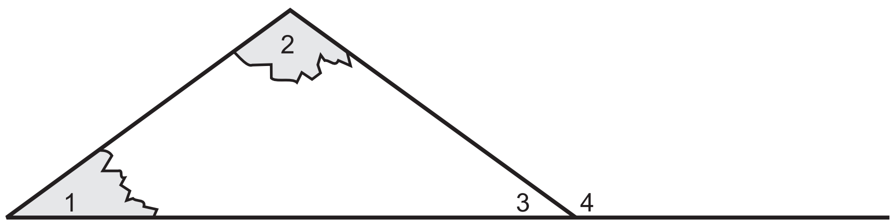
- Let learners sketch a triangle.
  - They should produce (extend) one side so that an exterior angle is formed
  - They should then number the angles 1 – 4.
  - They should all have a diagram that looks more OR less like the one shown.

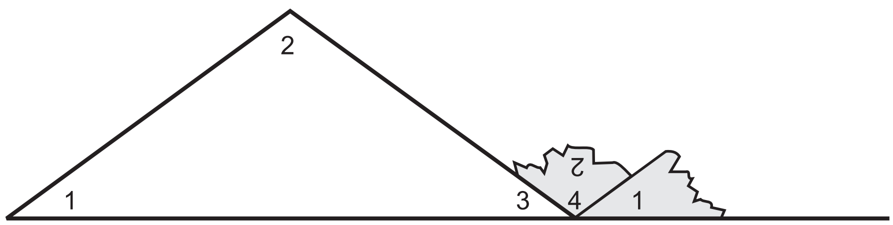


- Help the learners to identify:
  - Interior angles of the triangle (angles 1 and 2 on the diagram above);
  - The exterior angle (angle 4 on the diagram above);
  - The angle that is adjacent to the exterior angle (angle 3 on the diagram above);

- The angles that are not adjacent to the exterior angle (angles 1 and 2 on the diagram above are called the interior opposite angles).

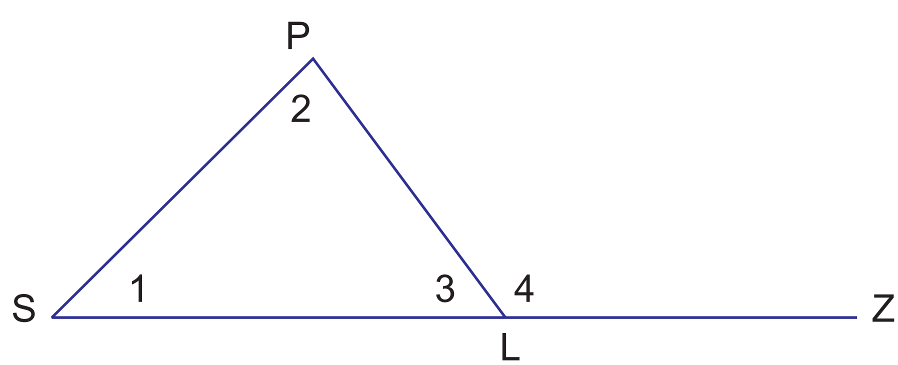
- The learners should then cut out angles 1 and 2 and paste them over angle 4 as shown in the figures.



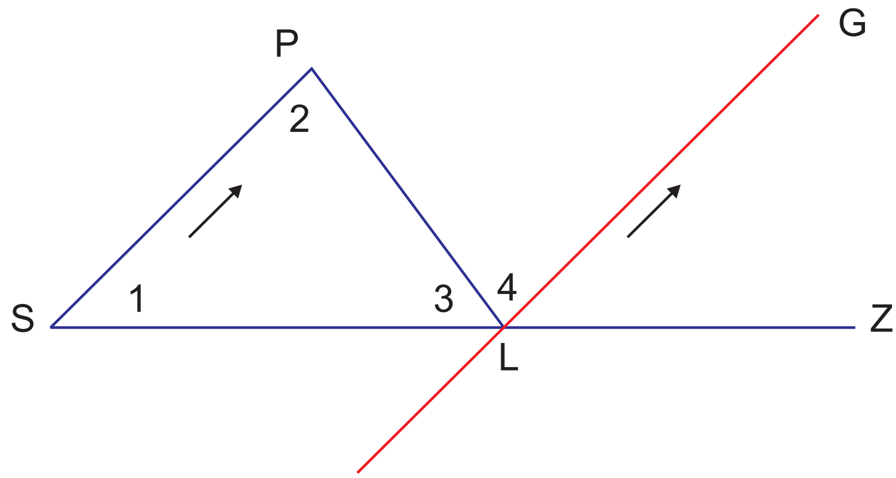


  - Learners should explain what they observe:
  - The observation is that the angles of a triangle add up to 180°. This can be seen because when you lay all three angles of a triangle on a straight line, they fit perfectly and make a straight angle – which is equal to 180°.
  - The other observation is that the exterior angle of a triangle is equal to the sum of the opposite interior angles of the triangle.
  - The experiment should be done more than once to consolidate these concepts, i.e. that the angles of a triangle add up to 180° and that the exterior angle of a triangle is equal to the sum of the interior opposite angles of the triangle.

- Let learners sketch a diagram like the one that follows.



- They should construct LG parallel to PS as shown.



- Help the learners identify equal angles.

Example

$\hat{S}_1 = G\hat{L}Z$ [Corresponding angles]

$\hat{P}_2 = G\hat{L}P$ [Alternate angles]

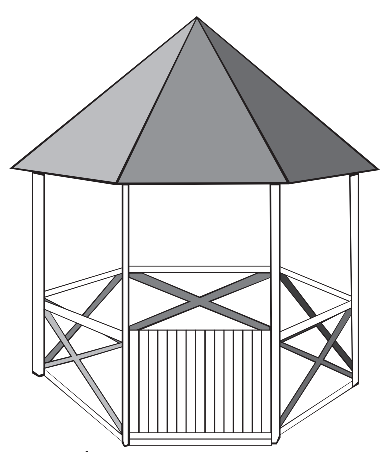
But $G\hat{L}Z + G\hat{L}P =$ Exterior angle of triangle PSL

$\therefore$ The exterior angle ($P\hat{L}Z$) = the sum of the interior opposite angles ($\hat{S}_1 + \hat{P}_2$).

Investigating the properties of an isosceles triangle

- Learners should be shown various structures with triangular parts.
- The type of triangle that they investigate in the following activity is an isosceles triangle. Learners will consolidate their knowledge about this triangle by working through the activity:
- Isosceles triangles have two sides and two angles of equal length.

Example



1). Describe the roof of the above gazebo.
   - It is made of isosceles triangles;
   - The base edge is along the edge of the roof;
   - The sides that join are all equal in length.

2). Sketch one section of the roof. Indicate equal parts. (The sketch will be of an isosceles triangle.)

3). Construct the following triangles:

a). ΔABC with $BC = 8\ cm$ and $\hat{B} = \hat{C} = 55°$.
- Measure the lengths of AB and AC.
- Comment on how the sides compare.
- Discuss: Sides AB and AC are equal in length.

b). ΔPTY with $TY = 6{,}5\ cm$ and $\hat{T} = \hat{Y} = 48°$.
- Measure the lengths of PT and PY.
- Comment on how the sides compare.
- Discuss: Sides AB and AC are equal in length.

c). ΔBMX with $MX = 7\ cm$ and $BM = BX = 5\ cm$.
- Measure the interior angles M and X.
- Comment on how the angles compare.
- Discuss: Angles M and X are equal in size.

d). ΔKFC with $FC = 6{,}5\ cm$ and $KF = KC = 8\ cm$.
- Measure the interior angles F and C.
- Comment on how the angles compare.
- Discuss: Angles F and C are equal in size.

Using measurements to discover axioms and theorems

- Learners could be given the following worksheets to complete in small groups.
- These activities consolidate the knowledge that the exterior angle of a triangle is equal to the sum of the interior opposite angles of the triangle.

Worksheet 1

In each figure, measure the angles marked 1, 2 and 4. Use the measurements to complete the table that follows.

Figure 1

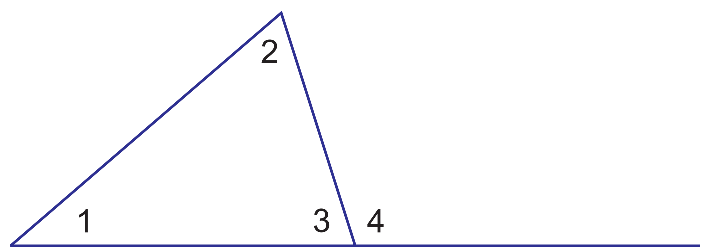


Figure 2

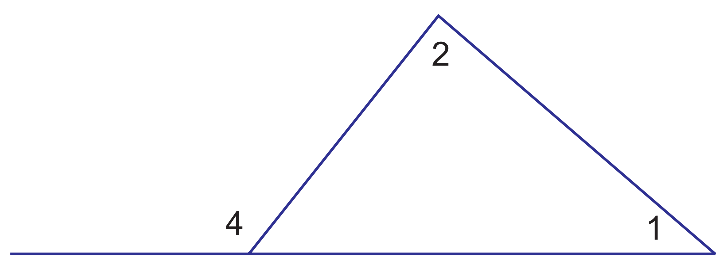


Figure 3

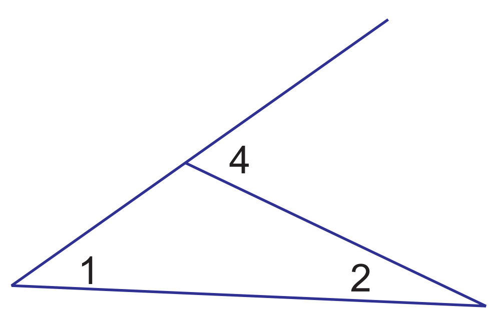


Figure	Angle 1	Angle 2	Angle 4	Angle 1 + Angle 2
1				
2				
3				

- Learners should compare the size of angle 4 to the sum of angle 1 and angle 2.

Worksheet 2

In each figure, measure BI, BN, interior angle I and interior angle N. Use the measurements to complete the table below.

Figure 1

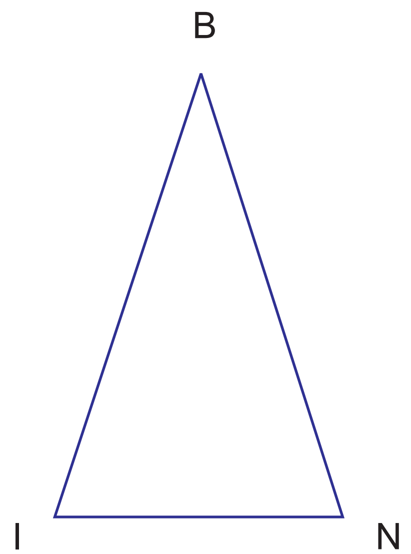


Figure 2

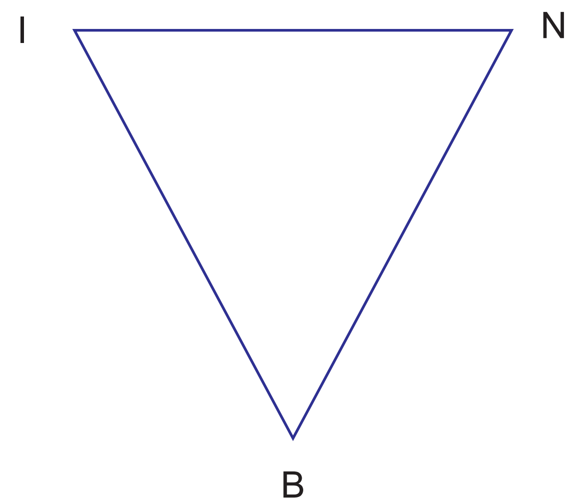


Figure 3

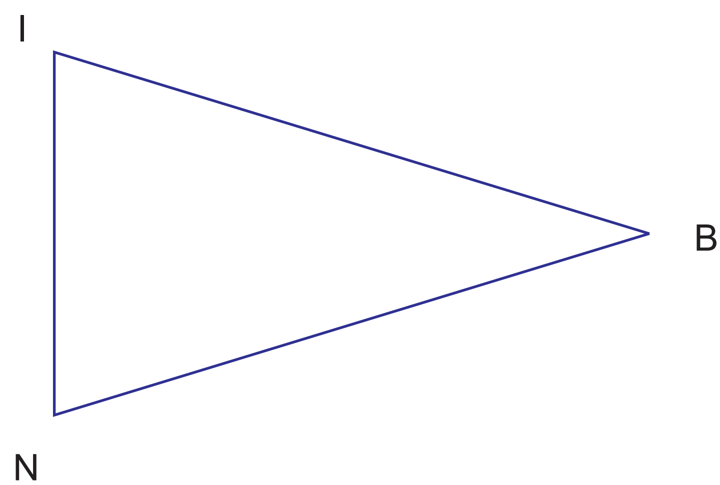
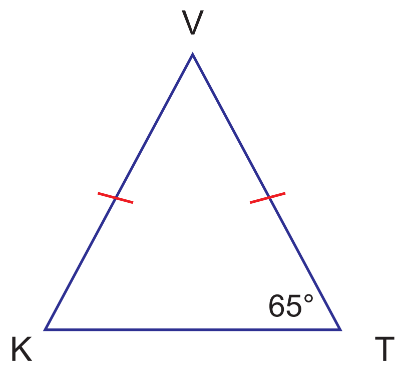


Figure	*BI*	*BN*	Angle *I*	Angle *N*

- Learners should compare BI and BN.
- They should compare angle I and angle N.
- They should describe equal sides OR equal angles.
- Learners should then be given practice exercises on application of the knowledge learnt above.

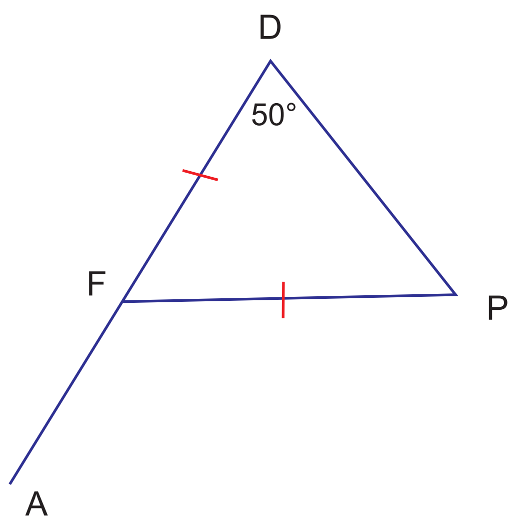
Examples

1). In ΔVKT, $VK = VT$ and $\hat{T} = 65°$.



Calculate the size of $\hat{K}$ and $\hat{V}$ and give reasons.

2). In ΔPDF, DF is produced to A. $DF = PF$ and $\hat{D} = 50°$.



Calculate the size of $P\hat{F}A$ and give reasons.

Solutions

1). $\hat{K} = \hat{T}$

Reason: ΔVKT is an isosceles triangle and $\hat{K}$ and $\hat{T}$ are opposite equal sides

Therefore $\hat{K} = 65°$

2). $\hat{P} = \hat{D}$

Reason: ΔPDF is an isosceles triangle and $\hat{P}$ and $\hat{D}$ are opposite equal sides

Therefore, $\hat{P} = 50°$

But $P\hat{F}A = \hat{P} + \hat{D}$

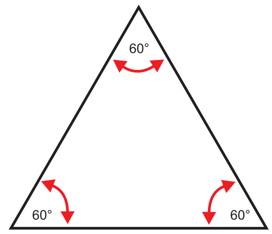
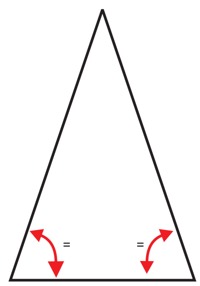
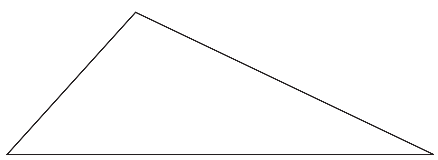
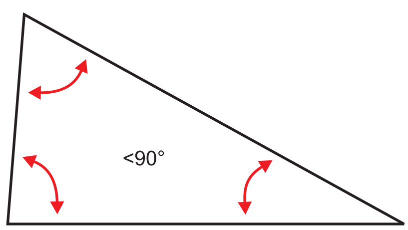
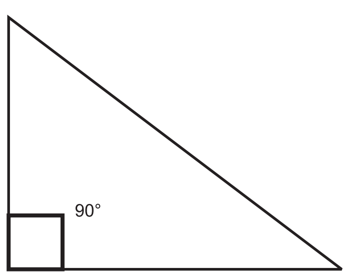
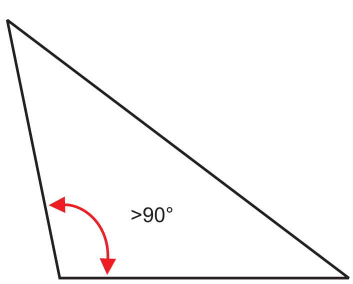
Reason: Exterior angle of a triangle = to the sum of the two interior opposite angles

Therefore $P\hat{F}A = 100°$

Consolidating properties of triangles

- Learners will benefit from sorting and comparing the different types of triangles, for example: equilateral; isosceles; scalene; and right angled triangles.
- Provide learners with concrete manipulatives OR drawings of the four types of triangles.
- The following worksheet may be used.
- Discuss the properties of each type of triangle with learners using the worksheet to assist you.

Notes:

	**Equilateral triangle** **Three** equal sides **Three** equal angles, always 60°
	**Isosceles triangle** **Two** equal sides **Two** equal angles
	**Scalene triangle** **No** equal sides **No** equal angles
	**Acute triangle** All angles are less than 90°
	**Right angled triangle** Has a right angle (90°)
	**Obtuse triangle** Has an angle more than 90°

Working with properties of triangles

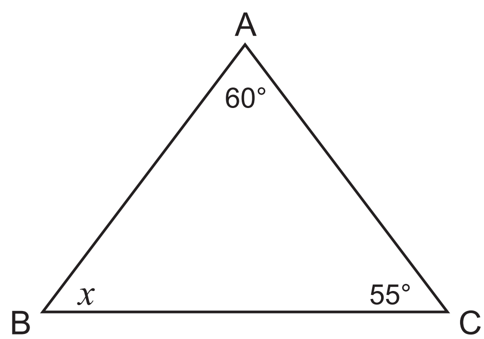
- Consolidate learners' knowledge by asking questions regarding the sum of the angles of a triangle.
- Allow learners to work on examples focusing on solving for unknown interior angles in a triangle.
- Once learners are confident with working on examples, allow learners to engage with an activity. Examples of activities that you could use follow.

Examples

Solving for interior angles of a triangle

Calculate the value of the unknown angles in the following examples:

1).



Solution

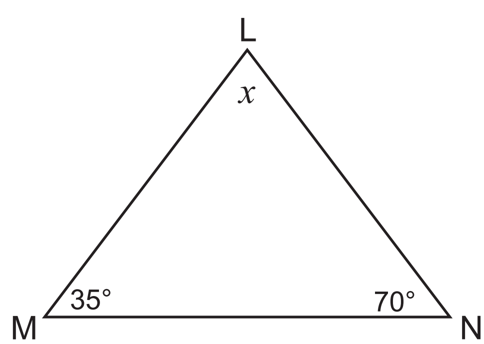
Sum of angles in a triangle $= 180°$

$x = 180° - (60° + 55°)$

$x = 180° - 115° = 65°$

$\hat{B} = 65°$

2).



Solution

Sum of angles of a triangle $= 180°$

$x = 180° - (70° + 35°)$

$x = 180° - 105° = 75°$

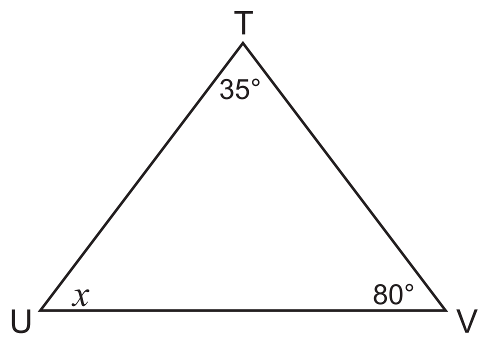
$\hat{L} = 75°$

Solving for interior angles of a triangle

The following activity sheet may be used to assist learners to consolidate their knowledge of working with interior angles of triangles.

Solve for the unknown angles in each of the following:

1).



Solution

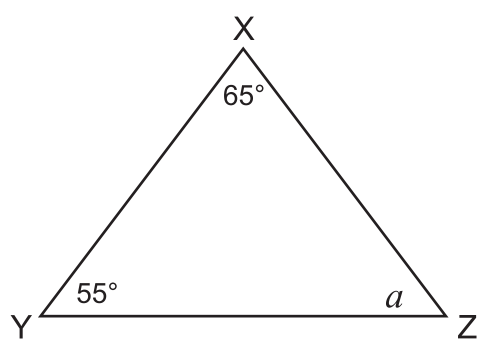
Sum of angles of a triangle $= 180°$

$x = 180° - (80° + 35°)$

$x = 180° - 115° = 65°$

$\hat{U} = 65°$

2).



Solution

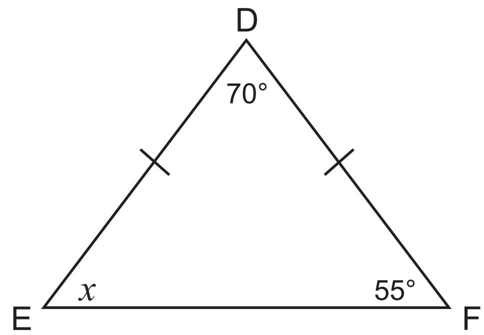
Sum of angles of a triangle $= 180°$

$a = 180° - (55° + 65°)$

$a = 180° - 120° = 60°$

$\hat{Z} = 60°$

3).



Solution

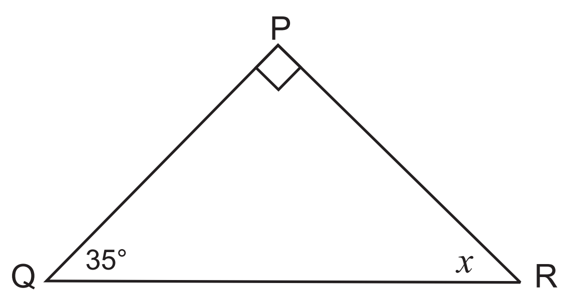
Sum of angles of a triangle $= 180°$

$x = 180° - (70° + 55°)$

$x = 180° - 125° = 55°$

$\hat{E} = 55°$

4).



Solution

Sum of angles of a triangle $= 180°$

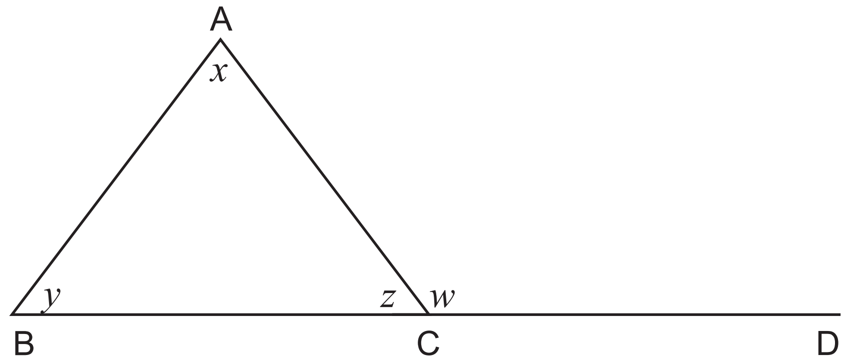
$x = 180° - (90° + 35°)$

$x = 180° - 125° = 55°$

$\hat{R} = 55°$

Working with exterior and interior angles of triangles

- Through demonstration and discussion explain exterior and interior angles of triangles and the relationship between them: The exterior angle of a triangle is equal to the sum of the interior opposite angles.



$\hat{x}, \hat{y}$ and $\hat{z}$ are interior angles of ΔABC and $\hat{w}$ = exterior angle of ΔABC

$\hat{w} = \hat{x} + \hat{y}$

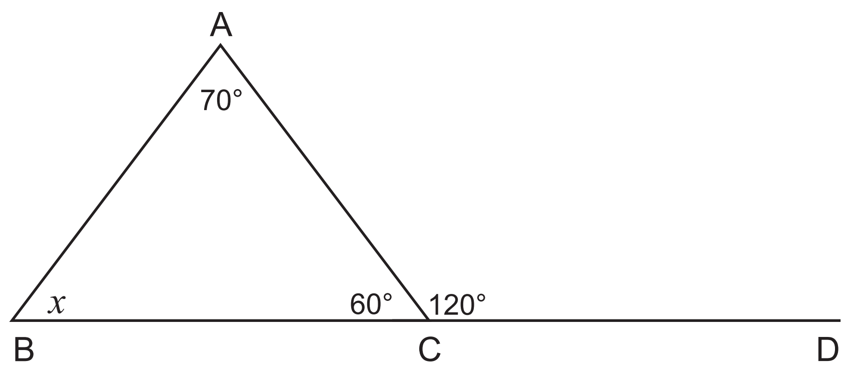
Use the following examples to assist you.

Examples

Solving for exterior angles of triangles

Solve for the unknown angle:

1).



Solution

Exterior angle of triangle = sum of interior opposite angles

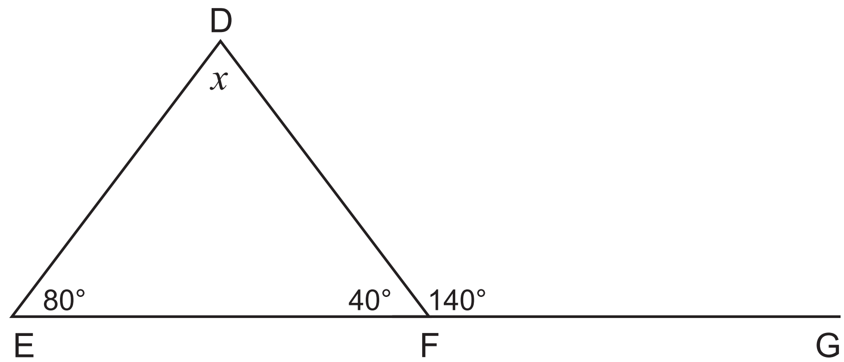
$120° = (70° + x)$

$120° - 70° = x$

$x = 50°$

$\hat{B} = 50°$

2).



Solution

Exterior angle of triangle = sum of interior opposite angles

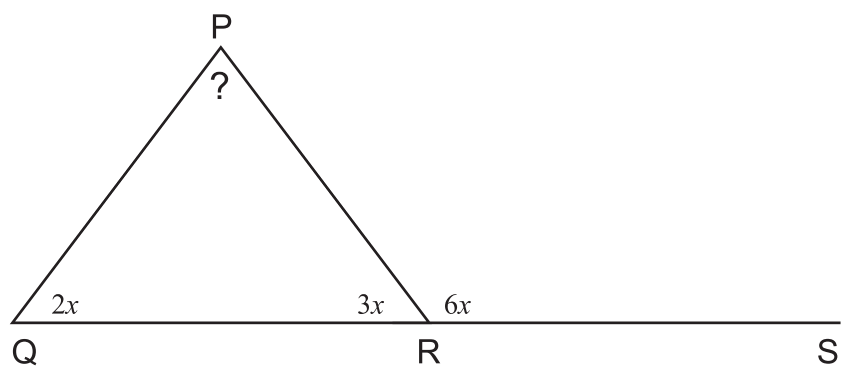
$140° = (80° + x)$

$140° - 80° = x$

$x = 60°$

$\hat{D} = 60°$

3).



Solution

Exterior angle of triangle = sum of interior opposite angles

$6x = (2x + ?)$

$6x - 2x = ?$

$? = 4x$

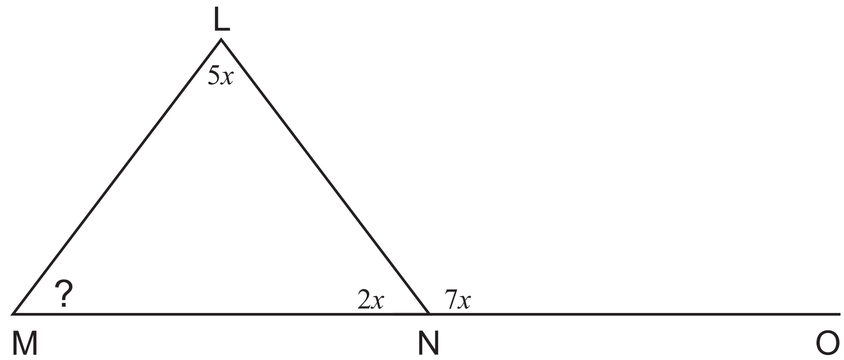
$\hat{P} = 4x$

- Once learners are confident with working on examples, allow learners to engage with an activity.
- The following activity may be used to assist learners to consolidate their knowledge of working with exterior angles of triangles.

Activity Sheet: Working with exterior and interior angles of triangles

Solve for the unknown angle:

1).



Solution

Exterior angle of triangle = sum of interior opposite angles

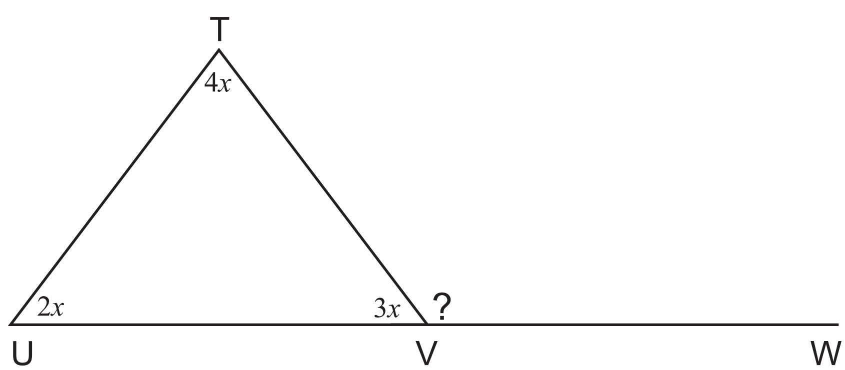
$7x = (5x + ?)$

$7x - 5x = ?$

$? = 2x$

$\widehat{M} = 2x$

2).



Solution

Exterior angle of triangle = sum of interior opposite angles

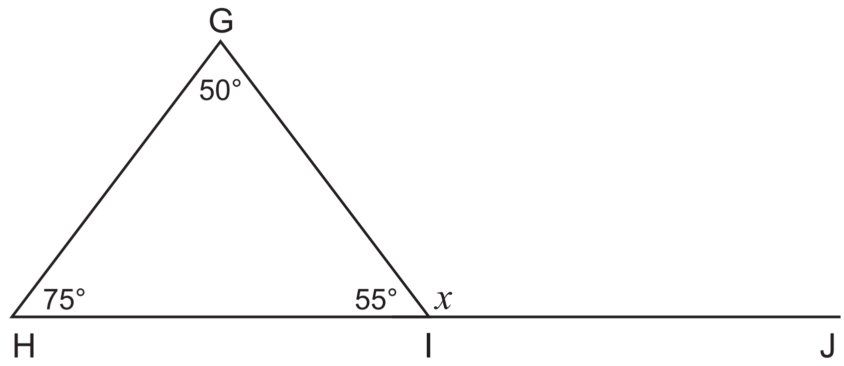
$6x = (2x + ?)$

$? = (4x + 2x)$

$? = 6x$

$T\hat{V}W = 6x$

3).



Solution

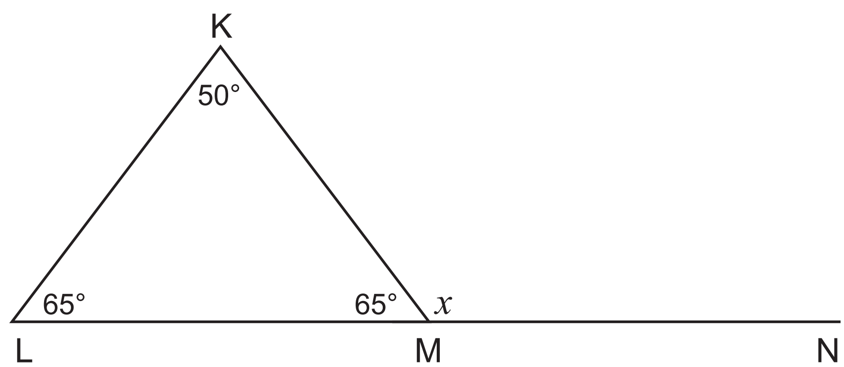
Exterior angle of triangle = sum of interior opposite angles

$x = (50° + 75°)$

$x = 125°$

$G\hat{I}J = 125°$

4).



Solution

Exterior angle of triangle = sum of interior opposite angles

$x = (50° + 65°)$

$x = 115°$

$K\widehat{M}N = 115°$

Establishing and investigating similarity

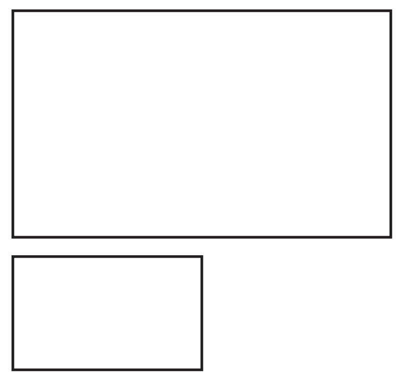
- Through demonstration, help learners to define similarity.
- Point out that to show that two polygons are similar we need to remember that:
  - All pairs of corresponding angles are equal; OR
  - All pairs of corresponding sides are in the same proportion.
  - The symbol used to show similarity is III
  - Use the following printable resource to assist with establishing minimum conditions for

similarity. This means that you will need to define similarity by using different examples. Use the following examples to assist you.

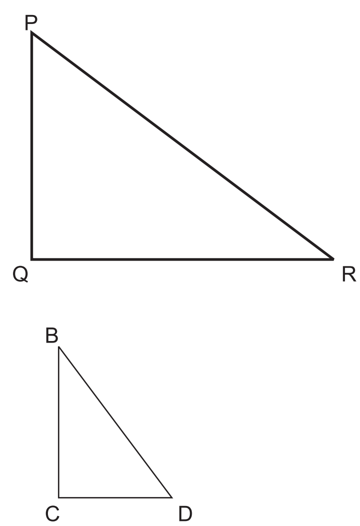
Examples: Similarity

Examine the polygons below. Are they similar? Explain your answer. You may use a ruler and protractor to assist you.

1).



2).



- For both 1 and 2 the learners may use protractors and rulers; by accurate measurement they will be able to deduce which pairs of shapes are similar.

Solution

1). The two rectangles are similar.
   - The two pairs of corresponding sides are in the same proportion. (AND/OR the pairs of corresponding angles are equal).

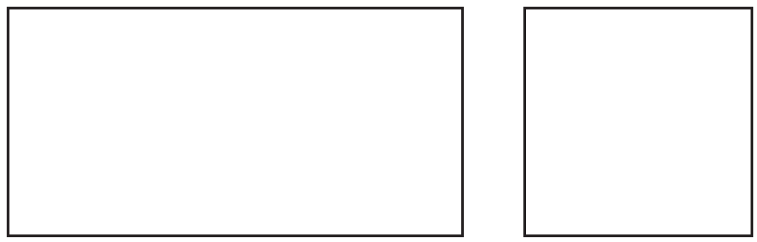
2). The two triangles are NOT similar.
   - The two pairs of corresponding sides are NOT in the same proportion. (AND/OR the pairs of corresponding angles are NOT equal).

Provide an activity to assist learners to consolidate what was learnt in this lesson/s. The following activity sheet may be used.

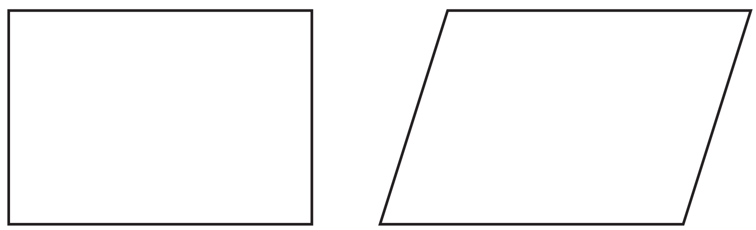
Activity sheet: Similarity

1). Are the following pairs of figures similar? Explain.

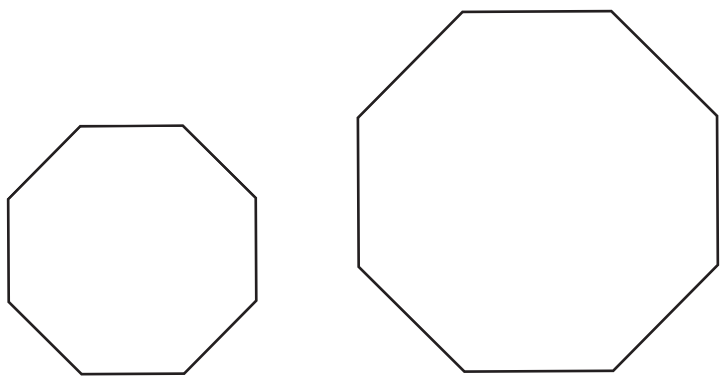
A.



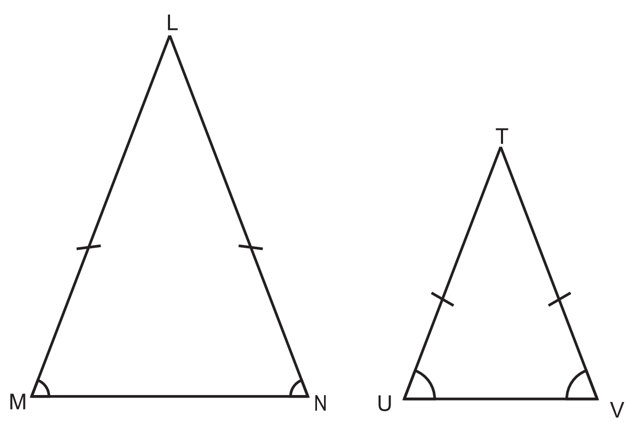
B.



C.



2). The following triangles are not drawn to scale.

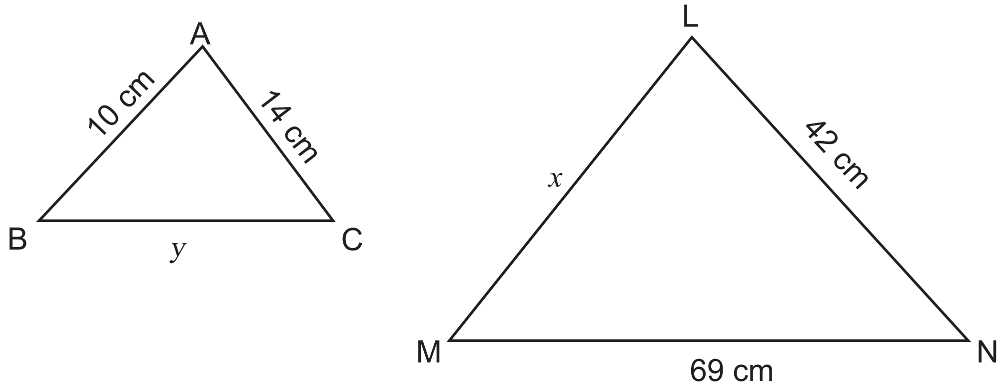


If in Δ LMN: $LM = 21\ cm$, $LN\ 21\ cm$, $MN = 15\ cm$, angle $L = 50$ degrees, angle $M = 65$ degrees, angle $N = 65$ degrees
and in Δ TUV: $TU = 7\ cm$, $TV = 7\ cm$, $UV = 5\ cm$, angle $T = 50$ degrees, angle $U = 65$ degrees, angle $V = 65$ degrees, is Δ LMN III to Δ TUV?

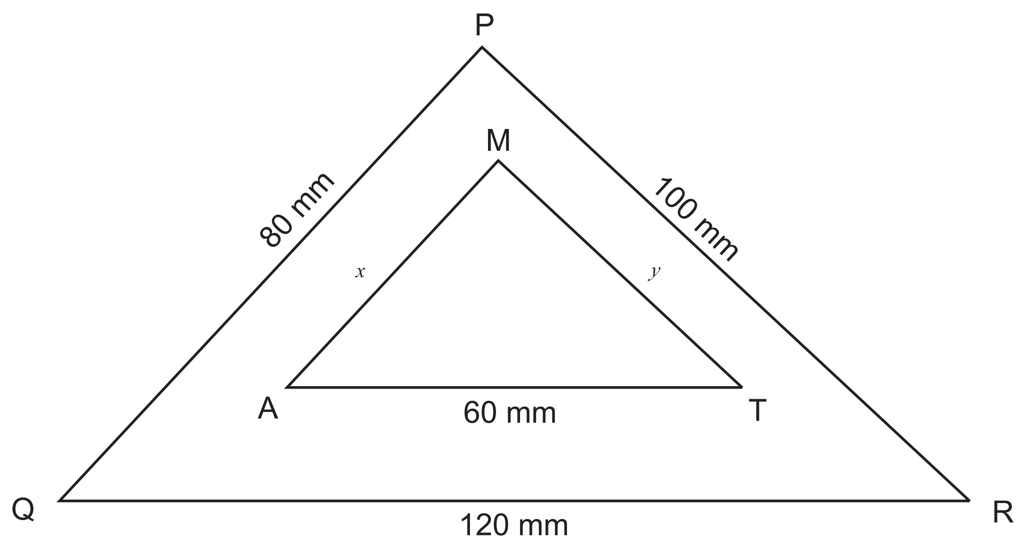
Explain.

3). Find the lengths of the unknown sides in the following pairs of similar triangles:

A.



B.



Solutions

1). In figures A and C the triangles are similar as all the corresponding sides in both pairs of triangles are in the same proportion.

In figure B the triangles are not similar as the corresponding sides in the pair of triangles are not in the same proportion.

2). The triangles in figure D are similar because the corresponding angles in each of the triangles are equal and all pairs of corresponding sides are in the same proportion.

3). In pair A both triangles are similar; therefore all pairs of corresponding sides are in the same proportion $x = 30\,cm\,(10\,cm \times 3)$.

In pair B both triangles are similar, therefore all pairs of corresponding sides are in the same proportion $x = 40\,cm\,(80\,cm \div 2)$.and $y = 50\,cm\,(100\,cm \div 2)$.

Establishing and investigating congruency

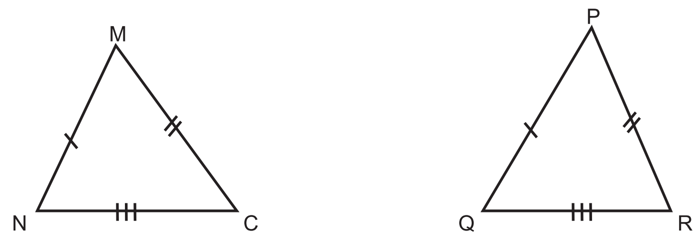
- Through demonstration help learners to define congruency.
- Remember the four conditions for any two triangles to be congruent are shown by the abbreviations: SSS (side, side, side), SAS (side, angle, side), AAS (angle, angle, side) and RHS (right angle/90°, hypotenuse, side).
- The symbol used to show congruency is $\equiv$.

- Use the following printable resource to assist with establishing the minimum conditions for congruency. This means that you will need to define congruency by using different examples. Use the following examples to assist you.

Notes:

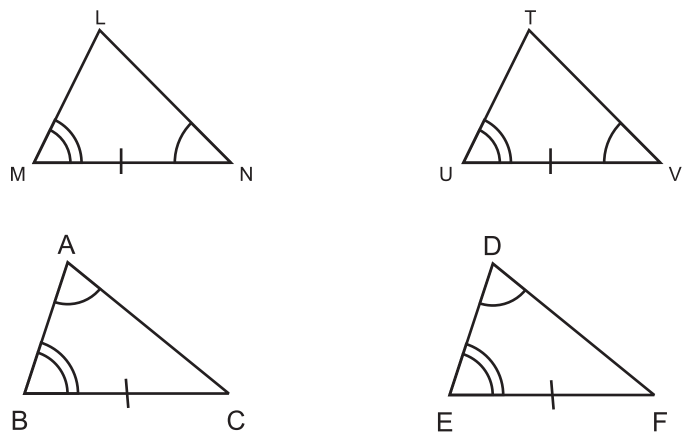
Cases of congruency

1). Three sides (SSS)



$\Delta MNC \equiv \Delta PQR \quad (SSS)$

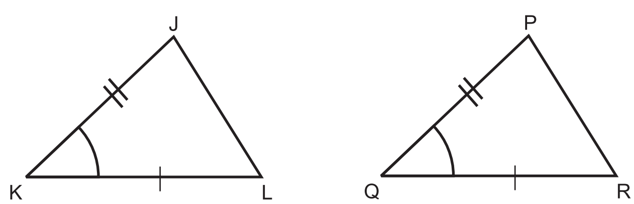
2). Two angles and one side (AAS OR ASA)



$\Delta LMN \equiv \Delta TUV \ (ASA)$

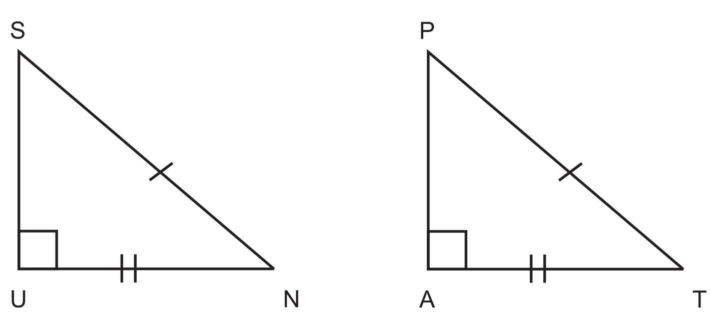
$\Delta ABC \equiv \Delta DEF \ (AAS)$

3). Two sides and one angle (SAS)



$\Delta JKL \equiv \Delta PQR \ (SAS)$

4). Right angle hypotenuse, side (RHS) OR ($90° \ HS$)

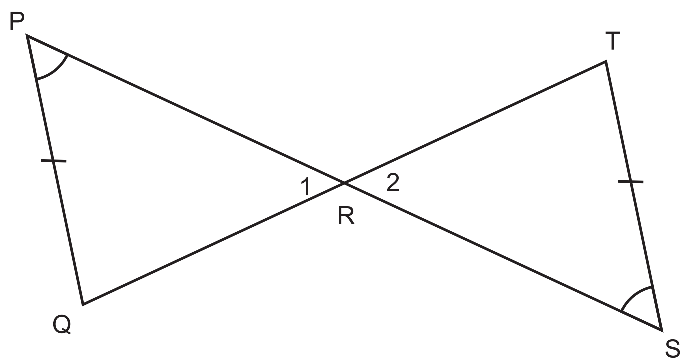


$\Delta SUN \equiv \Delta PAT$ ($90°$ HS or RHS)

- Provide examples and an activity to assist learners to consolidate what was learnt in this lesson/s. The following examples and activity may be used.

Examples

1). Prove that $\Delta PQR \equiv \Delta STR$

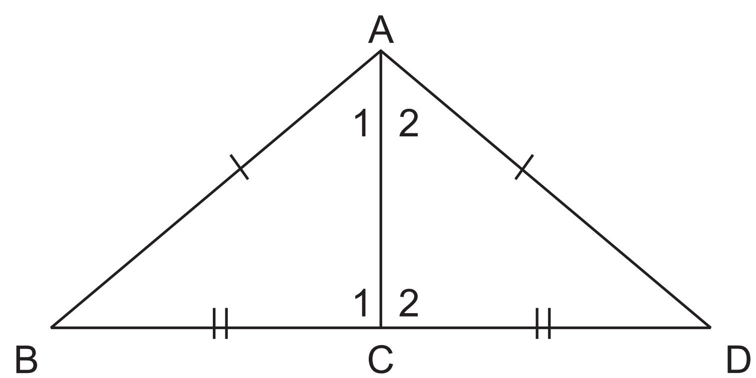


Proof: In ΔPQR and ΔSTR

$\hat{P} = \hat{S}$ [given]

$\therefore \Delta PQR \equiv \Delta STR$

2). Prove that $\Delta ABC \equiv \Delta ADC$



Proof: In ΔABC and ΔADC

$AB = AD$ [given]

$BC = CD$ [given]

$AC = AC$ [common]

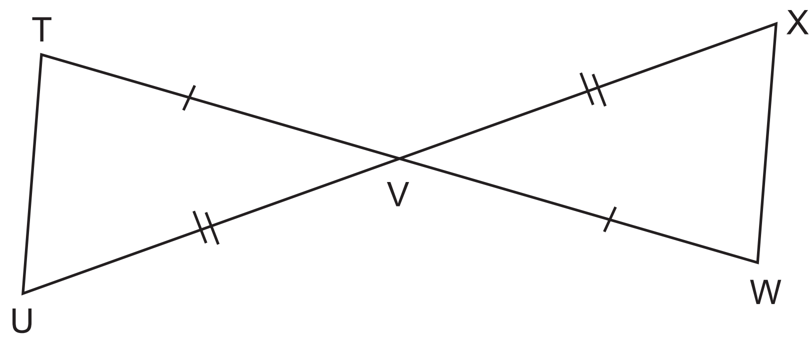
$\therefore \Delta ABC \equiv \Delta ADC$

Activity: Congruency

1). Using the diagram that follows:

a). Prove that $\Delta TUV \equiv \Delta WXY$.

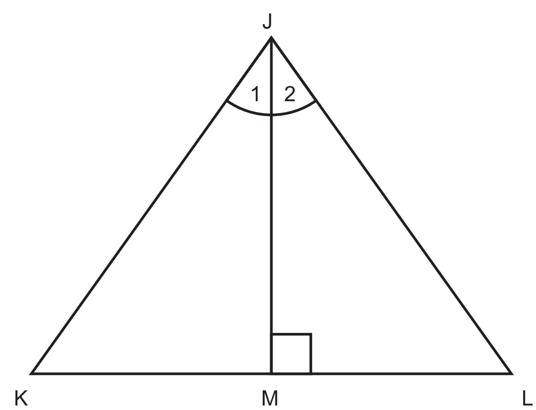
b). State all pairs of corresponding angles and sides that are equal.



2). In $\Delta JKL, JM$ bisects $\angle JKL, JM \perp KL$

a). Show that $\Delta JKM \equiv \Delta JLM$.

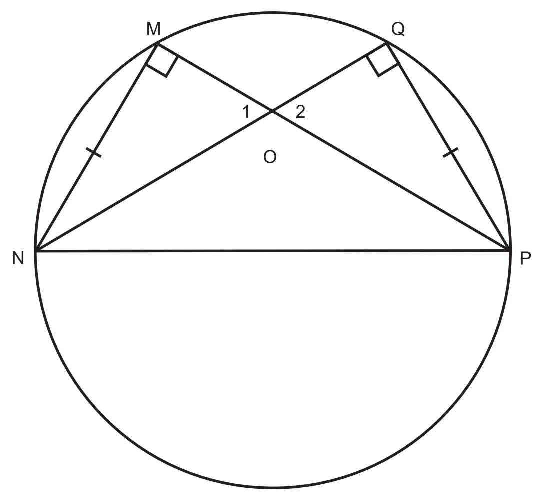
b). Calculate the value of KL if $JM = 16\,cm$ and $JL = 20\,cm$.



3). In the figure that follows $N\hat{M}P = N\hat{Q}P = 90°$ and $MN = QP$. Show that:

a). $\Delta NPQ \equiv \Delta PNM$.

b). $MP = QN$.



Solutions

1). a). Proof: In ΔTUV and ΔWXV

$\angle TVU = \angle WVX$ [vertically opposite angles]

$TV = WV$ [given]

$XV = UV$ [given]

$\therefore \Delta TUV \equiv \Delta WXV$ [S, A, S]

b). $TV = WV$

$TU = WX$

$\hat{T} = \widehat{W}$

$\hat{U} = \hat{X}$

2). a). $\hat{J}_1 = \hat{J}_2$ [$\hat{J}$ is bisected by JM]

JM is common

$\widehat{M}_1 = \widehat{M}_2 = 90°$ [given]

$\therefore \Delta JKM \equiv \Delta JLM$ [90 °, H, S or R, H, S]

b). Using the theorem of Pythagoras:

$ML^2 = JL^2 - JM^2$

$ML^2 = 20^2 - 16^2$

$ML^2 = 400 - 256$

$ML^2 = 144$

$ML = 12\ cm$

Since $KM = ML$ [$\Delta JKM \equiv \Delta JLM$]

$KL = ML + KM$

$KM = 24\ cm$

3). a). In ΔNPQ and ΔPNM

$NP = NP$ [common]

$NM = PQ$ [given]

$\widehat{M} = \hat{Q}$ [given]

$\therefore \Delta NPQ \equiv \Delta PNM$ [S, A, S]

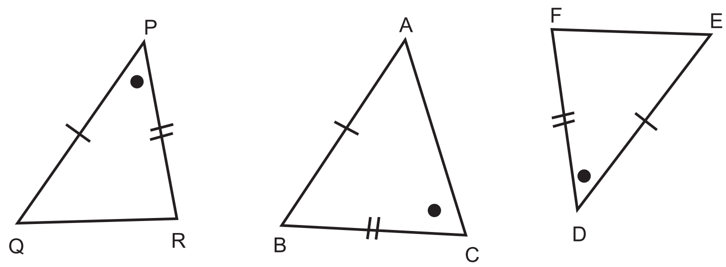
b). Since $\Delta NPQ \equiv \Delta PNM$

$\therefore MP = QN$ [remaining side]

Other examples of how to test geometry of 2-D shapes: Properties of triangles

ANA 2014 Grade 9 Mathematics Item 10.1

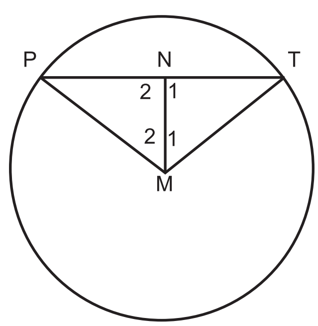
10.1



Which triangle is congruent to ΔPQR? [2]

ANA 2014 Grade 9 Mathematics Item 10.2

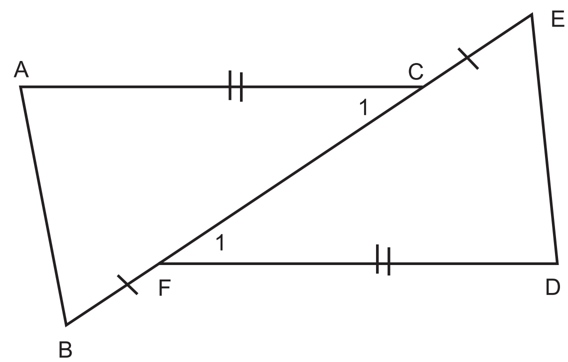
10.2 In the given figure, P and T are points on a circle with centre M. N is a point on a chord PT such that $MN \perp PT$.



Prove that $PN = NT$. [8]

ANA 2014 Grade 9 Mathematics Item 10.3.2

10.3

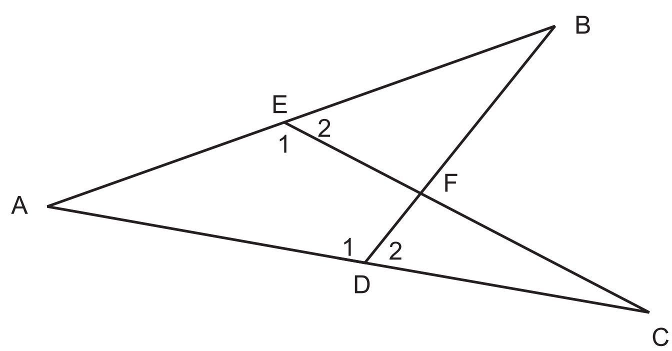


In the above diagram, $AC = DF$, $AB = DE$ and $BF = CE$.

10.3.2 Prove that $\Delta ABC \equiv \Delta DEF$. [5]

ANA 2014 Grade 9 Mathematics Items 10.4.1 and 10.4.2

10.4



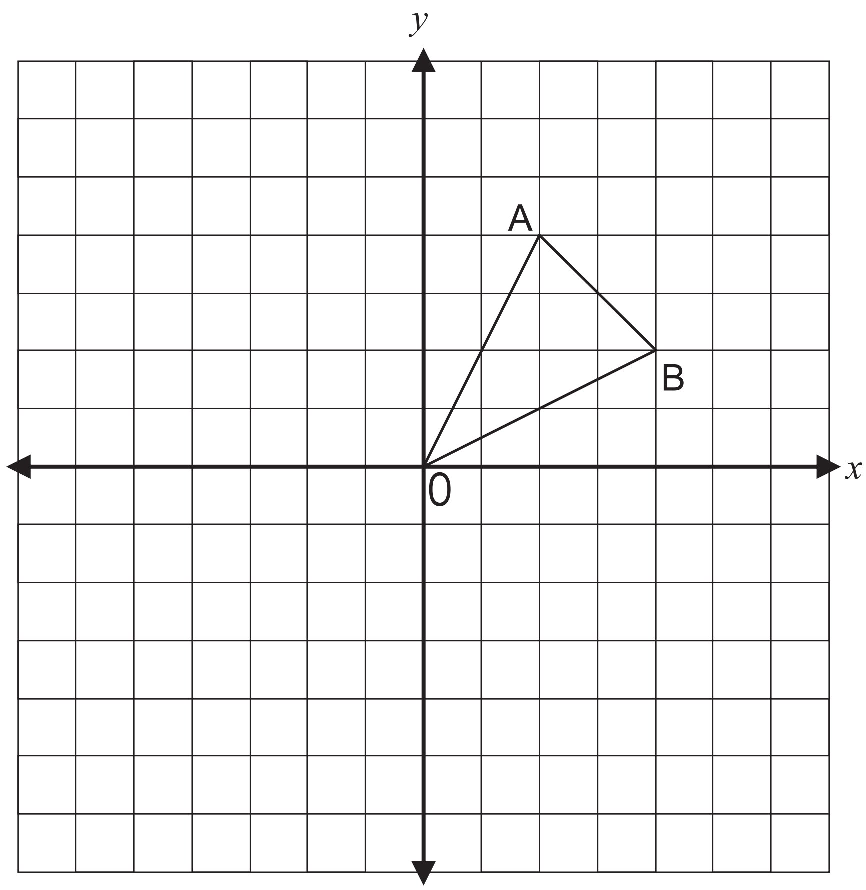
In the figure, $\hat{B} = \hat{C}$, $AD = 9\ cm$, $AE = 7\ cm$ and $CE = 21\ cm$.

10.4.1 Prove that $\Delta ABD \,|||\, \Delta ACE$. [6]

10.4.2 Calculate the length of BD. [5]

Transformations

ANA 2013 Grade 9 Mathematics Items 9.1, 9.2, 9.3 and 9.4



9.1. Use the given grid to draw $\Delta A'OB'$, the reflection of ΔAOB in the x-axis. [2]
9.2. Write down the coordinates of B', the image of B. [2]
9.3. On the same grid, draw the rotation of ΔAOB through $180°$ about the origin to form $\Delta A''OB''$. [2]
9.4. Hence, determine the length of $A'A''$. [2]

What should a learner know to answer these questions correctly?

Learners should be able to:

- Recognise, describe and perform transformations with points, line segments and simple geometric shapes in a co-ordinate plane;
- Reflect points or shapes on the x-axis or y-axis;
- Translate points or shapes within and across quadrants;
- Reflect points or shapes on the line $y = x$;
- Identify what the transformation of a point is if given the co-ordinates of its image.

Where is this topic located in the curriculum? Grade 9 Term 4

Content area: Space and shape (Geometry).

Topic: Transformation geometry.

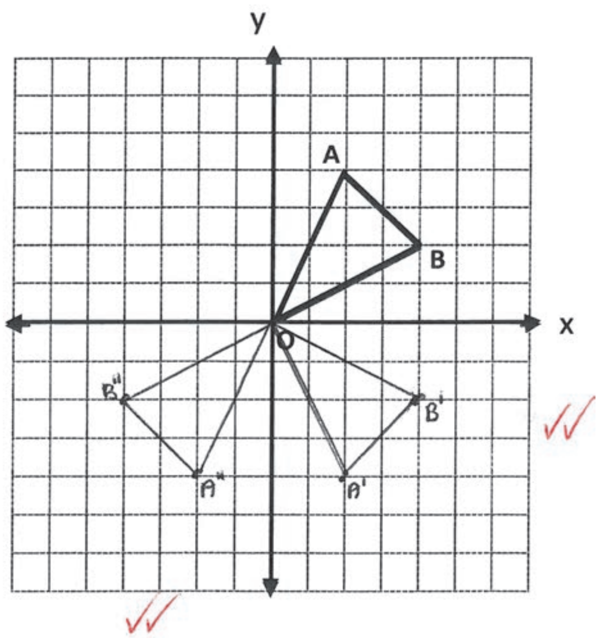
Concepts and skills:

- Recognise, describe and perform transformations with points, line segments and simple geometric figures on a co-ordinate plane focusing on:
  - Reflection on the x-axis or y-axis;
  - Translation within and across quadrants;
  - Reflection on the line $y = x$.
  - Rotation about the origin.
- Identify what the transformation of a point is if given the co-ordinates of its image.

What would show evidence of full understanding?

If the learner obtained the correct solution by sketching the appropriate figure, applying knowledge of coordinate geometry correctly or using the appropriate mathematical strategy.

Item 9.1 and 9.3



Item 9.2

9.2 Write down the coordinates of B', the image of B.

$B' = (4;-2)$ ✓

Item 9.4

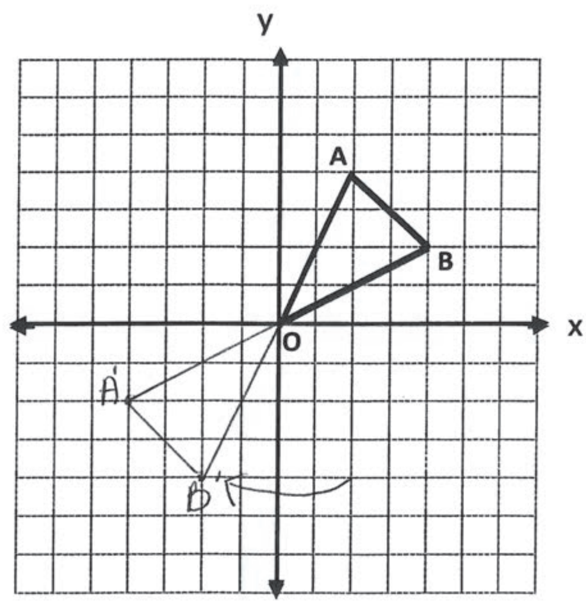
9.4 Hence, determine the length of $A'A''$.

$A'A''$ = 4cm or 4 blocks.

What would show evidence of partial understanding?

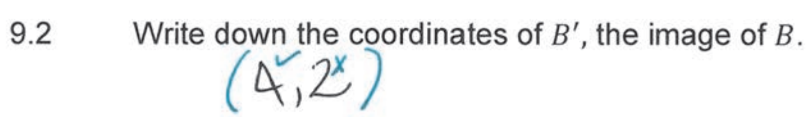
Item 9.1

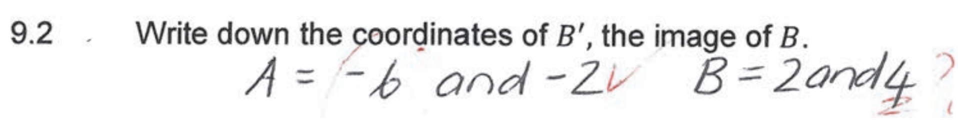
If the learner drew the reflection of ΔAOB on the coordinate plane, but not in the correct position: the reflection in this example is in the line $y = x$ rather than in the x-axis.



Item 9.2

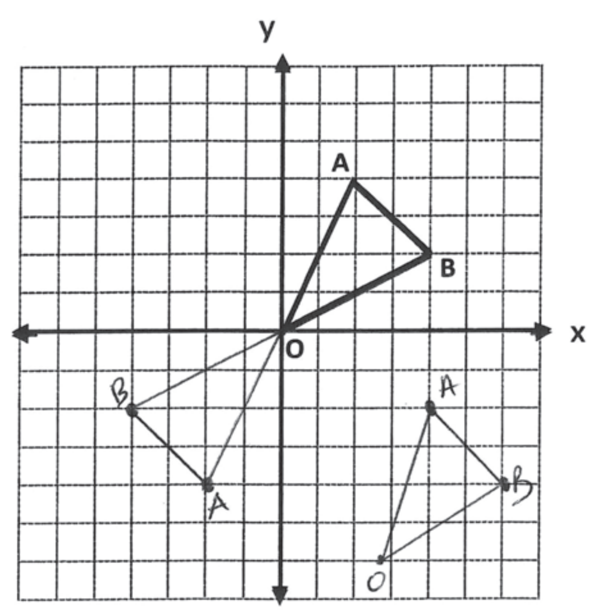
If the learner wrote down only one coordinate or gave a pair of coordinates in which only one of the coordinates was correct.



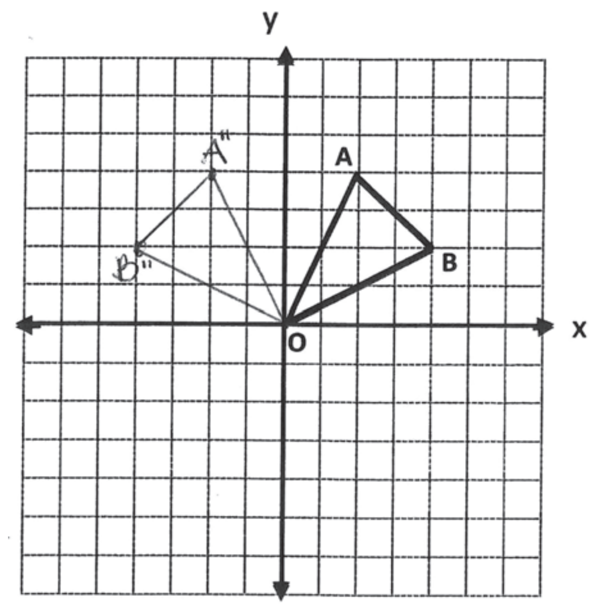


Item 9.3

- If the learner drew the rotation of ΔAOB on the coordinate plane, but did not label it according to the question specifications: the triangle meeting the original triangle in the sketch below at O is a correct rotation, but it is not correctly labelled;



- If the learner drew a reflection in the y-axis, but labelled it as the rotated figure.



Item 9.4

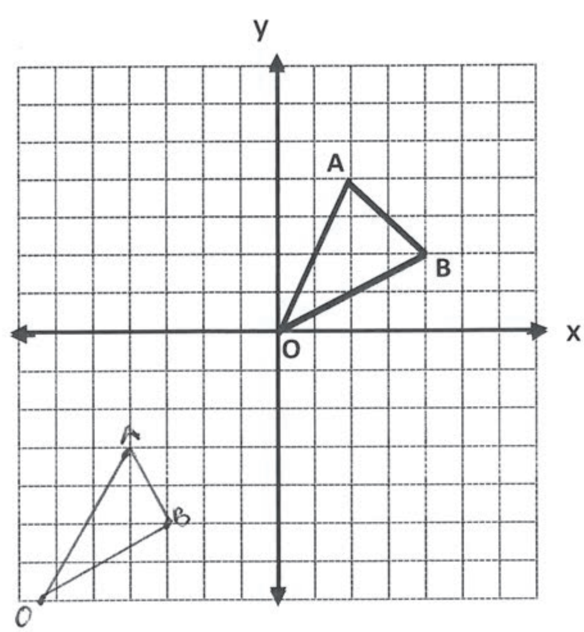
If the learner showed working using a mathematical method of finding the length of $A'A''$, but did not calculate the final answer correctly.

What would show evidence of no understanding?

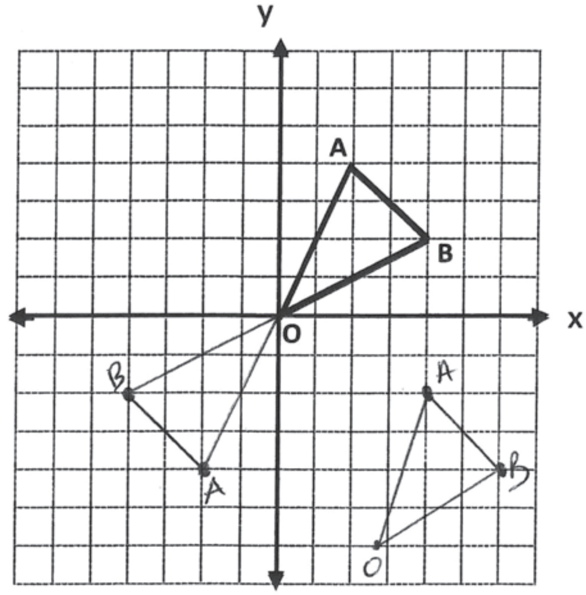
Item 9.1 and 9.3

If the learner drew image(s) of ΔAOB which did not show reflections or rotations correctly:

The triangle in the bottom left quadrant is neither a reflection nor a rotation of ΔAOB.



- The triangle in the bottom right quadrant is neither a reflection nor a rotation of ΔAOB.



Item 9.2

If the learner gave the incorrect coordinates of B' through:

- Ignoring the change in the sign of the y-coordinate;
- Interchanging the signs of the x- and y-coordinates;
- Using translation to determine the coordinates.

9.2 Write down the coordinates of B', the image of B. 3;3

9.2 Write down the coordinates of B', the image of B.

B' = 1;3 ✗

Item 9.4

If the learner incorrectly determined the length of $A'A''$ by using an inappropriate method (for example using angles or naming points instead of calculating a length).

9.4 Hence, determine the length of $A'A''$.

A'B'' = 180 ÷ 3
= 60°

9.4 Hence, determine the length of $A'A''$.

A'(-3;2) A''(4;-2)

9.4 Hence, determine the length of $A'A''$.

$$\frac{-4}{4}$$

What do the item statistics tell us?

Item 9.1

9% of learners answered the question correctly.

Item 9.2

4% of learners answered the question correctly.

Item 9.3

6% of learners answered the question correctly.

Item 9.4

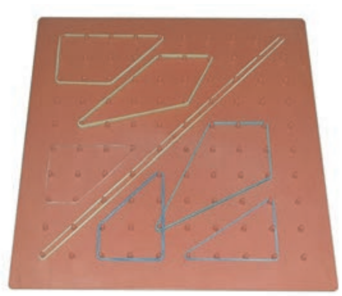
4% of learners answered the question correctly

Factors contributing to the difficulty of the items

- Learners may have poor understanding of the concepts and skills tested in this item such as: performing transformations (reflection, translation or rotation) with simple geometric shapes on a coordinate plane; correctly writing down coordinates of an image; and determining the lengths of line segments.
- The x-axis and y-axis are imposed on a grid: learners may not have realised this and so may have incorrectly identified the axes.

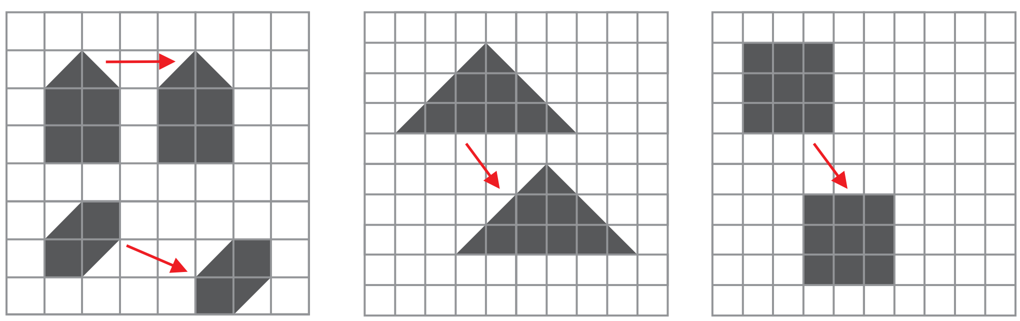
Teaching strategies

Transformations in a co-ordinate plane:

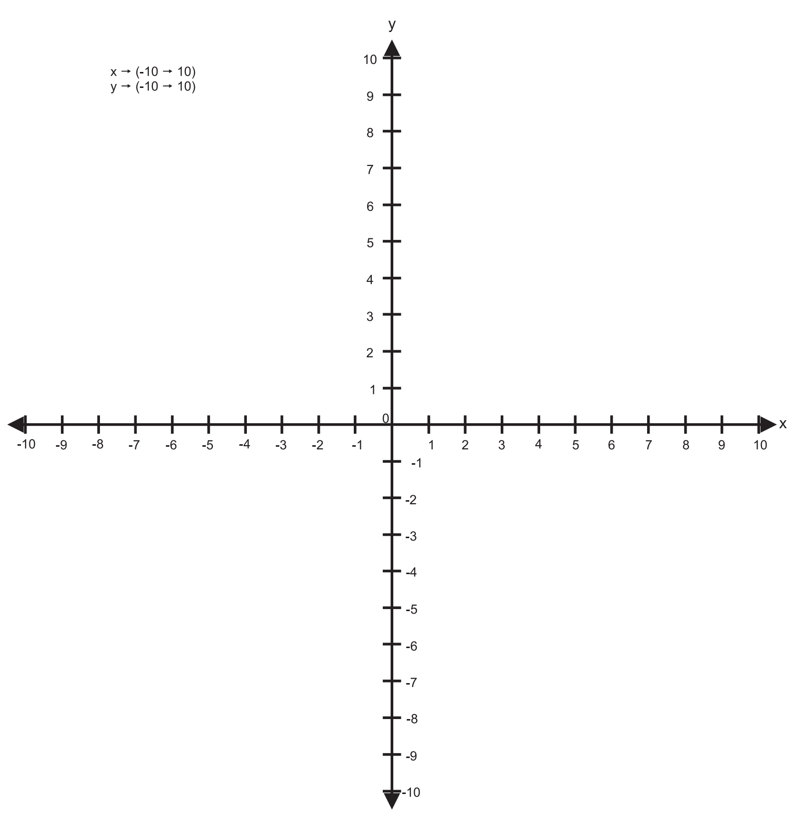


- Performing concrete transformations with simple geometric shapes on a coordinate plane would enable leaners to see what happens when shapes are transformed (reflected/translated/rotated). This would assist them to then develop the abstract concept of transformation of shapes on the coordinate plane.
- Provide learners with concrete manipulatives to give them this hands-on experience of transformation.
- You could use a geoboard – this is a board with nails on it that can be used to make shapes with elastic bands. Transformation of shapes can be demonstrated by moving the elastic bands.

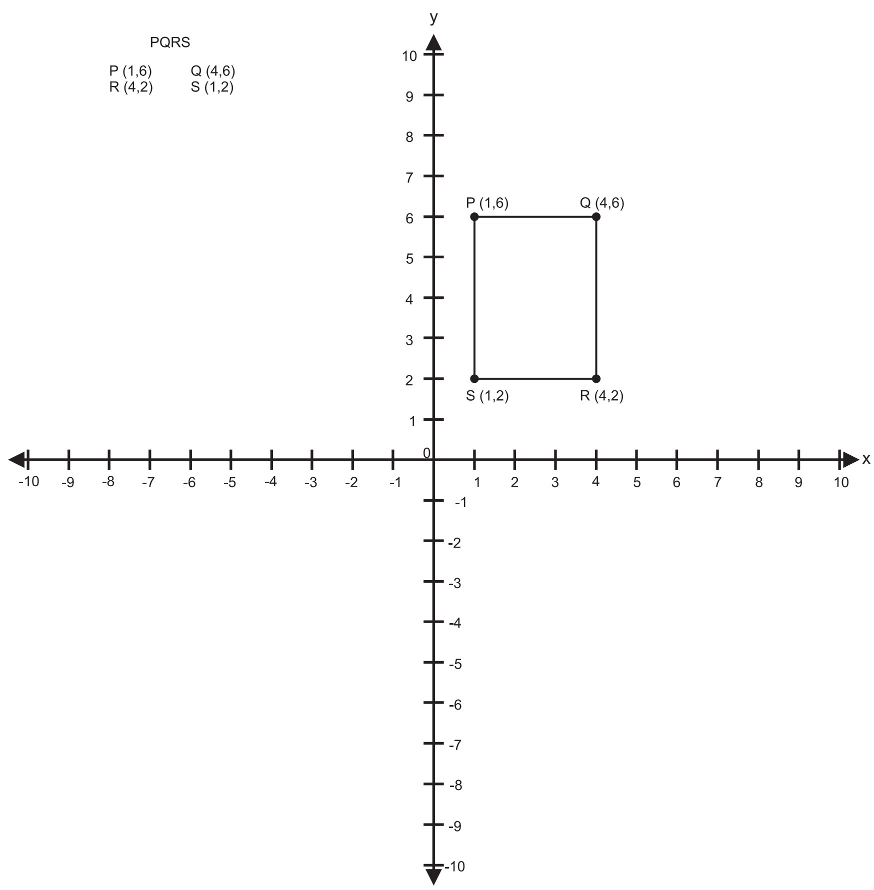
- You could also use tracing paper, grid paper or dotty paper if you don't have geoboards (see printables). Ask learners to draw a shape in one place on the paper and then move the shape by reflecting, translating or rotating it as shown in the diagrams that follow.
- Check that the learners do not change the shape when they move it – these transformations do not change the shape.
- Remind the learners that there are three main transformations:
  - Translation or a slide;
  - Rotation or a turn; and
  - Reflection or a flip.
- Remember to mention to the learners that after any of the transformations (turn, flip or slide) the shape still has the same size, area, angles and line lengths. For example, the shapes in the illustration have been translated. The arrows show the direction of the translation. The shapes have not changed, they have just moved, according to the translation.



- You could use a worksheet with a coordinate/Cartesian plane. Make sure your learners know how to draw and label the axes of a Cartesian plane (see the diagram).

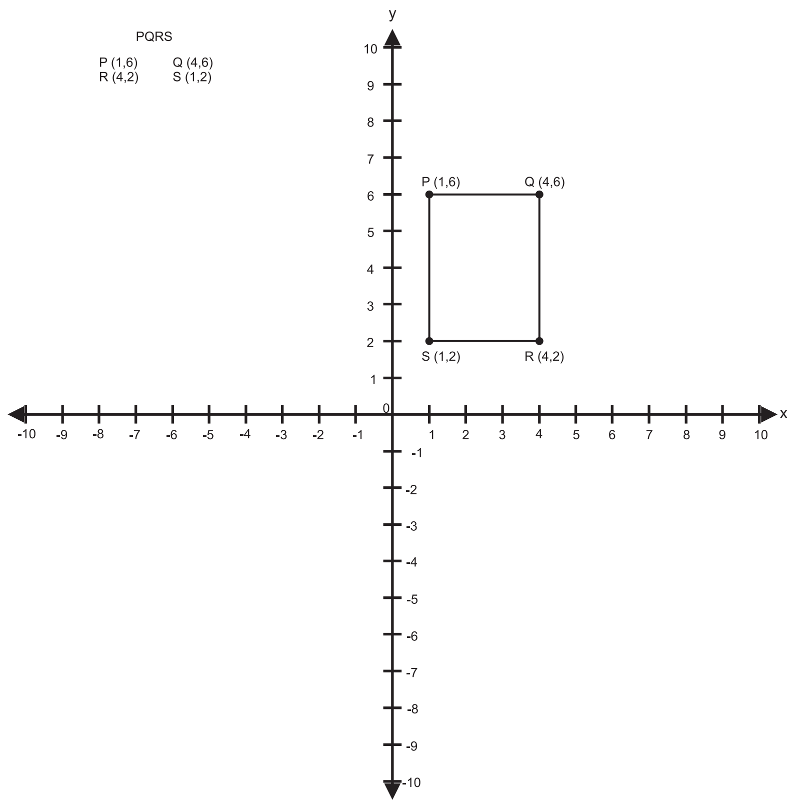


- Discuss transformations of the rectangle on a co-ordinate plane with your learners while they work through the following (or similar) activities.
- Give your learners the coordinates of a shape to draw on the plane.
- Make sure they know how to plot the points correctly on the plane.
- The example that follows shows the rectangle P (1; 6), Q (4; 6), R (4; 2), S (1; 2) plotted on the plane.

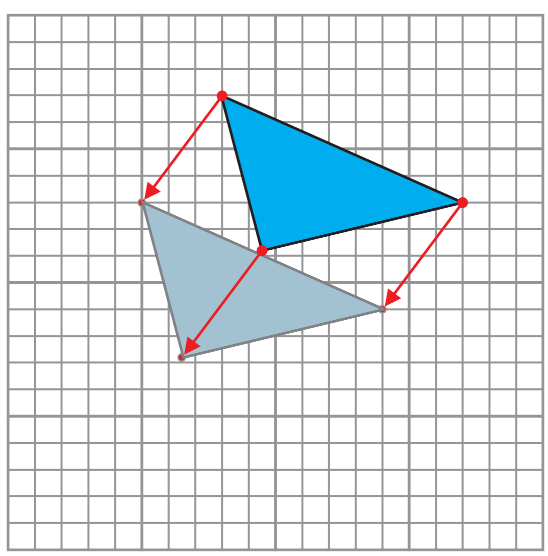


Example 1

- Plot points and draw the rectangle P (1; 6), Q (4; 6), R (4; 2), S (1; 2) on the plane. Use this as your

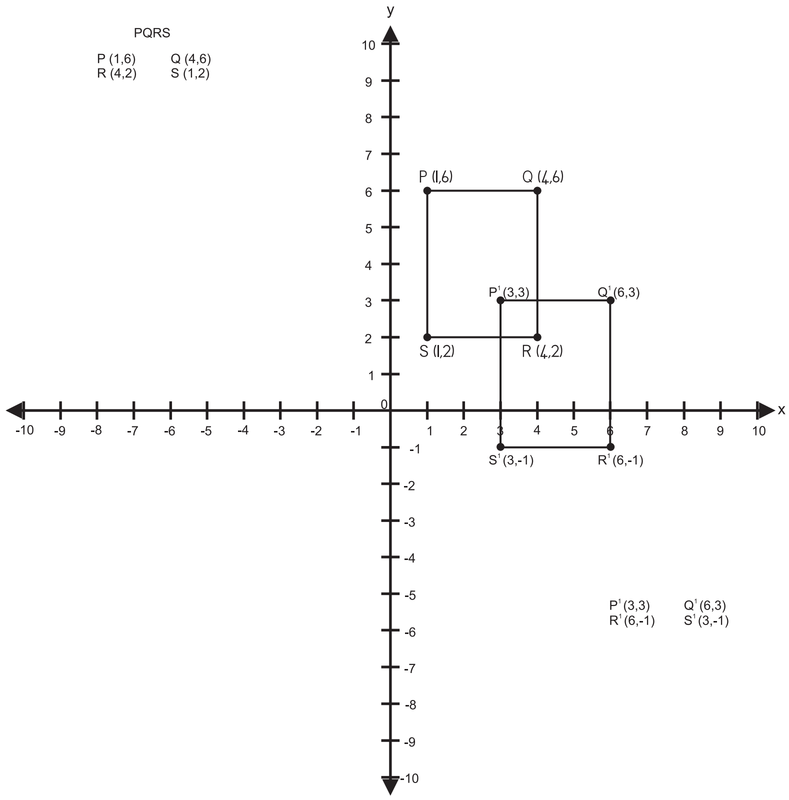


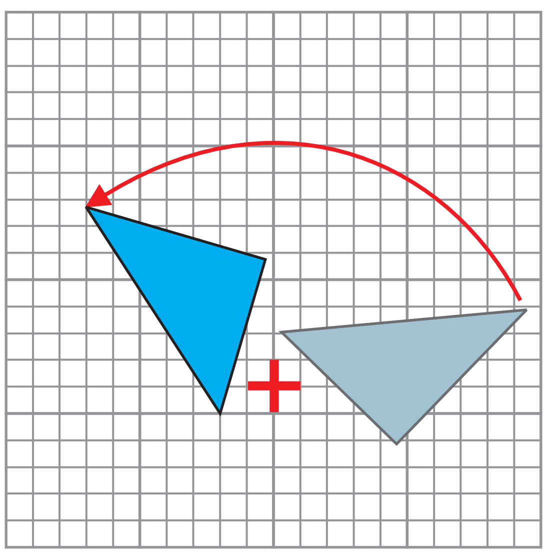
starting shape to draw transformations.



Translation or a slide

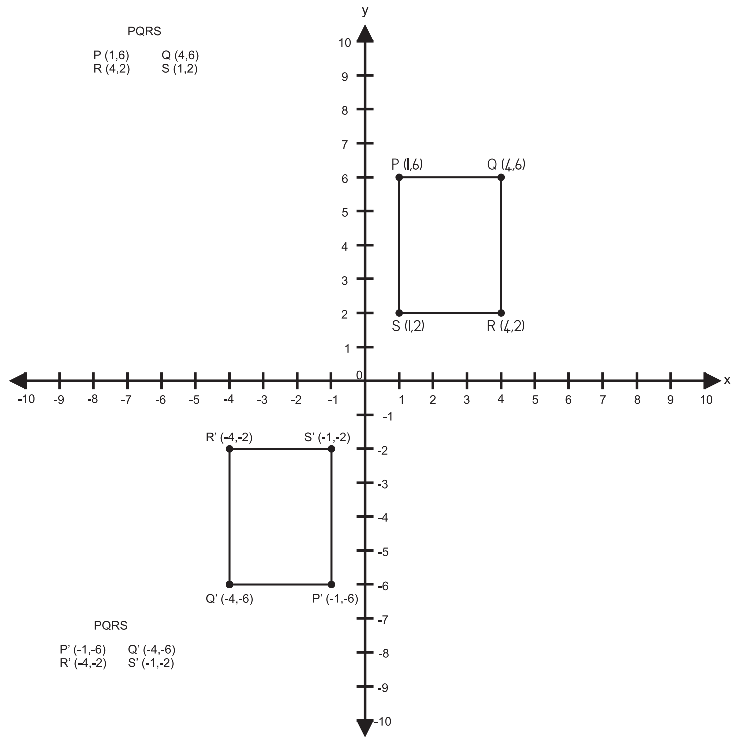
- Ask learners to draw a translation of the rectangle, for example, 2 units to the right and 3 units down.
- Draw the translated shape and work together with your learners to give the names of the coordinates of the translated shape.
- The coordinates are shown in the following sketch.





Rotation or a turn

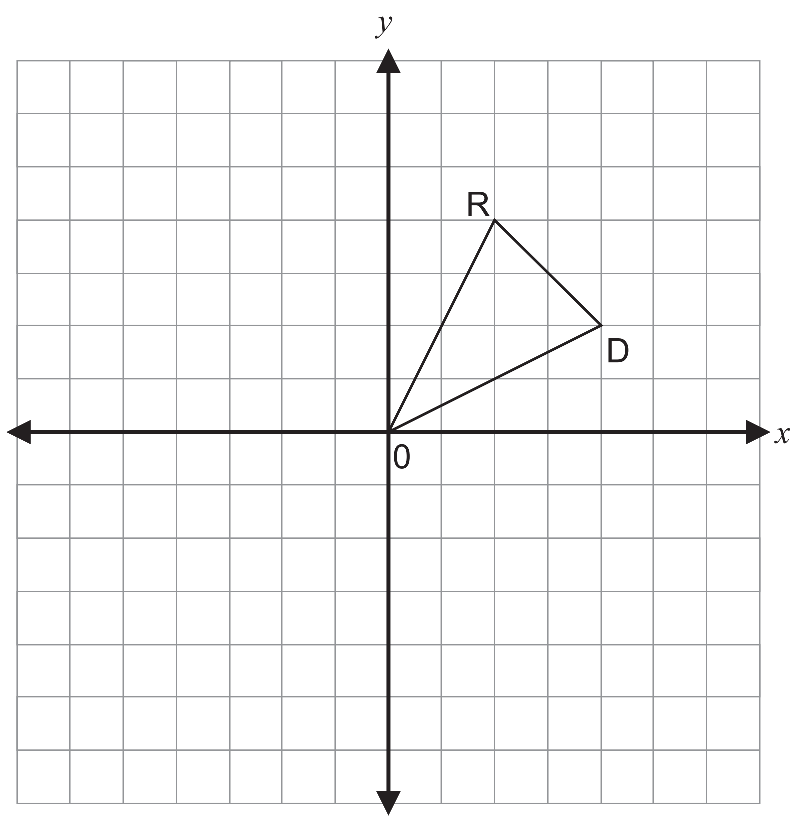
- Next ask the learners to use the same rectangle and rotate the shape 180° about the origin.
- The sketch that follows shows the original rectangle and the rotation of the rectangle 180° about the origin.
- Discuss the sketch with your learners. Give them the opportunity to name the coordinates of the shape in the rotated position.



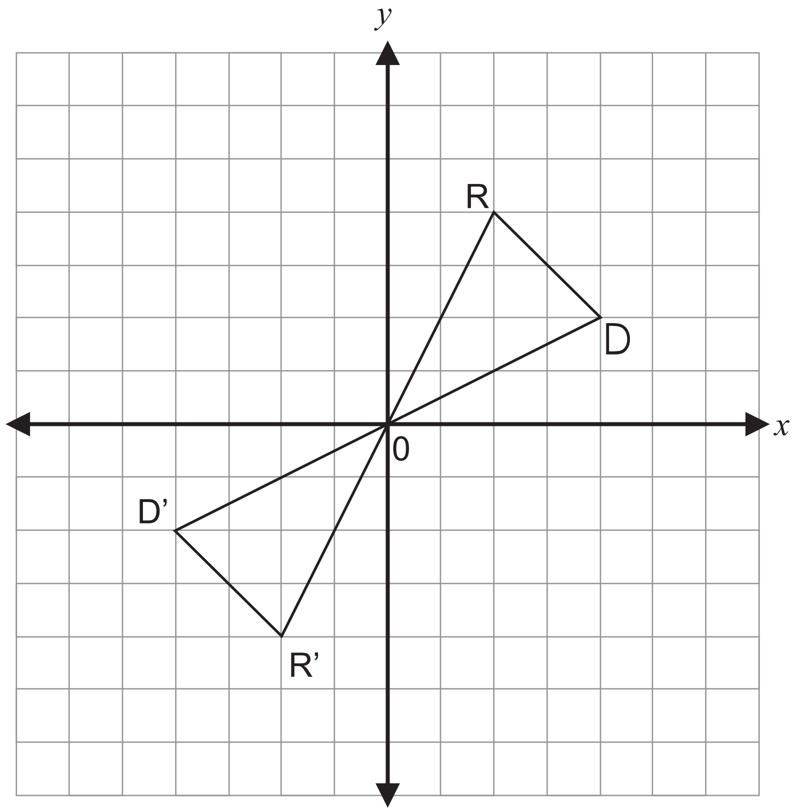
Example 2

Use another example to assist your learners with rotation about the origin:

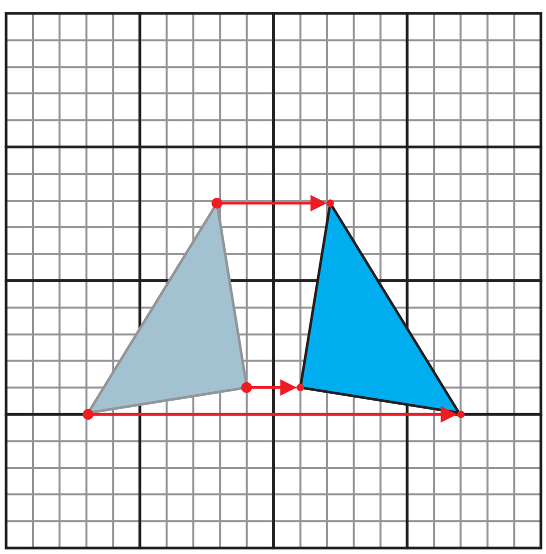
Rotate ΔROD $180°$ about the origin.



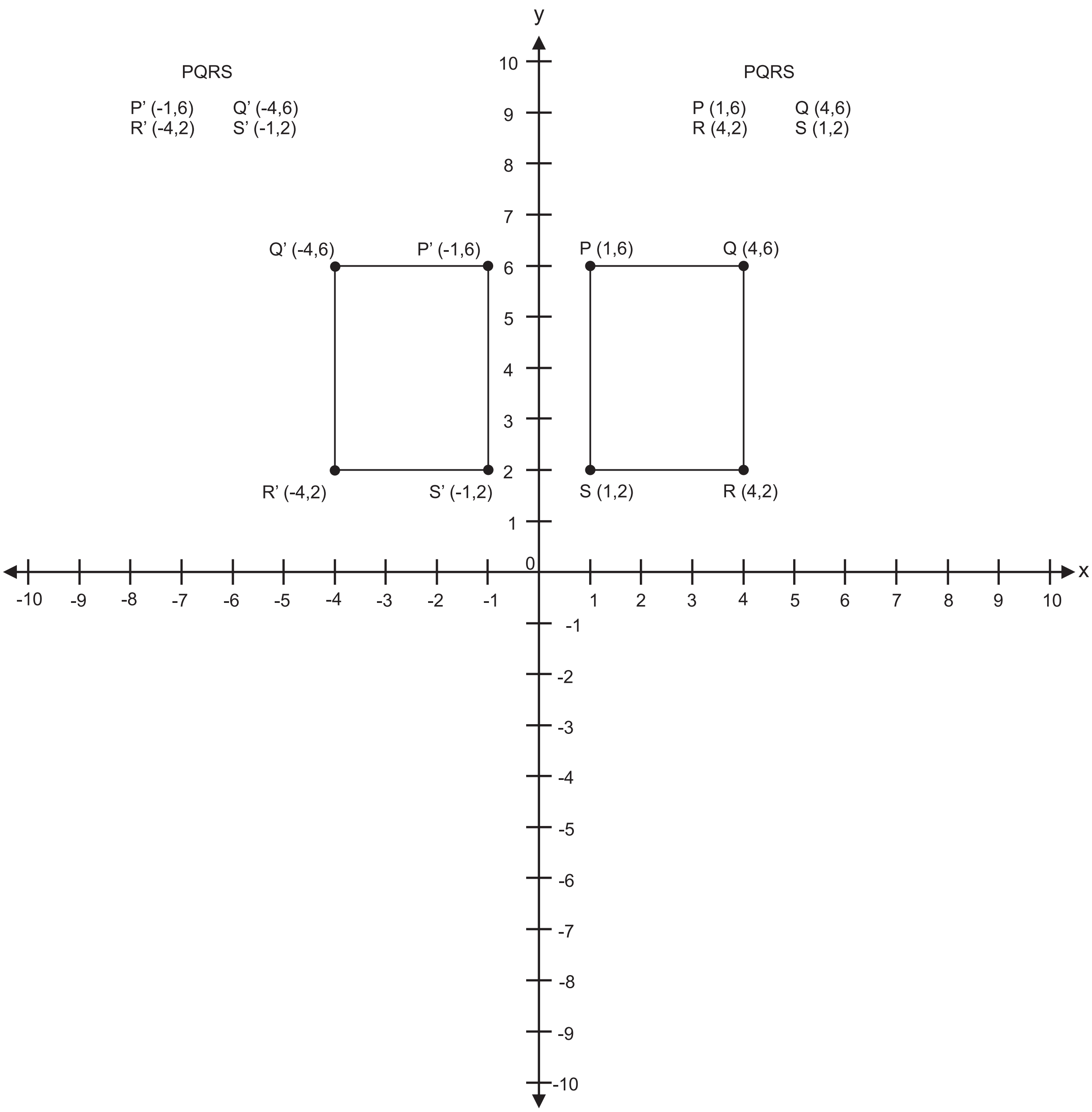
Solution



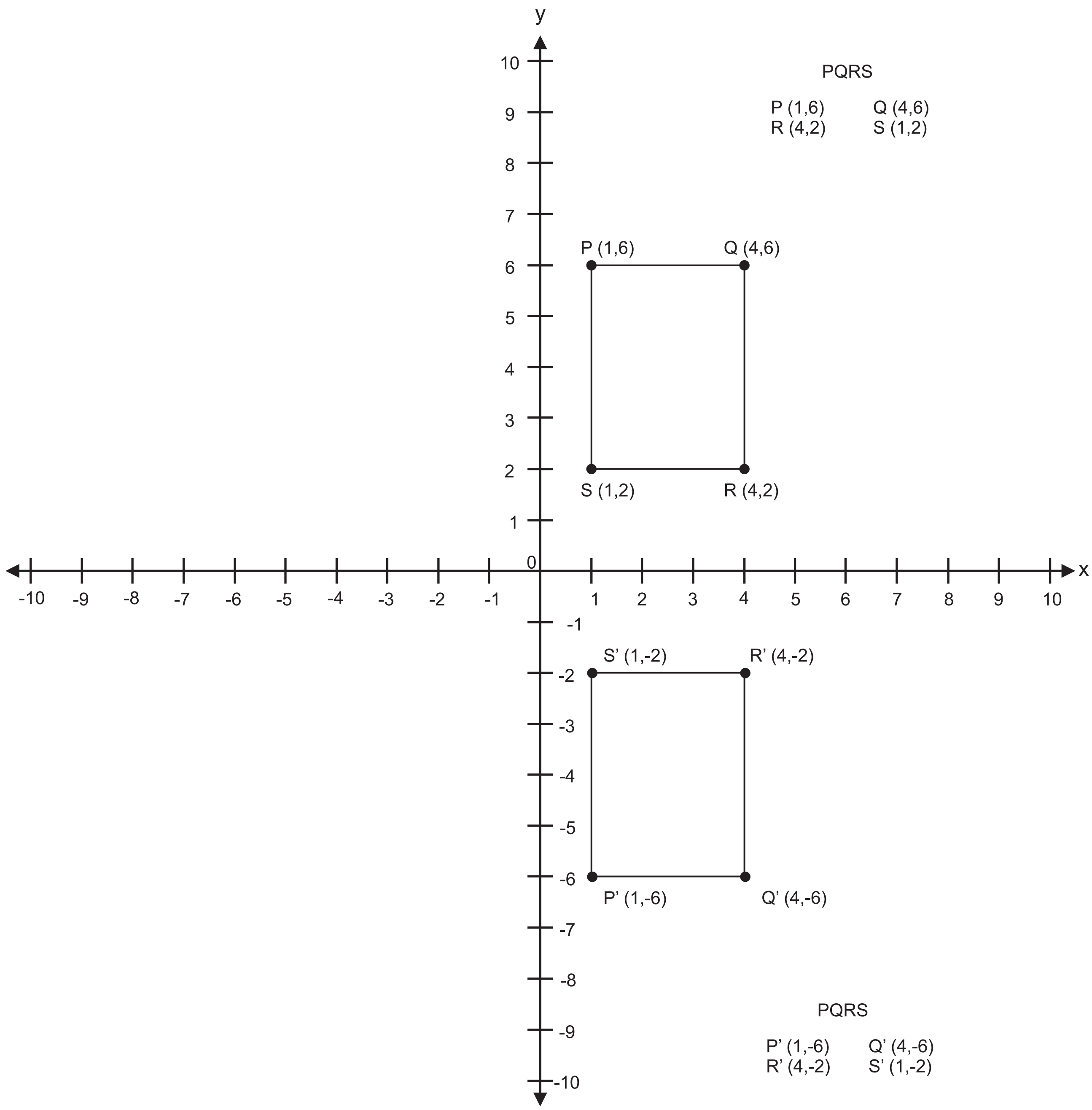
Reflection or a flip



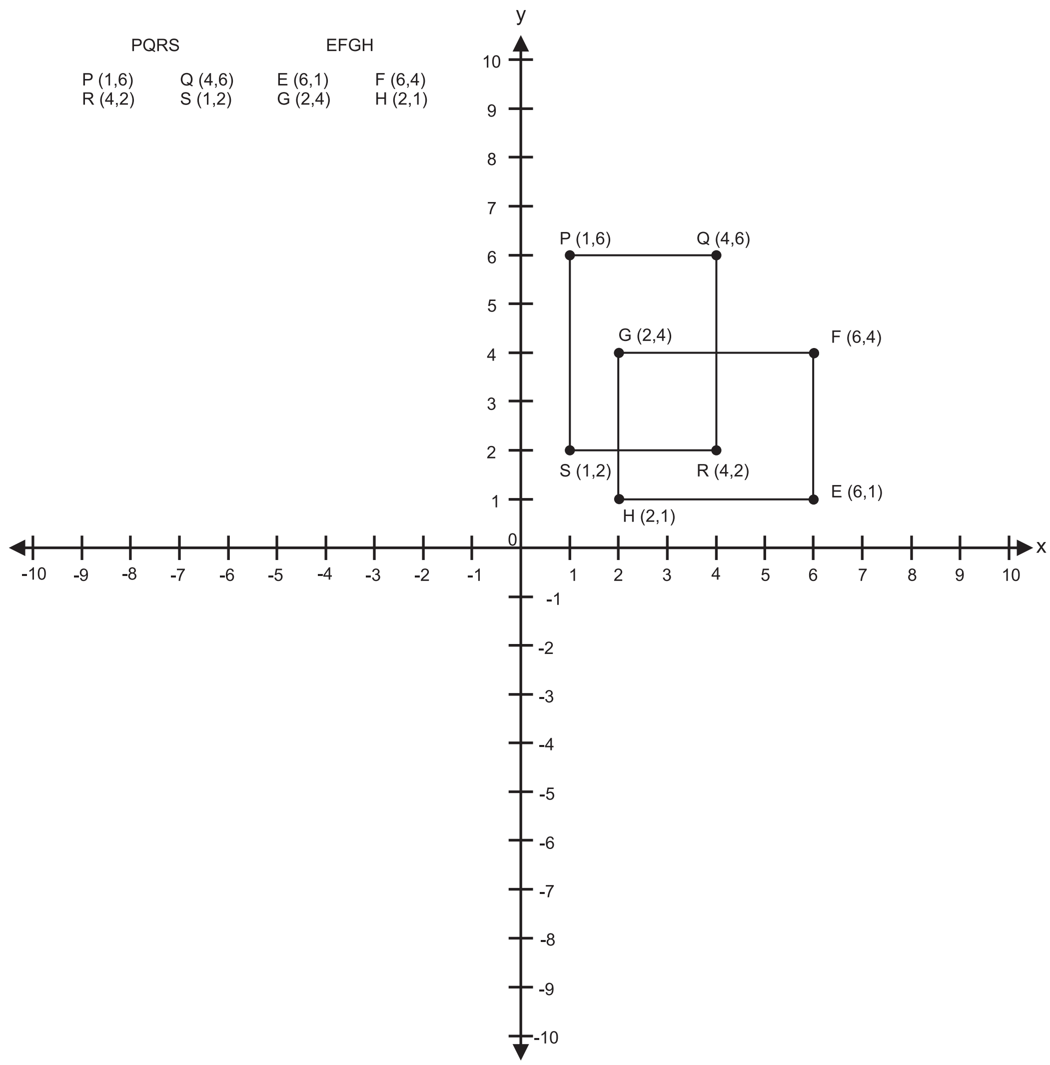
Next ask the learners to use the same rectangle and reflect the shape in the y-axis.



Next ask the learners to use the same rectangle and reflect the shape in the x-axis.



Now ask the learners to reflect the shape in the line $y = x$. Label the reflected shape $EFGH$.



- Consolidate this teaching by using the following rules when transforming shapes on a Cartesian plane.
- The rules help learners to remember how the coordinates change when a shape has undergone a transformation.

Reflection in the x-axis:
$T(x, y) = (x, -y)$

Reflection in the y-axis:
$T(x, y) = (-x, y)$

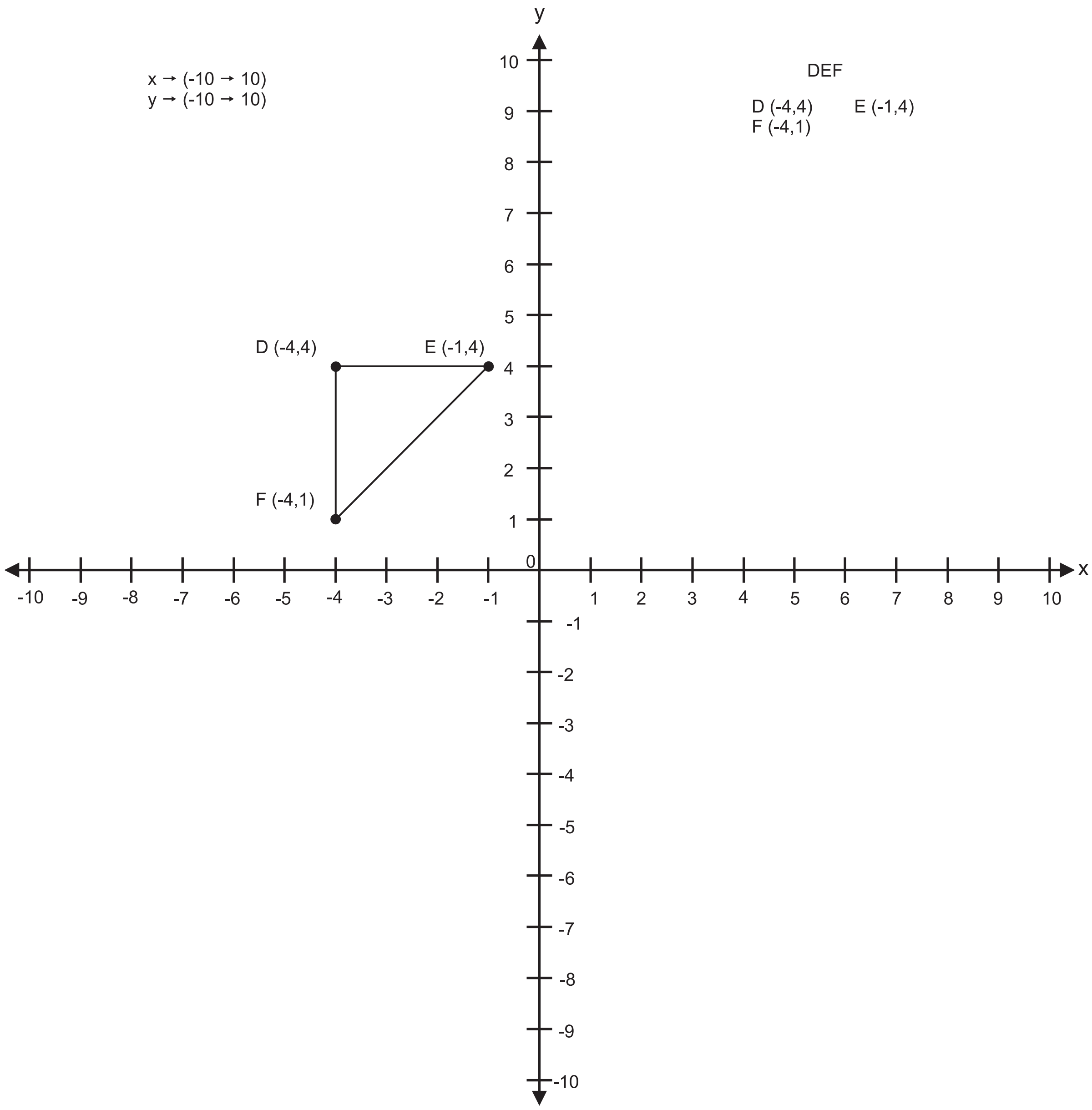
Reflection in the line $y = x$:
$T(x, y) = (y, x)$

Rotation 180° about the origin:
$T(x, y) = (-x, -y)$

Activities on transformations in a co-ordinate plane

- Use these activities to consolidate what was discussed.
- Point out the way in which the coordinates change when the shape is reflected. (The x-coordinates remain the same and the y-coordinates change sign.)

Activity: What are the coordinates of the triangle which is a reflection of Δ DEF in the y-axis?

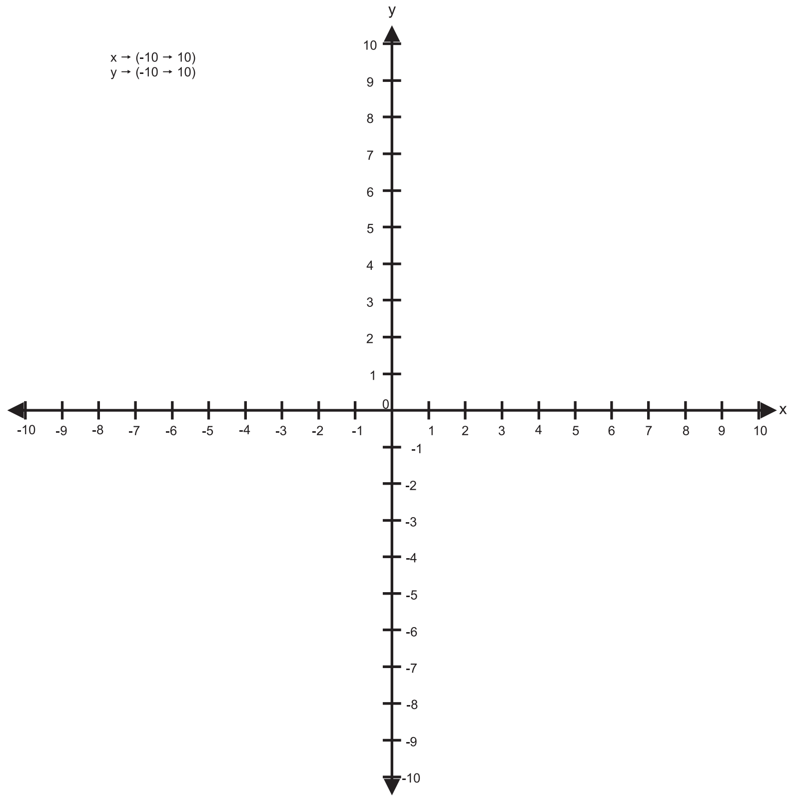


Solution

The reflection of Δ DEF in the y-axis has coordinates

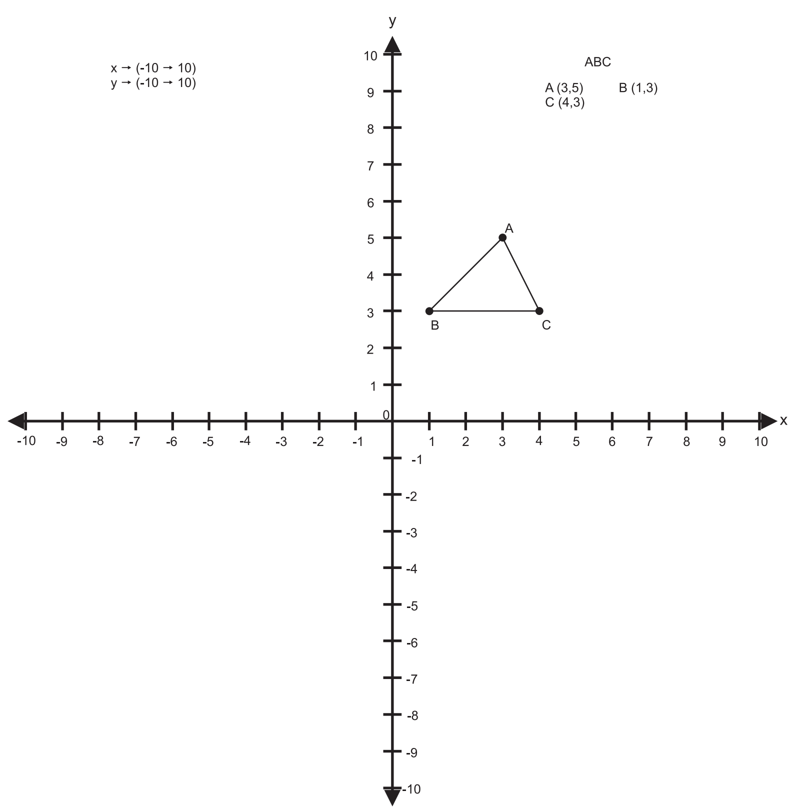
D' (4; 4), E' (1; 4), F' (4; 1)

- Draw the Cartesian plane on the chalkboard
- Remember to indicate the x- and y-axis.



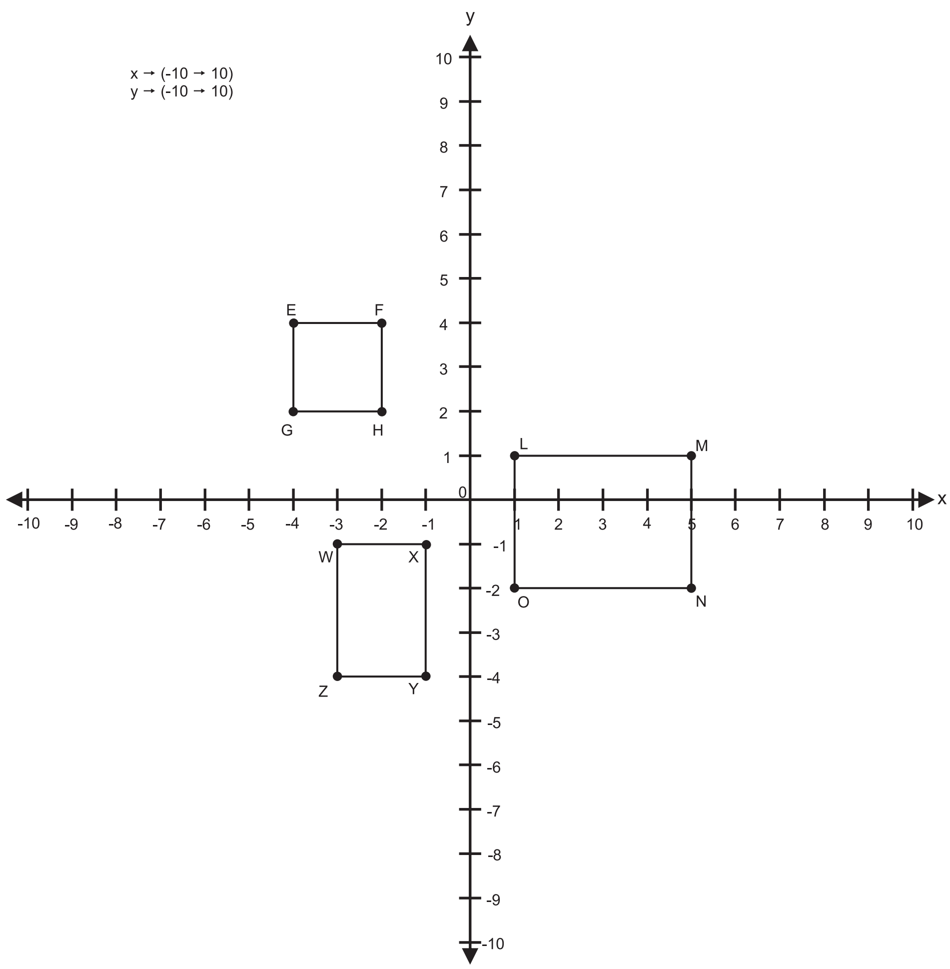
- Through demonstration help learners to transform shapes, for example, a triangle, square or rectangle on the Cartesian plane.
- Discuss with learners how to write down co-ordinates of points of the simple shape drawn on a Cartesian plane.

Activity: What are the coordinates of ΔABC?



- Discuss ordered pairs with the learners, for example, the value of the x is always written first in an ordered pair: (x, y). Thus in the example, A (3, 5), B (1, 3) and C (4, 3).
- Let learners complete the activity of writing out the coordinates of the points of simple shapes in a co-ordinate plane on their own or in pairs.
- Use the following printable activity sheet to assist with the writing of co-ordinates of points in a Cartesian plane.

Activity: What are the coordinates of Rectangle LMNO, Square EFGH and Rectangle WXYZ?



Solution

Rectangle LMNO: L (1, 1), M (5, 1), N (5,-2), O (1,-2)

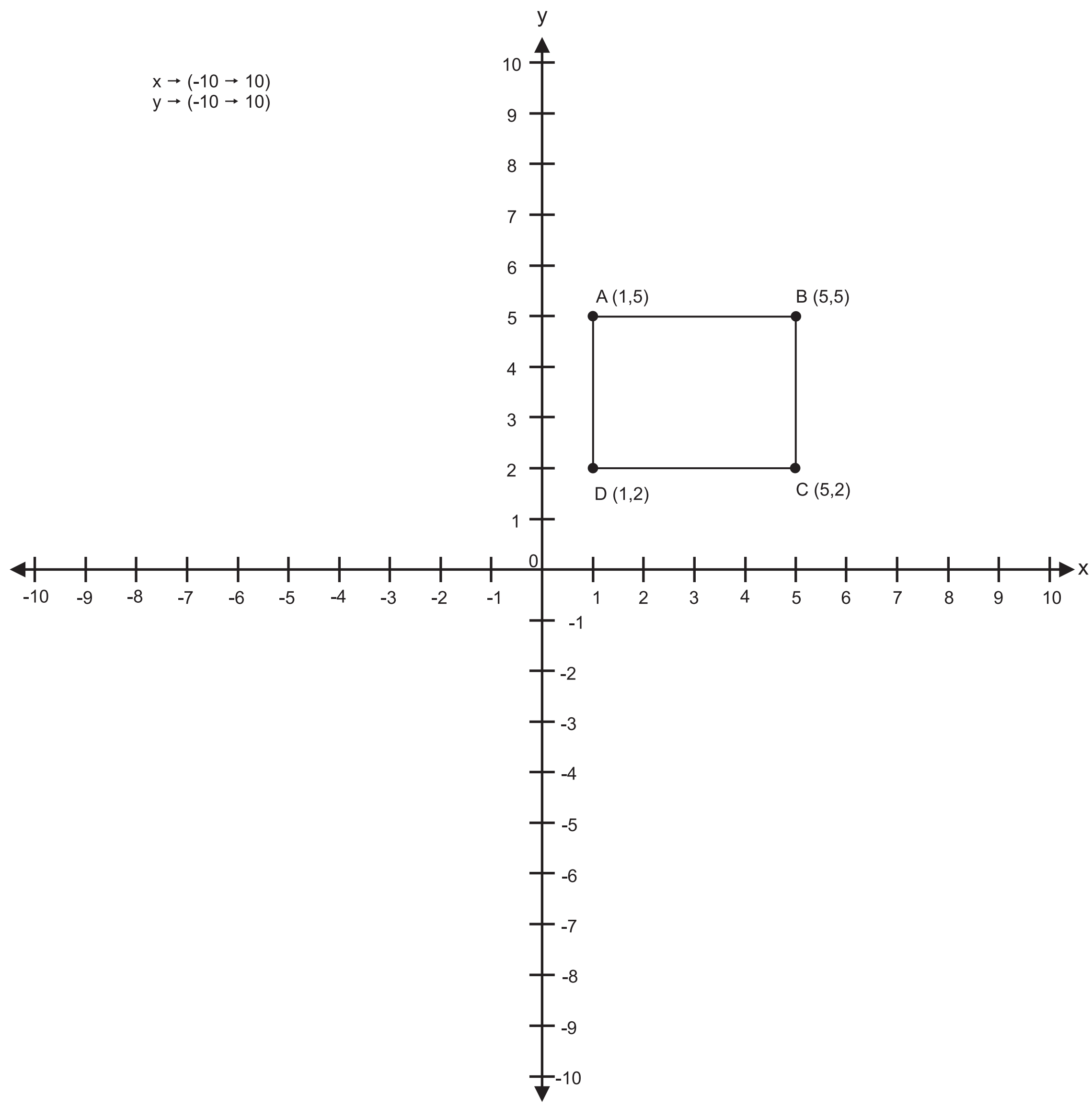
Square EFGH: E (-4, 4), F (-2, 4), G (-2, 2), H (-4, 2)

Rectangle WXYZ: W (-3,-1), X (-1,-1), Y (-1,-4), Z (-3, -4)

Finding the length of line segments on a Cartesian plane

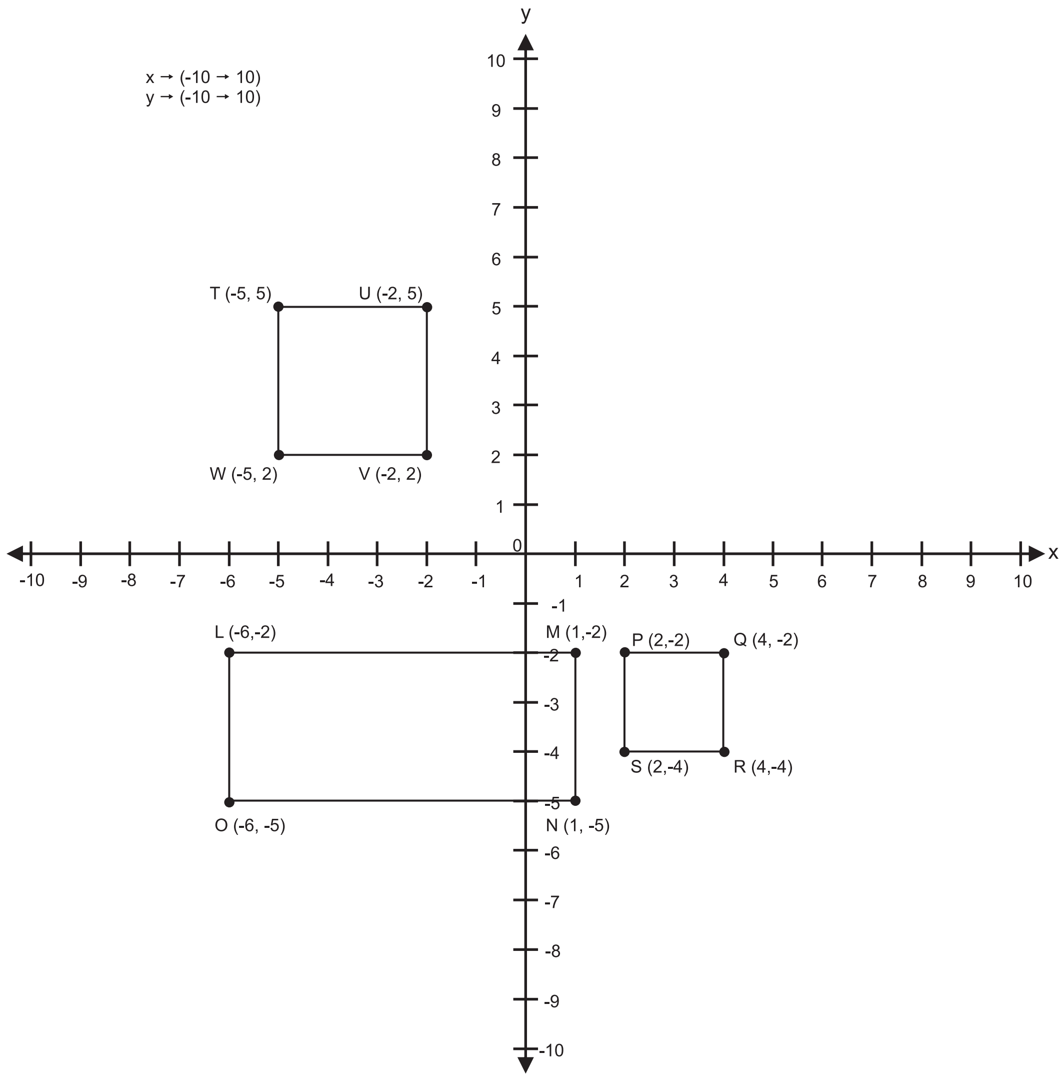
Activity

- Provide learners with graph/grid paper **(see printables)**.
- Next ask learners to draw a simple shape on the 1 cm by 1 cm grid paper or on a fully labelled Cartesian plane.
- Assist the learners to label the points of the shape as follows.



- Through demonstration, help learners to identify the four line segments by using the diagram of the rectangle in the illustration, for example, the line segments would be AB, BC, CD and DA.
  - Now demonstrate how to find the lengths of the line segments, for example, work out how many 1 cm blocks are along the lengths of the line segments or how many units each line segment measures on the Cartesian plane.

  - Use the following activity sheet to assist with establishing learners' confidence in determining the lengths of line segments.
  - Ask learners to find the lengths of each line segment in each of the shapes.



Solution

In square PQRS:

PQ = 2 units/2 cm

QR = 2 units/2 cm

RS = 2 units/2 cm

SP = 2 units/2 cm

In rectangle LMNO:

LM = 7 units/7 cm

MN = 3 units/3 cm

NO = 7 units/7 cm

OL = 3 units/3 cm

In square TUVW:

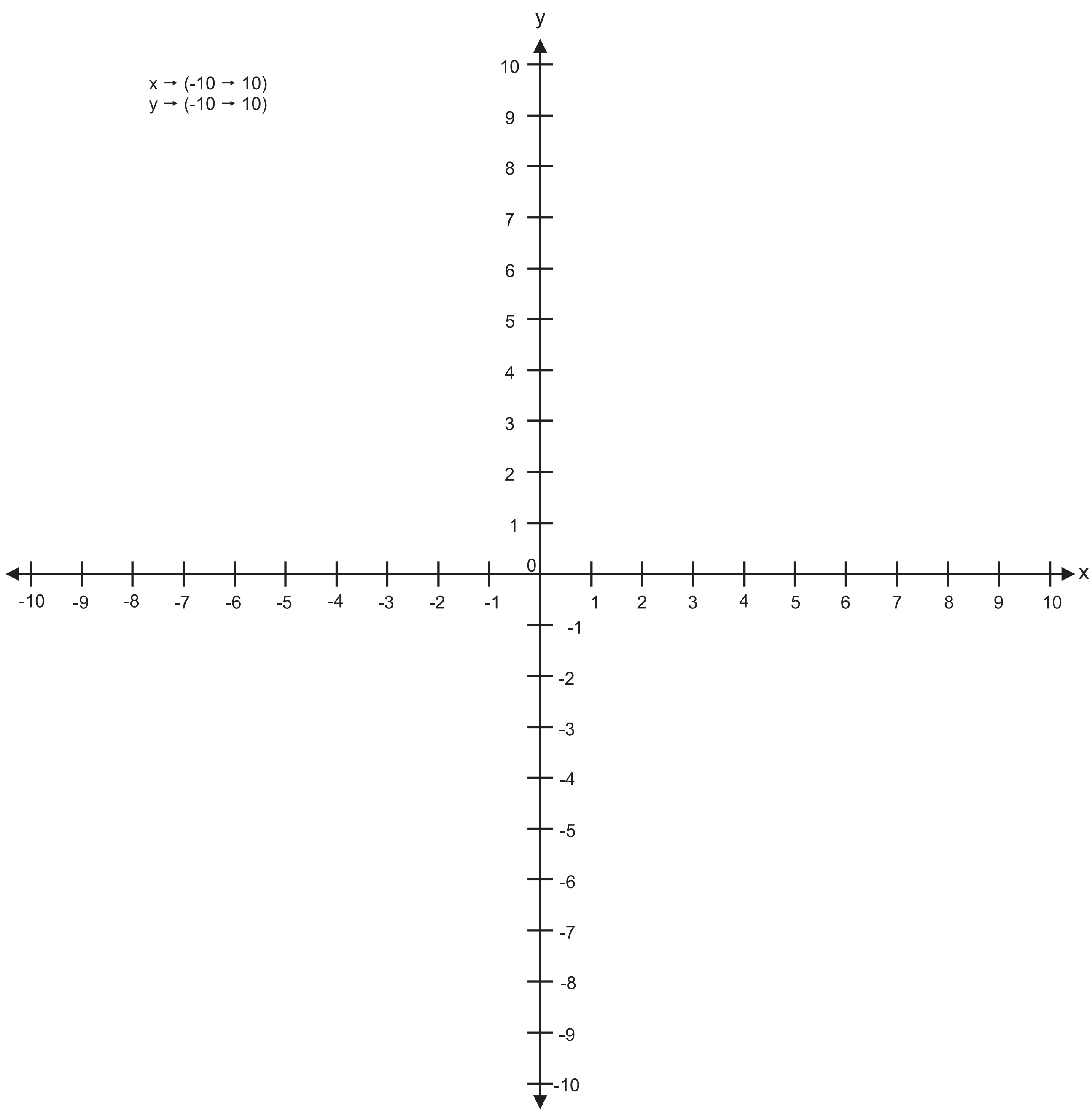
TU = 3 units/3 cm

UV = 3 units/3 cm

VW = 3 units/3 cm

WT = 3 units/3 cm

Printable: Cartesian plane

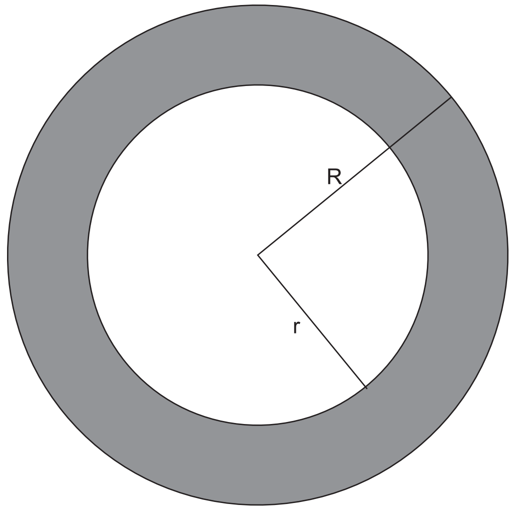


Printable: Grid paper (1 cm x 1 cm)

Printable: Dotty paper

Measurement: Area and perimeter of 2-D shapes

ANA 2013 Grade 9 Mathematics Item 10.1



10.1.1	Show that the area of the shaded section of the circle is equal to $\pi(R^2 - r^2)$.	[2]
10.1.2	Determine the area of the shaded ring in terms of π if R = 14 cm and r = 8 cm.	[2]

What should a learner know to answer this question correctly?

Learners should be able to:

- Recall formulae for calculating and working with the area of circles;
- Show that the area of a shaded ring between two circles is equal to $\pi(R^2 - r^2)$;
- Use appropriate formulae to solve problems and calculate the area of circles.

Where is this topic located in the curriculum? Grade 9 Term 3

Content area: Measurement

Topic: Area and perimeter of 2-D shapes

Concepts and skills:

- Use appropriate formulae and conversion between SI units to solve problems and calculate the perimeter and area of polygons and circles.

What would show evidence of full understanding?

If the learner obtained the correct solution by using an appropriate mathematical strategy.

Item 10.1.1

10.1.1 Show that the area of the shaded ring is equal to $\pi(R^2 - r^2)$.

$A = \pi R^2 - \pi r^2$

$= \pi(R^2 - r^2)$

Item 10.1.2

10.1.2 Determine the area of the shaded ring in terms of π if

$R = 14$ cm and $r = 8$ cm.

$A = \pi(14^2) - \pi(8^2)\text{cm}^2$

$= 132\pi\ \text{cm}^2$

What would show evidence of partial understanding?

Item 10.1.1

- If the learner used substitution in the correct formula (partially correctly) and did some computation but did not find a final answer;

10.1.1 Show that the area of the shaded ring is equal to $\pi(R^2 - r^2)$.

$3{,}142\ (14\ \text{cm} - 8\ \text{cm})$

$43{,}988 - 25{,}136 =$

- If the learner wrote down the correct formula as the final answer but did not give an accurate proof or use mathematically correct simplification to derive the proof.

10.1.1 Show that the area of the shaded ring is equal to $\pi(R^2 - r^2)$.

$R^2 - r^2 = R^4$

$= \pi(R^2 - r^2)$

Item 10.1.2

- If the learner wrote down the correct formula but did not complete the substitution;

10.1.2 Determine the area of the shaded ring in terms of π if $R = 14$ cm and $r = 8$ cm.

Area $= \pi(R^2 - r^2)$

=

- If the learner wrote down only part of the formula and did not substitute correctly into the formula.

10.1.2 Determine the area of the shaded ring in terms of π if $R = 14$ cm and $r = 8$ cm.

πr^2

14,0cm × 8,0cm²

= 896cm

What would show evidence of no understanding?

Item 10.1.1

- If the learner did not attempt the question;
- If the learner used a method that does not show any relation to the formula given in the question:

10.1.1 Show that the area of the shaded ring is equal to $\pi(R^2 - r^2)$.

$360 \times \frac{22}{7}$

$= \frac{7920}{7}$ ∴ 1131,40

Item 10.1.2

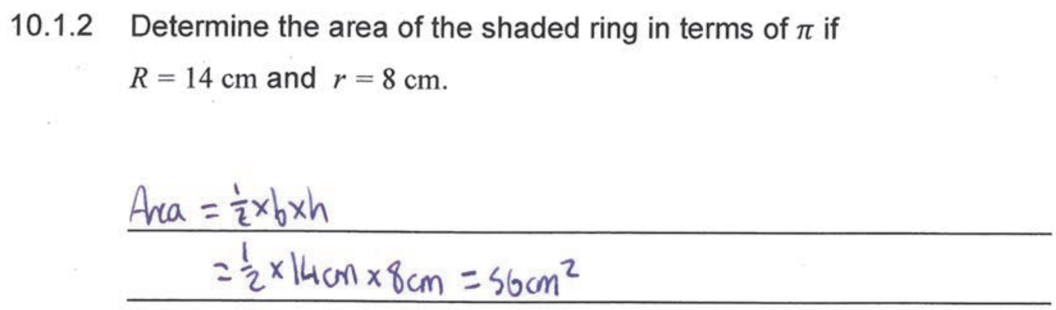
- If the learner did not attempt the question;
- If the learner used a formula that is not standard or related to the question;
- If the learner substituted values without showing an understanding of what they represent.

10.1.2 Determine the area of the shaded ring in terms of π if $R = 14$ cm and $r = 8$ cm.

~~AR~~ $\pi \times R$

$= \frac{22}{7} \times \frac{4}{8} = \frac{11}{7}$ = 1,571

- If the learner used the incorrect formula for area.


10.1.2 Determine the area of the shaded ring in terms of π if $R = 14$ cm and $r = 8$ cm.

Area = $\frac{1}{2} \times b \times h$

= $\frac{1}{2} \times 14cm \times 8cm = 56cm^2$

What do the item statistics tell us?

Item 10.1.1

4 % of learners answered the question correctly.

Item 10.1.2

5 % of learners answered the question correctly.

Factors contributing to the difficulty of the item

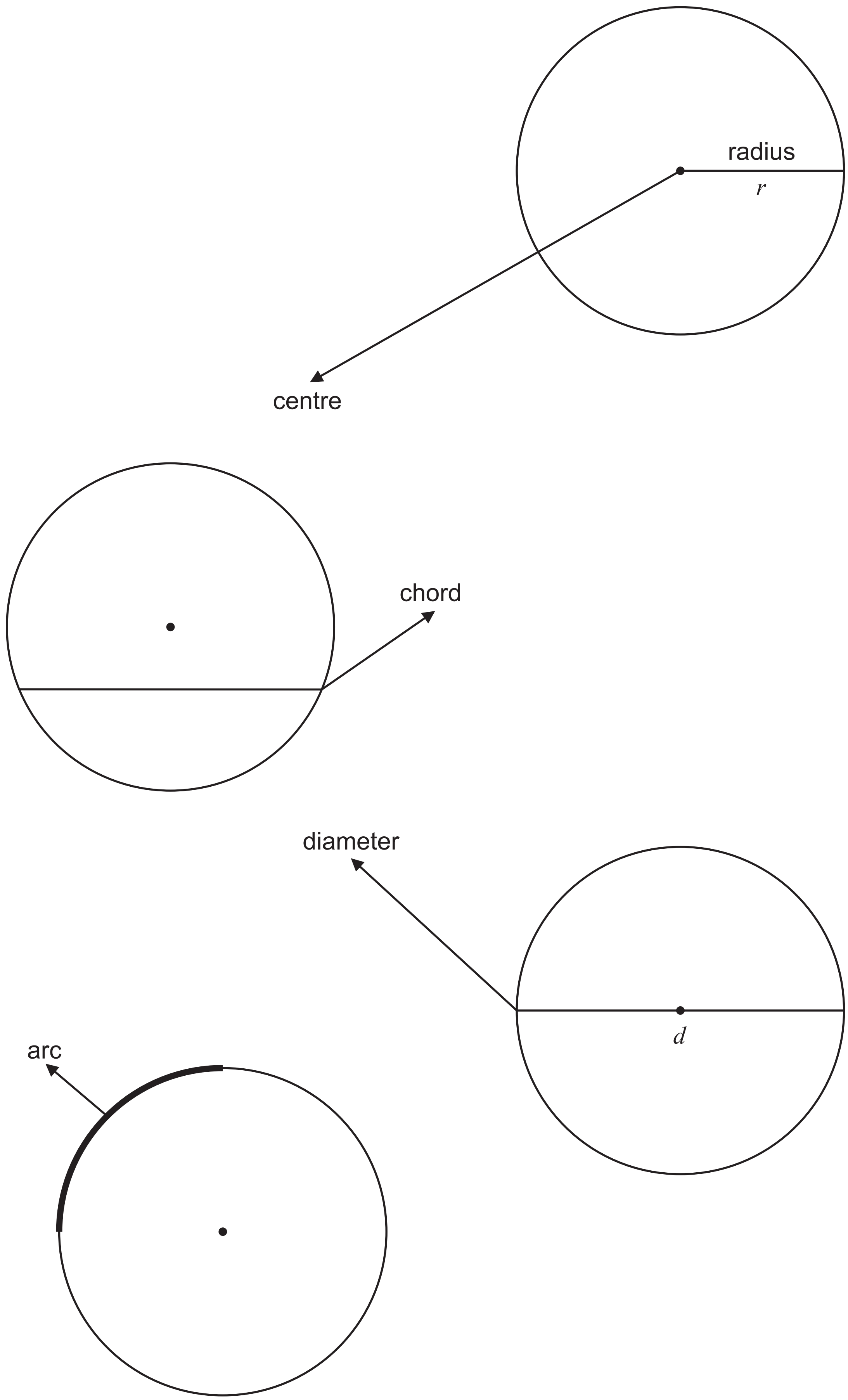
- Learners may have poor understanding of the concepts and skills tested in this item, such as being able to identify or use the correct formula for determining the area of a circle.
- Learners may have poor mathematical skills and not be able to substitute values correctly in an equation.
- Learners may confuse the concept of perimeter with that of area and interchange the formulae for the two concepts.
- Learners may confuse the formulae for areas of other polygons with the formula for the area of a circle.

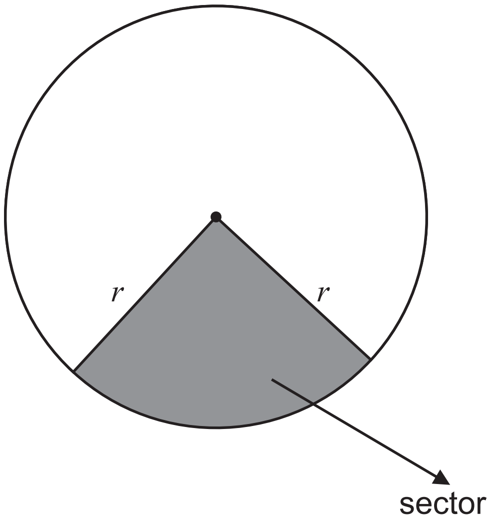
Teaching strategies

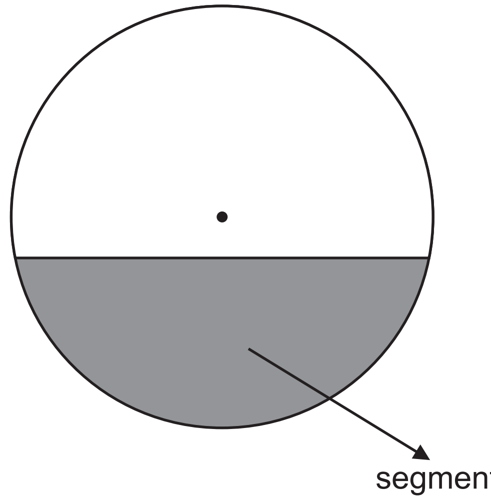
Revision of circle terminology

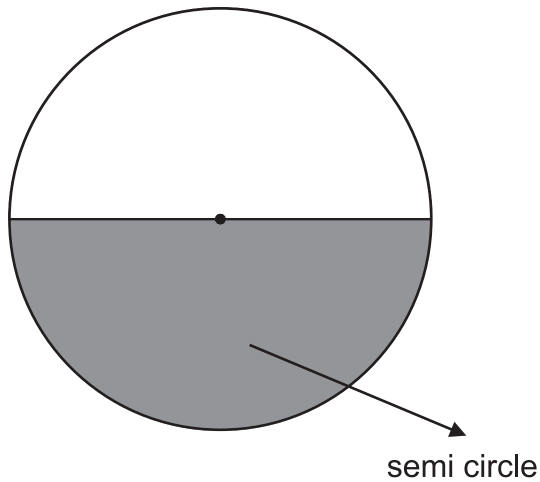
- Start basic revision of circles with your learners.
- Learners ought to be able to identify the different parts of a circle.
- The following terms are used commonly in mathematics when working with problem solving involving circles. Use the diagrams to assist you when you explain the meaning of each term to your learners.
- Draw the shapes and write the terms on the board.
- Allow the learners the opportunity to discuss the terms. Let them use all the terms to explain to each other (in pairs) what words we use when we speak about circles in mathematics.

Useful terminology when working with circles

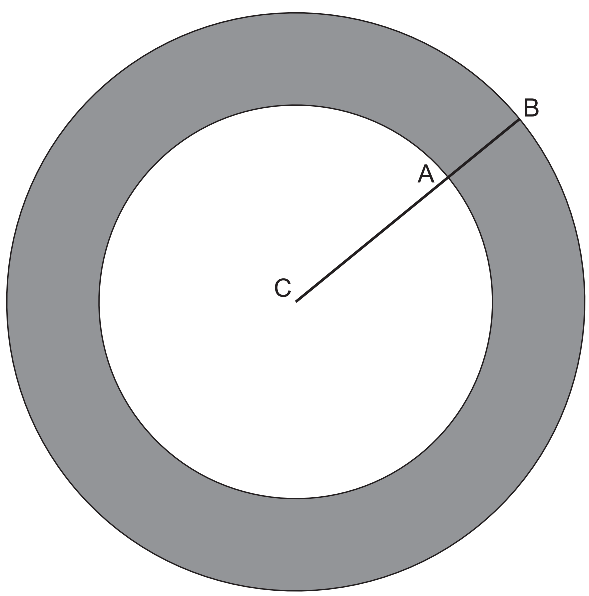






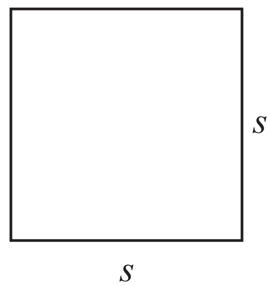


- Concentric circles share the same centre.

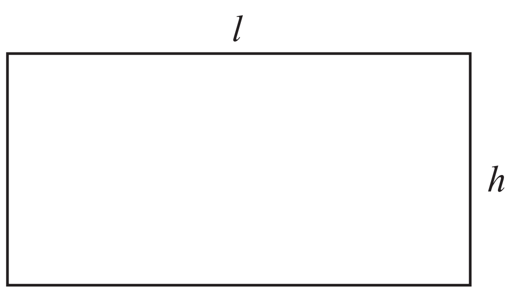


Working with the perimeter of polygons and the circumference of circles:

- Remind your learners of the meaning of the word perimeter:
- Perimeter is the distance around a shape.
- The following formulae may be used to assist your learners to remember perimeter:



  - Perimeter of a square $= 4s$ or $4l$ where $s =$ side and $l =$ length
  - Perimeter of a rectangle: $P = 2(l + b)$ or $P = 2l + 2b$ where $l =$ length and $b =$ breadth



- When we talk about the distance around a circle we use the word circumference.
  - The circumference of a circle is calculated using the following formula: $C = 2\pi r$ or $C = \pi d$

- Use examples based on calculating the perimeter of polygons and the circumference of circles to assist your learners to recall the concepts of perimeter and circumference. The following examples may be used.

Examples

1). If the length of one side of a square is 6 cm calculate the perimeter of the square.

$P = 4s$

$P = 4 \times 6\,cm$

$P = 24\,cm$

2). If the perimeter of a square is 28 cm what is the value of the length of one side of the square?

$P = 4s$

$28\,cm = 4s$

$s = 28 \div 4$

$s = 7\ cm$

3). In rectangle STUV the length is 12 mm and the breadth is 5 mm. Calculate the perimeter of rectangle STUV.

$P = 2(l + b)$

$P = 2(12 + 5)\ mm$

$P = 2(17\ mm)$

$P = 34\ mm$

4). If the perimeter of a rectangle is 300 cm and the length of one side of the rectangle is 85 cm calculate the breadth of the rectangle.

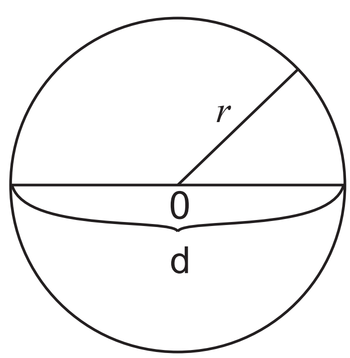
$P = 2(l + b)$

$300 = 2(85 + b)$

$85 + b = 150\ cm$

$b = 65\ cm$

5). The following circle with centre O has a diameter of 10 cm. What is the circumference of this circle?



$C = \pi d$

$C = \pi \times 10\ cm$

$C = 10\pi\ cm$ or $31{,}4\ cm$

6). A circle with centre O has a radius of 4 cm. What is the circumference of this circle?

$C = 2\pi r$

$C = 2\pi \times 4\ cm$

$C = 8\pi\ cm$ or $25{,}1\ cm$

The following activity sheet may be used to consolidate your discussion on perimeter and circumference.

Activities

1). If the length of one side of a square is 11 cm calculate the perimeter of the square.

2). If the perimeter of a square is 64 cm what is the value of the length of one side of the square?

3). In rectangle ABCD the length is 14 mm and the breadth is 6 mm. Calculate the perimeter of rectangle ABCD.

4). If the perimeter of a rectangle is 200 cm and the length of one side of the rectangle is 60 cm calculate the breadth of the rectangle.

5). A circle with centre O has a diameter of 12 cm. What is the circumference of this circle?

6). A circle with centre O has a radius of 8 cm. What is the circumference of this circle?

Solutions

1). $P = 4s$

$P = 4 \times 11\ cm$

$P = 44\ cm$

2). $P = 4s$

$64\ cm = 4\ s$

$s = 64 \div 4$

$s = 16\ cm$

3). $P = 2(l + b)$

$P = 2(14 + 6)\ mm$

$P = 2(20)\ mm$

$P = 40\ mm$

4). $P = 2(l + b)$

$200 = 2(60 + b)$

$60 + b = 100\ cm$

$b = 40\ cm$

5). $C = \pi d$

$C = \pi \times d$

$C = 12\pi$ or $37,7\ cm$

6). $C = 2\pi r$

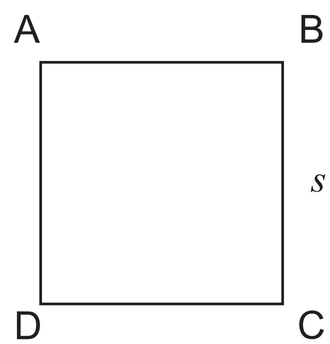
$C = 2\pi \times 8\ cm$

$C = 16\pi$ or $50,3\ cm$

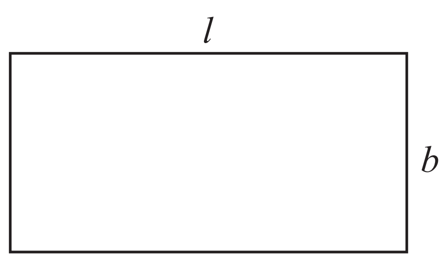
Working with the area of polygons and the area of circles:

- Discuss the concept area with your learners.
- Use the following formulae to assist you when discussing area.

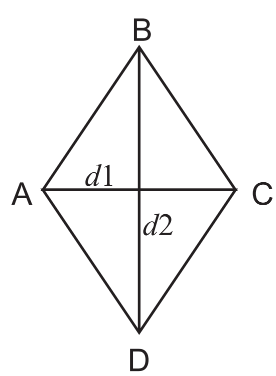
- Area of a square: $A = 1^2$ or $A = s^2$ where s = side and l = length



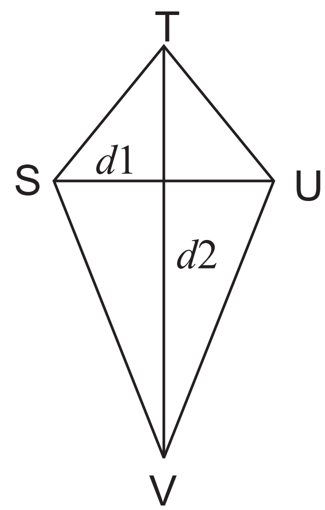
- Area of a rectangle: $A = 1 \times b$ where l = length and b = breadth



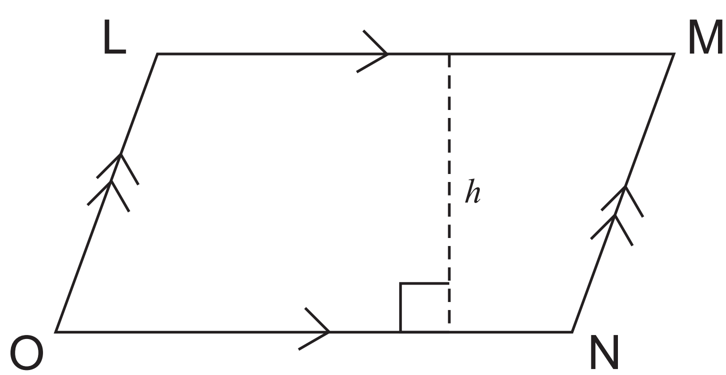
- Area of a rhombus: $A = \frac{1}{2}$ $(diagonal\ 1 \times diagonal\ 2)$



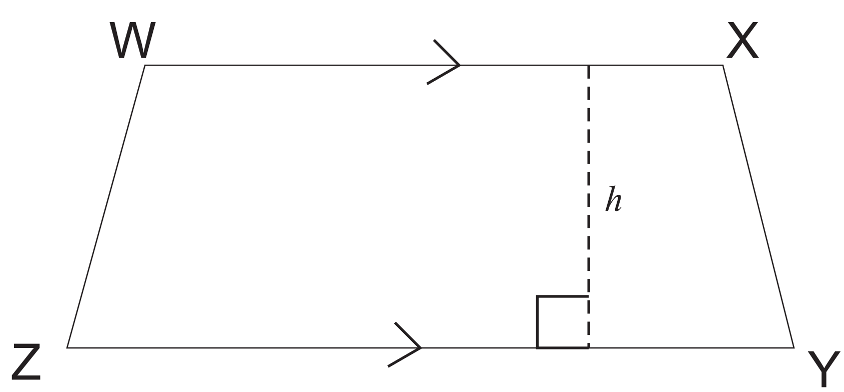
- Area of a kite: $A = \frac{1}{2}\,(diagonal\,1 \times diagonal\,2)$



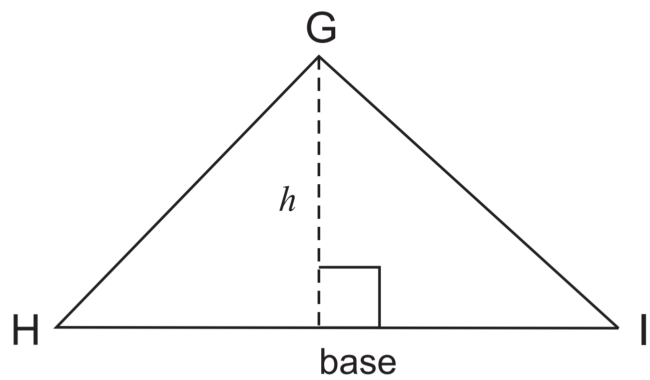
- Area of a parallelogram: $A = base \times height$



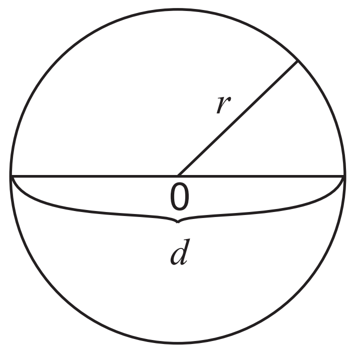
- Area of a trapezium: $A = \frac{1}{2}\,(sum\ of\ parallel\ sides) \times height$



- Area of a triangle: $A = \frac{1}{2} \times base \times height$



- Area of a circle: $A = \pi r^2$



- Use examples based on calculating the area of polygons and the area of circles to assist your learners to recall the concept of area.
- The following examples may be used.

Examples

1). Calculate the area of a square if one side of the square is 9 cm.

$A = s^2$

$A = 9\ cm \times 9\ cm$

$A = 81\ cm^2$

2). If the area of a square is 49 cm^2 calculate the length of one side of the square.

$A = s^2$

$s^2 = 49\ cm^2$

$s = 7\ cm$

3). Calculate the area of a rectangle if the length of the rectangle is 10 cm and the breadth is 6 cm.

$A = l \times b$

$A = 10\,cm \times 60\,cm$

$A = 60\,cm^2$

4). If the area of a rectangle is 400 cm^2 and the length is 40 cm calculate the breadth of the rectangle.

$A = l \times b$

$400\,cm^2 = 40\,cm \times b\,cm$

$b = 10\,cm$

5). Calculate the area of a triangle if the base is 12 cm and the height is 8 cm.

$A = \frac{1}{2} \times base \times height$

$A = \frac{1}{2} \times 12\,cm \times 8\,cm$

$A = \frac{1}{2} \times 96\,cm^2$

$A = 48\,cm^2$

6). If the area of a triangle is 360 cm^2 and its base is 30 cm what is the height of this triangle?

$A = \frac{1}{2} \times base \times height$

$360 = \frac{1}{2} \times 30\,cm \times h\,cm$

$360\,cm^2 \times 2 = 30\,cm \times h\,cm$

$720\,cm^2 \div 30\,cm = h$

$h = 24\,cm$

7). Calculate the area of a circle with centre O and radius 3 cm.

$A = \pi r^2$

$A = \pi 3^2$

$A = 9\pi\,cm^2$ or $28{,}3\,cm^2$

8). If the area of a circle is 36 cm² what is the radius of this circle?

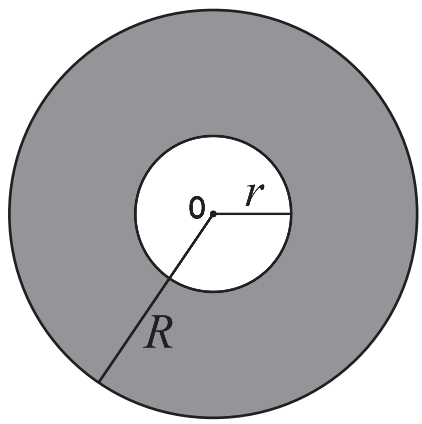
$A = \pi r^2$

$36\,cm^2 = \pi r^2$

$r^2 = 36 \div \pi\,cm^2$ or $r^2 \approx 11{,}5\,cm^2$

$r \approx 3{,}39\,cm$

9). In the figure below calculate the area of the shaded portion of the circle.



The big circle has a radius R; the smaller circle has a radius r.

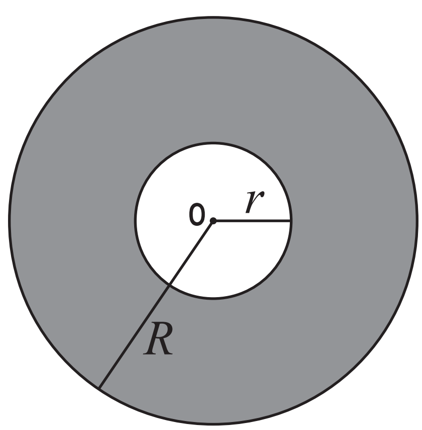
Area of big circle $= \pi R^2$

Area of small circle $= \pi r^2$

Area of shaded portion $= \pi R^2 - \pi r^2$

Area of shaded portion $= \pi(R^2 - r^2)$

10). In the figure below calculate the area of the shaded portion if the big circle has a radius R = 12 cm and the smaller circle has a radius r = 4 cm.



Area of big circle $= \pi R^2$

Area of small circle $= \pi r^2$

Area of shaded portion $= \pi R^2 - \pi r^2$

Area of shaded portion $= \pi(R^2 - r^2)$

Area of the shaded portion $= \pi(12^2 - 4^2)\, cm^2$

Area of shaded portion $= \pi \times (144 - 16)\, cm^2$

Area of shaded portion $= \pi \times (128)\, cm^2$

Area of shaded portion $= 128\pi\, cm^2$ or $402{,}1\, cm^2$

The following activity sheet may be used to consolidate your discussion on working with area of polygons and circles.

Activities

1). Calculate the area of a square if one side of the square is 12 cm.

2). If the area of a square is 169 cm^2 calculate the length of one side of the square.

3). Calculate the area of a rectangle if the length of the rectangle is 15 cm and the breadth is 9 cm.

4). If the area of a rectangle is 600 cm^2 and the length is 60 cm calculate the breadth of the rectangle.

5). Calculate the area of a triangle if the base is 18 cm and the height is 6 cm.

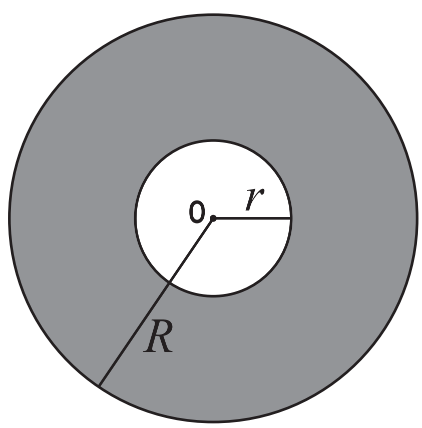
6). If the area of a triangle is 420 cm^2 and its base is 40 cm what is the height of this triangle?

7). Calculate the area of a circle with centre O and radius 8 cm.

8). If the area of a circle is 64 cm^2 what is the radius of this circle?

9). In the figure below calculate the area of the shaded portion of the circle. The big circle has a radius R; the smaller circle has a radius r.

10). In the figure below calculate the area of the shaded portion if the big circle has a radius R = 16 cm and the smaller circle has a radius r = 7 cm.



Solutions

1). $A = s^2$

$A = 12\, cm \times 12\, cm$

$A = 144\, cm^2$

2). $A = s^2$

$s^2 = 169\, cm^2$

$s = 13\, cm$

3). $A = l \times b$

$A = 15\ cm \times 9\ cm$

$A = 135\ cm^2$

4). $A = 1 \times \mathrm{b}$

$600\ cm^2 = 60\ cm \times b\ cm$

$b = 10\ cm$

5). $A = \frac{1}{2} \times base \times height$

$A = \frac{1}{2} \times 18\ cm \times 6\ cm$

$A = \frac{1}{2} \times 108\ cm^2$

$A = 54\ cm^2$

6). $A = \frac{1}{2} \times base \times height$

$420\ cm^2 = \frac{1}{2} \times 40\ cm \times h\ cm$

$420\ cm^2 \times 2 = 40\ cm \times h\ cm$

$840\ cm^2 \div 40\ cm = h$

$h = 21\ cm$

7). $A = \pi r^2$

$A = \pi 8^2$

$A = 64\pi\ cm$ or $201{,}1\ cm$

8). Area $= \pi r^2$

$64\ cm^2 = \pi r^2$

$r^2 = 64 \div \pi\ cm^2$ or $r^2 \approx 20{,}4\ cm^2$

$r \approx 4{,}51\ cm$

9). Area of big circle $= \pi R^2$

Area of small circle $= \pi r^2$

Area of shaded portion $= \pi R^2 - \pi r^2$

Area of shaded portion $= \pi(R^2 - r^2)$

10). Area of big circle $= \pi R^2$

Area of small circle $= \pi \mathrm{r}^2$

Area of shaded portion $= \pi R^2 - \pi r^2$

Area of shaded portion $= \pi(R^2 - r^2)$

Area of the shaded portion $= \pi(16^2 - 7^2)$

Area of shaded portion $= \pi \times (256 - 49)\ cm^2$

Area of shaded portion $= \pi \times (207)\ cm^2$

Area of shaded portion $= 207\pi\ cm^2$ or $650{,}3\ cm^2$

Another example of how to test area and perimeter of 2-D shapes

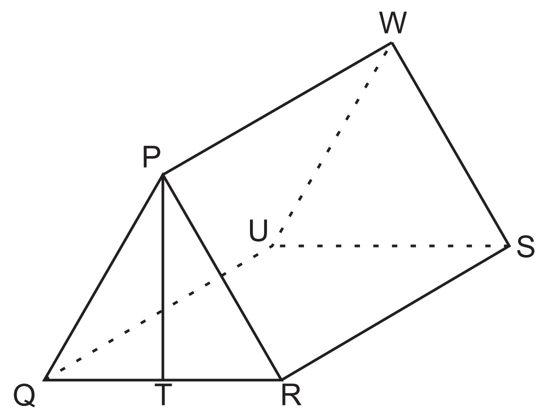
ANA 2014 Grade 9 Mathematics Item 11.3

11.3	The circumference of a circle is 52 cm. Calculate the area of the circle correct to 2 decimal places.	[4]

Notes:

Measurement: Surface area and volume of 3-D objects

ANA 2013 Grade 9 Mathematics Item 10.2



10.2.1. Determine the length of QT if QR = 48 cm. (Give a reason for your answer). [2]

10.2.2. If PQ = PR = 25 cm, show that PT = 7 cm. [4]

10.2.3. Hence, calculate the area of ΔPQR. [3]

10.2.4. Calculate the volume of the prism if RS = 80 cm. [2]

10.2.5. Calculate the surface area of the prism. [5]

What should a learner know to answer this question correctly?

Learners should be able to:

Item 10.2.1

- Describe, sort, name and compare triangles according to their sides and angles;
- Solve geometric problems involving unknown sides and unknown angles in triangles and quadrilaterals;
- Solve geometric problems involving the properties of congruent and similar triangles.

Item 10.2.2:

- Recall the theorem of Pythagoras that was taught in Grade 8;
- Solve problems based on the theorem of Pythagoras.

Item 10.2.3

Recall the area formula for calculating the area of triangles;

Use the area formula to calculate the area of ΔPQR.

Item 10.2.4

- Recall the formula for calculating the volume of prisms;

- Use the formula for calculating the volume of prisms effectively and accurately.

Item 10.2.5:

- Calculate the area of 2-D shapes, for example, polygons and circles;
- Define and work with prisms;
- Construct and work with nets of prism and cylinders;
- Define and work with the surface area of prisms and cylinders;
- Write down the formulae for calculating the surface area of prisms and cylinders;
- Substitute correctly within the formulae for calculating the surface area of prisms and cylinders.

Where is this topic located in the curriculum? Grade 9, Term 3

Content area: Measurement.

Topic: Surface area and volume of 3-D objects.

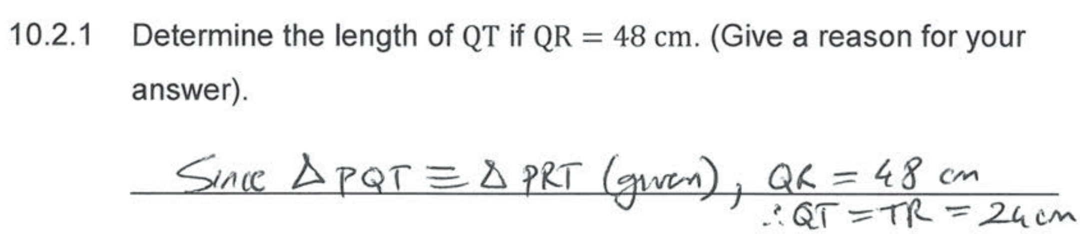
Concepts and skills:

- Use appropriate formulae and conversions between SI units to solve problems;
- Calculate the surface area, volume and capacity of cubes, rectangular prisms, triangular prisms and cylinders.

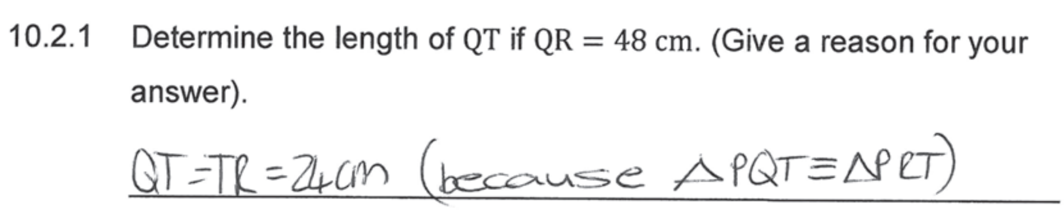
What would show evidence of full understanding?

If the learner obtained the correct solution by using an appropriate mathematical strategy, the correct formula, correct substitution and correct SI units as shown in the following examples.

Item 10.2.1


10.2.1 Determine the length of QT if QR = 48 cm. (Give a reason for your answer).

Since $\Delta PQT \equiv \Delta PRT$ (given), QR = 48 cm
∴ QT = TR = 24 cm


10.2.1 Determine the length of QT if QR = 48 cm. (Give a reason for your answer).

QT = TR = 24 cm (because $\Delta PQT \equiv \Delta PRT$)

Item 10.2.2

10.2.2 If $PQ = PR = 25$ cm, show that $PT = 7$cm.

~~$PQ^2 - PQ$~~ $25^2 = 44^2 - PT^2$

$625 = 576 - PT^2$

$PT^2 = 625 - 576$

$PT = \sqrt{49} = 7cm$

10.2.2 In ΔPQT:

$PT^2 = (25^2 - 24^2)\ cm^2$ (Pythagoras) ✓✓**M**

$= (625 - 576)\ cm^2$ ✓**M or** $(25+24)(25-24)\ cm^2$ ✓**M**

$= 49\ cm^2$

$PT = 7\ cm$✓**M**

Item 10.2.3

10.2.3 Hence, calculate the area of ΔPQR.

Area = ½ b x h

Area = ½ 48 x 7

Area = 24 x 7

Area = 168 cm²

10.2.3 Area $\Delta PQR = \frac{base \times height}{2}$ ✓**M** or $= \frac{1}{2}(base \times height)$

$= \frac{(48)(7)}{2}\ cm^2$✓**M**

$= (24)(7)\ cm^2$

$= 168\ cm^2$✓**CA**

Item 10.2.4

10.2.4 Volume = Area of base × height✓**M**

$= 168\ cm^2 \times 80\ cm$

$= 13\ 440\ cm^3$✓**CA**

Item 10.2.5

10.2.5 Surface area $= 2(Area\ \Delta PQR) + 2(Area\ PRSW) + Area\ QRSU$ ✓M

$= 2(168)\ cm^2 + 2(80 \times 25)\ cm^2 + 80(48)\ cm^2$ ✓✓✓M

$= 336\ cm^2 + 4\ 000\ cm^2 + 3\ 840\ cm^2$

$= 8\ 176\ cm^2$ ✓CA

What would show evidence of partial understanding?

Item 10.2.1

If the learner obtained the correct answer, but provided no valid mathematical reason for why the length of QT should be half of QR.

10.2.1 Determine the length of QT if $QR = 48$ cm. (Give a reason for your answer).

It is 24 because it is half of QR & thats half of 48.

Item 10.2.2

If the learner showed that PT = 7 cm, but did not use the correct mathematical process.

10.2.2 If $PQ = PR = 25$ cm, show that $PT = 7$cm.

PQ = PR = 25

PT = 7cm

= Righ angle hypotenuse side.

Item 10.2.3

If the learner used the correct formula but substituted incorrectly.

10.2.3 Hence, calculate the area of ΔPQR.

Area = ½ b×h×L

Area = 24×25×7

Area = 4200cm³

Item 10.2.4

If the learner used the correct formula, but substituted incorrectly.

10.2.4 Calculate the volume of the prism if $RS = 80$ cm.

Volume prism = l×h×b

Volume prism = 48×25×80

Volume prism = 960 000 cm³

10.2.4 Calculate the volume of the prism if $RS = 80$ cm.

V = L×B×H

= 7×48×24

= 8064

Item 10.2.5

If the learner calculated the area of some of the faces of the prism, but failed to calculate the area of all the faces of the prism.

10.2.5 Calculate the surface area of the prism.

A = ½ l×h.

A = ½ × 80×25

A = 2000/2.

A = 1000

What would show evidence of no understanding?

Item 10.2.1

- If the learner did not attempt the question;
- If the learner gave the measurement of an angle instead of finding the length of the line QT;

10.2.1 Determine the length of QT if $QR = 48$ cm. (Give a reason for your answer).

QR is gonna be 90° because its a acute angle.

10.2.1 Determine the length of QT if QR = 48 cm. (Give a reason for your answer).

180-132=48 . The right angle is equals=180.

- If the learner solved the problem incorrectly.

10.2.1 Determine the length of QT if QR = 48 cm. (Give a reason for your answer).

48 QT = QR = 48cm (given)(RHS),

Item 10.2.2

- If the learner did not answer the question;
- If the learner wrote out the correct answer (which was known since it was part of the question) without providing logical mathematical reasons. The learner made up some arbitrary working to produce the solution;

10.2.2 If $PQ = PR = 25$ cm, show that $PT = 7$cm.

25 - 7

18cm - 7cm - 11 - cm

4 + 3 = 7cm

10.2.2 If $PQ = PR = 25$ cm, show that $PT = 7$cm.

~~27~~ PQ = PR

25 - 18

= 7cm

∴ PT = 7cm

- If the learner answered the question incorrectly, giving random working with no mathematical backing.

10.2.2 If $PQ = PR = 25$ cm, show that $PT = 7$cm.

25 + 25 = 50

7 + 7 = 14

50 + 14 = 64cm

6

Item 10.2.3

- If the learner did not answer the question;
- If the learner calculated the area incorrectly by using the incorrect formula;

10.2.3 Hence, calculate the area of ΔPQR.

~~48cm~~ area = ½ + ½

= ~~48cm~~ 48cm + 48cm

= 96 cm²

OR

A = L + B + H

48 + 25 + 7

80 cm

OR

w × b × h

25 + 25 + 25

= 75

∴ PQR 25

- If the learner calculated the area incorrectly without using/showing a formula.

10.2.3 Hence, calculate the area of ΔPQR.

48 × 25 × 7

= 8400 cm²

10.2.3 Hence, calculate the area of ΔPQR.

48cm + 25cm + 7cm

= 80

10.2.3 Hence, calculate the area of ΔPQR.

48cm*25cm +7cm

= 80

Item 10.2.4

- If the learner did not answer the question;
- If the learner attempted to answer the question with no evidence of any mathematical thought.

10.2.4 Calculate the volume of the prism if $RS = 80$ cm.

RS = 80cm

= l is × $\frac{1}{2}$ +

Item 10.2.5

- If the learner did not attempt the question;
- If the learner used the incorrect formula and/or substituted incorrectly.

10.2.5 Calculate the surface area of the prism.

A = L×B

A = 48+25

A = 48×25

A = 1200

10.2.5 Calculate the surface area of the prism.

Surface area = $\frac{1}{2}$ ×b×h ×L

= 700cm² × 25cm × 7cm

= 1225 000 cm ÷ 7000

= 175cm³

What do the item statistics tell us?

Item 10.2.1

12% of learners answered the question correctly.

Item 10.2.2

3% of learners answered the question correctly.

Item 10.2.3

5% of learners answered the question correctly.

Item 10.2.4

4% of learners answered the question correctly.

Item 10.2.5

1% of learners answered the question correctly.

Factors contributing to the difficulty of the item

- Learners may have poor understanding of the concepts and skills tested in this item.
- Learners may not be able to sort, name and compare triangles according to their sides and angles.
- Learners may not be able to identify and describe the properties of congruent and similar shapes.
- Learners may not be able to solve geometric problems involving unknown sides and unknown angles in triangles and quadrilaterals.
- Learners may not be able to solve geometric problems involving the properties of congruent and similar triangles.
- Learners may not be able to use the theorem of Pythagoras appropriately.
- Learners may not be able to use the formulae for the surface area of polygons.
- Learners may not be able to use the formulae for the volume of polygons.

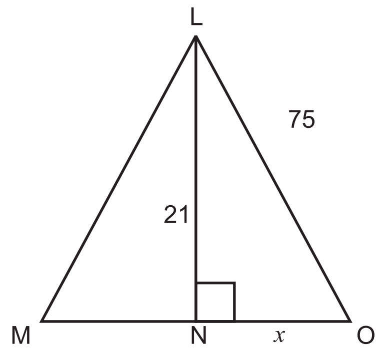
Teaching strategies

Theorem of Pythagoras

- The theorem of Pythagoras states that in a right angled triangle the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides.
- The theorem can be written as an equation relating the lengths of the sides a, b and c in a triangle ABC: $a^2 = b^2 + c^2$, where side a is the side opposite the right angle. This is often called the Pythagorean equation.
- Learners need to be able to use Pythagoras's equation to find unknown lengths in triangles.
- Use the following example to assist your learners with the theorem of Pythagoras. Work through the example giving explanations such as those that follow to help your learners to use the theorem correctly.
- You should draw the diagram on the board to facilitate the whole class discussion on the activity.

Example

In the following triangle calculate the value of the unknown side x.



- Identify the right angle (90°) in the triangle.

  In ΔLNO

  $Angle\ N = 90°$ [given]

- Since you have identified a right angle in the triangle, you may use Pythagoras's theorem to find the length of the side that is not known.

- The first step is to write out the Pythagorean equation, using the sides of the triangle in the given problem.

  $LO^2 = LN^2 + NO^2$

- The next step is to isolate the unknown side, so that you can solve for the unknown length.

  $NO^2 = LO^2 - LN^2$

- Now you can substitute the given values into the Pythagorean equation and calculate the length of side NO.

- Check that your learners know how to work out squares of numbers (e.g. 75^2) correctly with their calculators so that they can find the correct answer to the question.

  $NO^2 = 75^2 - 21^2$

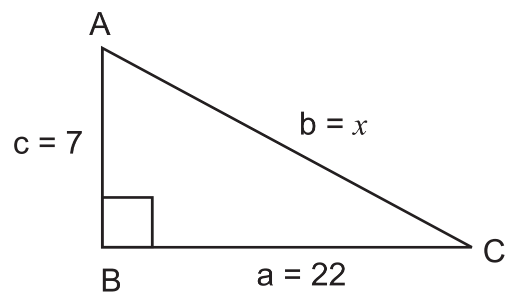
  $NO^2 = 5\ 184$

  $NO = 72$

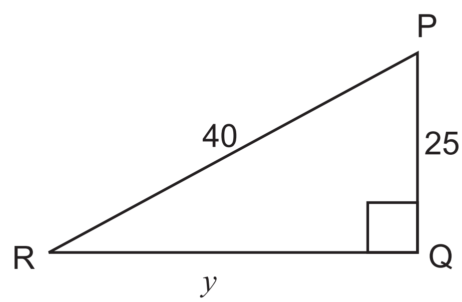
Activity

Use the theorem of Pythagoras to calculate the length of the unknown side in the questions that follow. Please note that the diagrams are not drawn to scale.

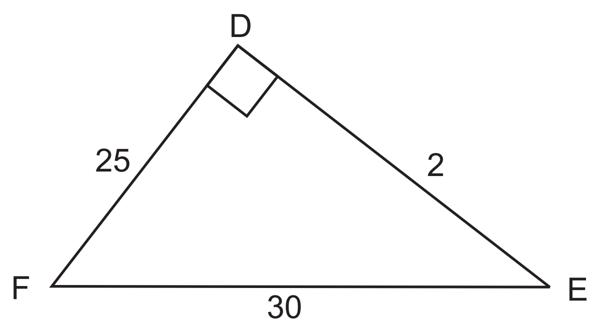
1).



2).



3).



Solutions

1). $AC^2 = AB^2 + BC^2$

$AC^2 = 7^2 + 22^2$

$AC^2 = 533$

$AC = 23{,}1$

2). $PR^2 = PQ^2 + RQ^2$

$RQ^2 = PR^2 - PQ^2$

$RQ^2 = 40^2 - 25^2$

$RQ^2 = 975$

$RQ = 31{,}2$

3). $EF^2 = DF^2 + DE^2$

$DE^2 = EF^2 - DF^2$

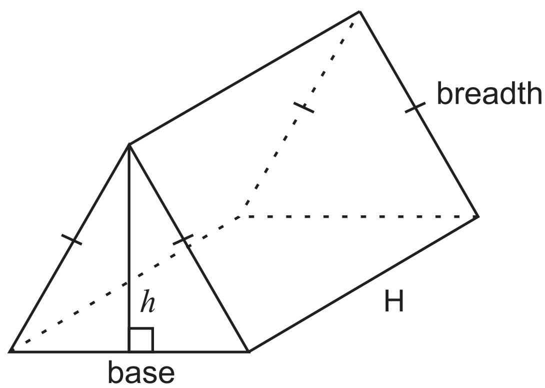
$DE^2 = 30^2 - 25^2$

$DE^2 = 275$

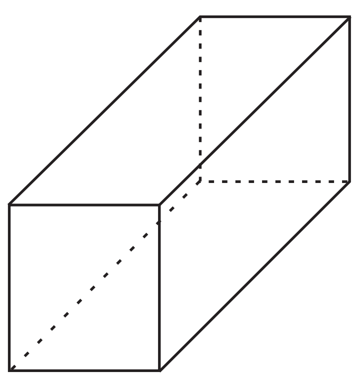
$DE = 16,6$

Working with the volume of prisms and cylinders

- When working with volume provide your learners with concrete manipulatives[1] to work with, for example, cereal boxes, tissue boxes or empty cool-drink cans.
- Discuss the definition of a prism with your learners. A **prism** gets its name from the shape it is based on (this shape could be any polygon). All the other faces of a prism are rectangular.
- For example:
- A triangular prism is a prism with two triangular faces that are the same and all of the other side faces are rectangles which join the two triangular faces together.

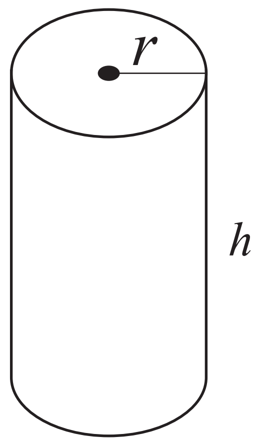


- A square prism is a prism with two square faces that are the same and all of the other side faces are rectangles which join the two square faces together.



[1] Manipulatives refers to any concrete objects that assist with the conceptualisation of mathematical concepts.

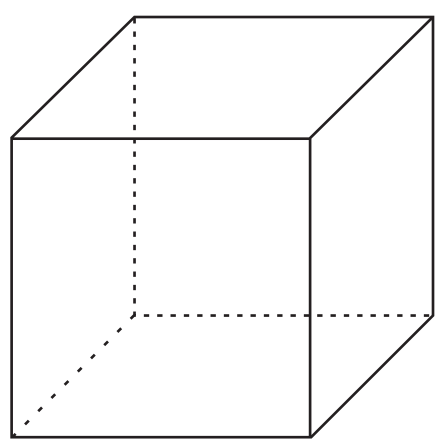
- Discuss the definition of a **cylinder** with your learners. A cylinder is a shape which has two circular faces which are joined by a curved surface. When you cut the cylinder open the curved surface can be made flat into a rectangle.



- Revise the formulae for the volume of common prisms and the cylinder with the learners.
- The following general formulae may be used to assist you when revising the concept of volume. **Volume = area of the base x height**.

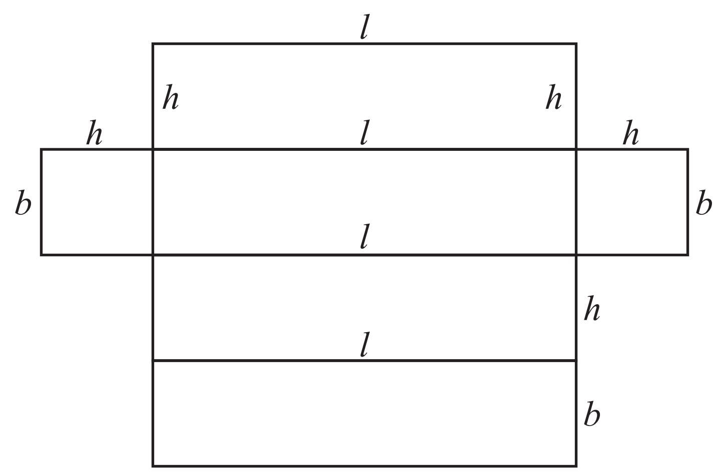
- There are different variations on this general formula for the special types of prisms your leaners need to know about. The general rule also applies to cylinders.

- Volume of a **cube/cuboid prism** $= l \times l \times l = l^3$ where l is the length of one side. All sides in a cube/cuboid are of equal length.

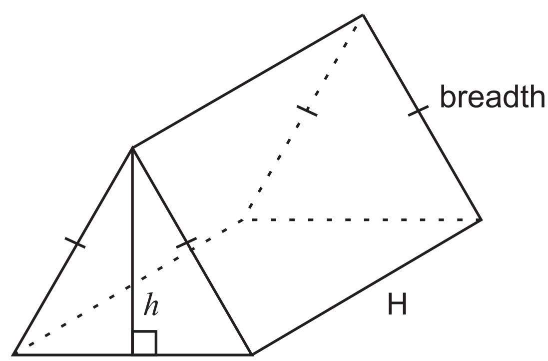


l cube

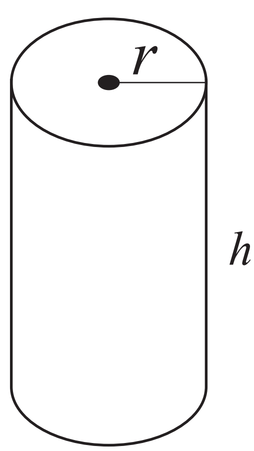
- Volume of a **rectangular prism** $= l \times b \times h$ where l is the length, b is the breadth and h is the height of the prism.



- Volume of a triangular prism = $(\frac{1}{2} \times b \times h) \times H$
  where b is the base of the triangular face, h is the height of the triangle and H is the height of the prism.



- Volume of a **cylinder** $= \pi r^2 \times h$ where r is the radius of the base of the cylinder and h is the height of the cylinder.

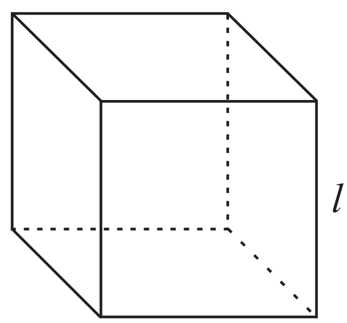


Use the following examples to assist your learners when working with calculating the volume of prism and volume of cylinders.

- You should work through the examples step by step. Each time you do an example:
- Draw the diagram on the board;
- Identify the lengths of the sides that you will use in your calculation;
- Discuss the choice of the correct formula to be used for the shape in the question;
- Work through the substitution into the chosen formula showing your learners how it should be done; and
- Complete the calculation with the whole class, making sure that they can all do the necessary steps in the calculation correctly.

Examples

1). Calculate the volume of the following cube. The length of the sides of the cube is 5 cm each.



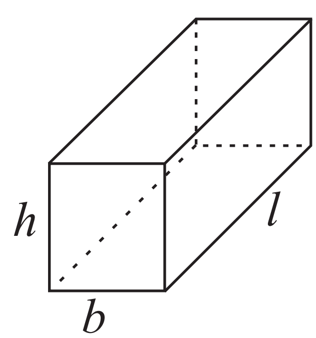
Volume of cube

$= l \times l \times l = l^3$

$= 5\ cm \times 5\ cm \times 5\ cm$

$= 125\ cm^3$

2). Calculate the volume of the rectangular prism that follows.
The length of the prism is 6 cm, the breadth is 4 cm and the height is 10 cm.



Volume of rectangular prism

$= l \times b \times h$

$= 6\ cm \times 4\ cm \times 10\ cm$

$= 240\ cm^3$

3). Calculate the volume of the triangular prism with the following dimensions:

The base of the triangular face is 10 cm, the height of the triangle is 8 cm and the height of the triangular prism is 15 cm.

Volume of triangular prism

$= (\frac{1}{2} \times b \times h) \times H$

$= (\frac{1}{2} \times 10\,cm \times 8\,cm) \times 15\,cm$

$= (\frac{1}{2} \times 80\,cm) \times 15\,cm$

$= 600\,cm^3$

4). Calculate the volume of a cylinder with the following dimensions:

The radius of the circular base is 6 cm and the height of the cylinder is 12 cm.

Volume of a cylinder

$= \pi r^2 \times h$

$= \pi 6^2 \times 12$

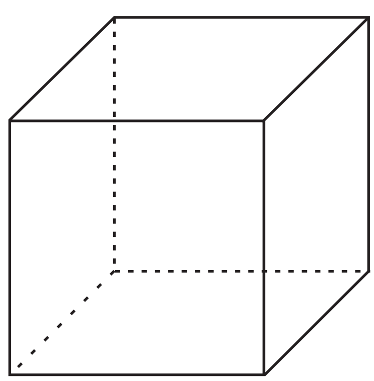
$= 432\,\pi \text{ or } 1\,357{,}2\,cm^3$

Use the following activity sheet to consolidate the examples and your discussion on the volume of prisms and cylinders.

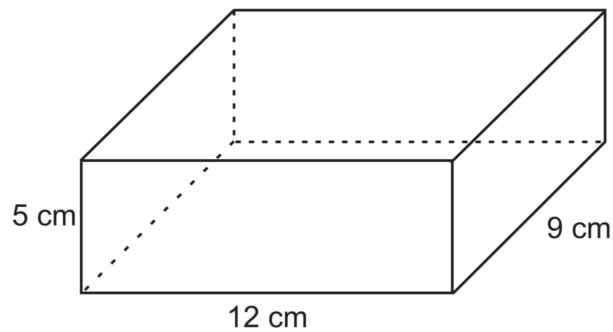
Activity

Using the formula for volume of different prisms and the formula for calculating the volume of cylinders, solve the following problems. Please note that the diagrams are not drawn to scale.

1). Calculate the volume of the following cube. The length of the sides of the cube is 8 cm.



2). Calculate the volume of the rectangular prism that follows. The length of the prism is 9 cm, the breadth is 12 cm and the height is 5 cm.



3). Calculate the volume of the triangular prism with the following dimensions. First draw the shape, marking in the given information.

The base of the triangular face is 12 cm, the height of the triangle is 6 cm and the height of the triangular prism is 18 cm.

4). Calculate the volume of a cylinder with the following dimensions. First draw the shape, marking in the given information.

The radius of the circular base is 8 cm and the height of the cylinder is 15 cm.

Solutions

1). Volume of cube

$= l \times l \times l = l^3$

$= 8\,cm \times 8\,cm \times 8\,cm$

$= 512\,cm^3$

2). Volume of rectangular prism

$= l \times b \times h$

$= 9\,cm \times 12\,cm \times 5\,cm$

$= 540\,cm^3$

3). Volume of triangular prism

$= (\frac{1}{2} \times b \times h) \times H$

$= (\frac{1}{2} \times 12\,cm \times 6\,cm) \times 18\,cm$

$= (\frac{1}{2} \times 72\,cm) \times 18\,cm$

$= 648\,cm^3$

4). Volume of a cylinder

$= \pi r^2 \times h$

$= \pi 8^2 \times 15$

$= 960\pi$

or

$3\,015{,}9 \text{ cm}^3$

Working with surface area of prisms and cylinders

- Construct nets of various 3-D objects.
- You could use the printable nets at the end of this section. Some of the nets provided are for shapes for which we are not required to find the surface area or volume according to the curriculum. It is still interesting to work with and discuss these shapes to familiarise learners with a variety of shapes **(see printables)**.
- Assist the learners to construct the various 3-D objects on their own or in pairs.

- Explain the concept of surface area to your learners.
- The surface area of an object is the sum of the area of all its faces.

- Assist the learners to work with nets of common prisms: for example, a cube, a rectangular prism and a triangular prism.
- Point out all the faces in each of these prisms. While you do this help the learners to see what surface area means in relation to the particular shape they are manipulating.
- Provide the formulae for calculating the surface area of a cube/cuboid, a rectangular prism and a triangular prism.

Surface area of a cube:

A cube has 6 equal faces

The area of 1 face $= s \times s$

$\therefore$ total surface area of a cube $= 6 \times s \times s$

Surface area of rectangular prism:

The rectangular prism has 6 faces

3 pairs of faces are equal

$\therefore$ total surface area of a rectangular prism

$$= 2(l \times b) + 2(l \times h) + 2(b \times h)$$

Surface area of a triangular prism:

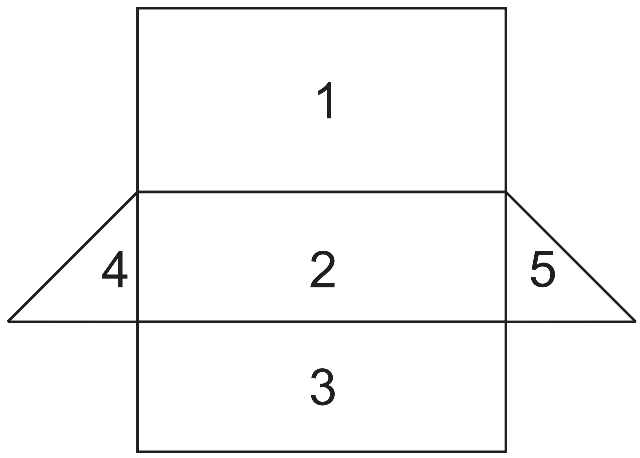
The triangular prism has 5 faces

There are 3 rectangular faces and 2 congruent triangular faces

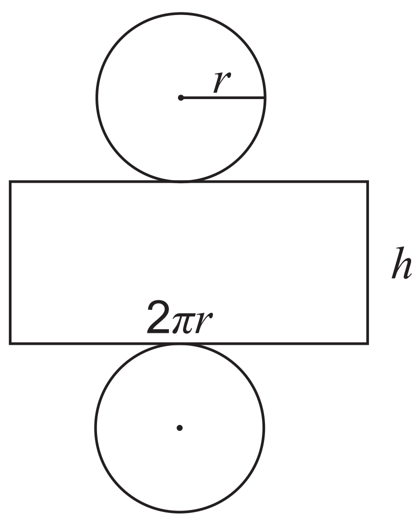
∴ total surface area = area 1 + area 2 + area 3 + area 4 + area 5

Surface area of a triangular prism

$=(s1+s2+s3)\times H+2(\frac{1}{2}b\times \perp h)$



- Explain how to calculate the surface area of a cylinder by using the net of a cylinder **(see printables)**.



Surface area of a cylinder:

The cylinder is made up of two circular bases and one rectangular body.

The surface area of a cylinder = 2 × area of circular base + area of rectangular body.

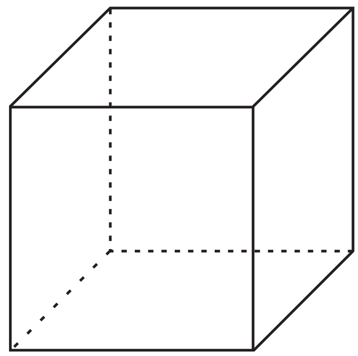
∴ total surface area of a cylinder $=2\times \pi r^2+2\pi r h$.

Explain the calculation of surface area to your learners using examples.

Use the following examples to assist you.

Examples

1). Calculate the surface area of a cube whose sides have a length of 8 cm each.

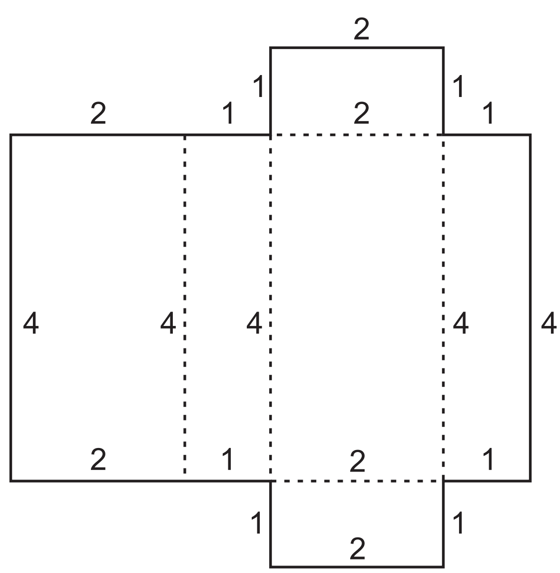


Surface area of cube

$= 6 \times s \times s$

$= 6 \times 8 \times 8$

$= 384\, cm^2$



2). Using the net shown here calculate the surface area of this rectangular prism.

Surface area of rectangular prism

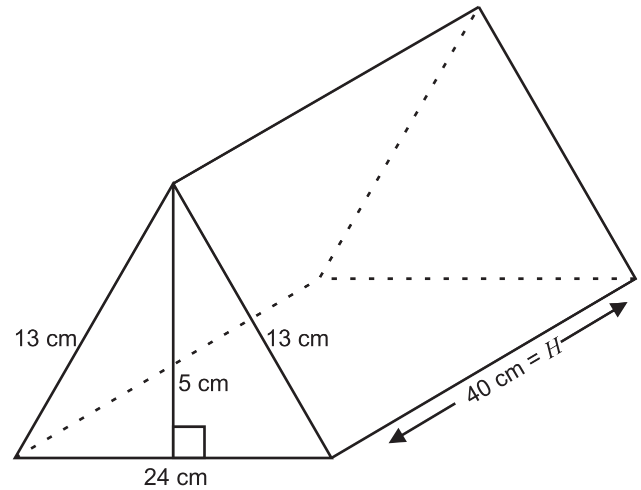
$= 2(l \times b) + 2(l \times h) + 2(b \times h)$

$= 2(2 \times 1) + 2(4 \times 2) + 2(4 \times 1)$

$= 2(2) + 2(8) + 2(4)$

$= 4 + 16 + 8 = 36\, cm^2$

3). Using the diagram below calculate the surface area of the triangular prism.



- The base of the triangular face is 24 cm;
- The $\perp$ height of the triangle is 5 cm;
- The height of the triangular prism is 40 cm;
- The triangular faces are isosceles triangles with sides measuring 13 cm each.

The surface area of a triangular prism

$$= (s1 + s2 + s3) \times H + 2(\tfrac{1}{2} \times b \times \perp h)$$
$$= (13 + 13 + 24) \times 40 + 2(\tfrac{1}{2} \times 24 \times 5)$$
$$= 50 \times 40 + 2(60)$$
$$= 2\,120\ cm^2$$

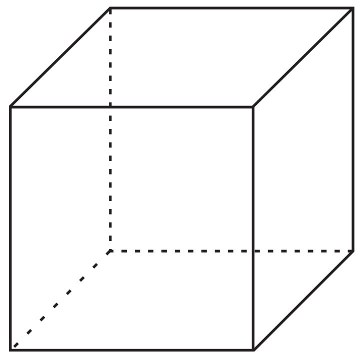
4). Calculate the surface area of a cylinder with the following dimensions: the radius of the circular base is 7 cm and the height of the cylinder is 22 cm.

Surface area of cylinder

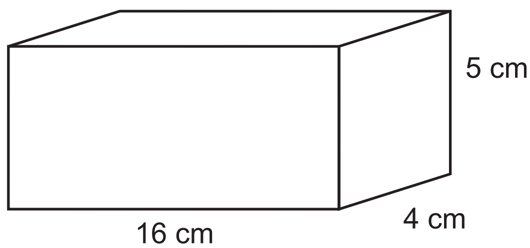
$$= 2 \times \pi r^2 + 2\pi r H$$
$$= 2 \times \pi 7^2 + 2\pi 7 \times 22$$
$$= 2 \times 49\pi + 308\pi$$
$$= 98\pi + 308\pi$$
$$= 406\pi\ cm^2 \text{ or } 1\,275{,}5\ cm^2$$

Activity

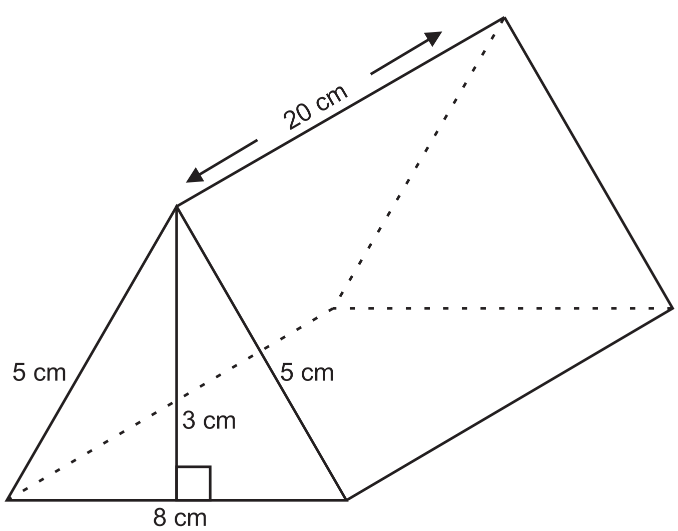
1). Calculate the surface area of a cube which has sides of a length of 11 cm each.



2). Using the next diagram calculate the surface area of this rectangular prism.



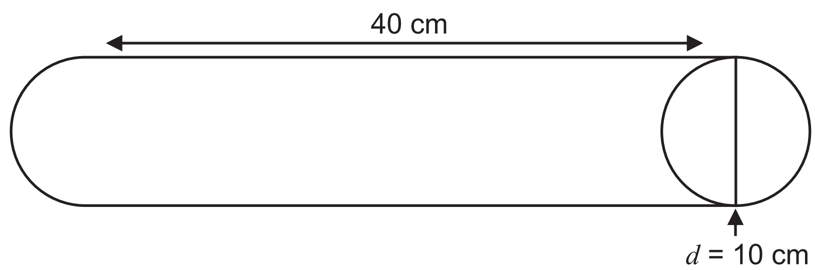
3). Using the following diagram calculate the surface area of the triangular prism. The base of the triangular face is 8 cm, the ⊥ height of the triangle is 3 cm and the height of the triangular prism is 20 cm. The triangular faces are isosceles triangles with base sides measuring 5 cm each.



4). Using the diagram that follows calculate the surface area of this cylinder. The diameter of the circular base is 10 cm and the height of the cylinder is 40 cm.

Solutions

1). Surface area of cube



$= 6 \times s \times s$

$= 6 \times 11 \times 11$

$= 762\ cm^2$

2). Surface area of rectangular prism

$= 2(l \times b) + 2(l \times h) + 2(b \times h)$

$= 2(5 \times 4) + 2(16 \times 4) + 2(16 \times 5)$

$= 2(20) + 2(64) + 2(80) = 40 + 128 + 160 = 328\ \text{cm}^2$

3). Surface area of triangular prism

$= (s1 + s2 + s3) \times H + 2(\frac{1}{2} \times b \times \perp h)$

$= (5 + 5 + 8) \times 20 + 2(\frac{1}{2} \times 8 \times 3)$

$= 18 \times 20 + 2(12)$

$= 384\ cm^2$

4). Surface area of cylinder $= 2 \times \pi r^2 + 2\pi r H$

But $d = 2r \therefore r = \frac{d}{2} = \frac{10}{2} = 5$ i

$\therefore$ Surface area of cylinder

$= 2 \times \pi 5^2 + 2\pi 5 \times 40$

$= 2 \times 25\pi + 400\pi$

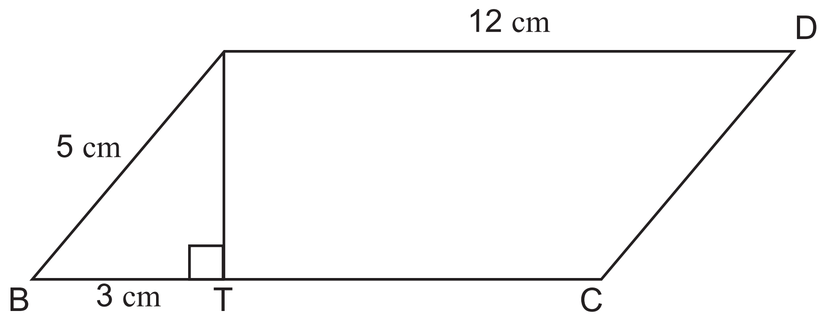
$= 50\pi + 400\pi$

$= 450\pi\ cm^2$ or $1\,413{,}7\ cm^2$

Another example of how to test surface area and volume of 3-D objects

ANA 2014 Grade 9 Mathematics Item 11.1

QUESTION 11



In parallelogram $ABCD$,
$AB = 5\ cm, AD = 12\ cm, BT = 3\ cm$ and $AT \perp BC$.

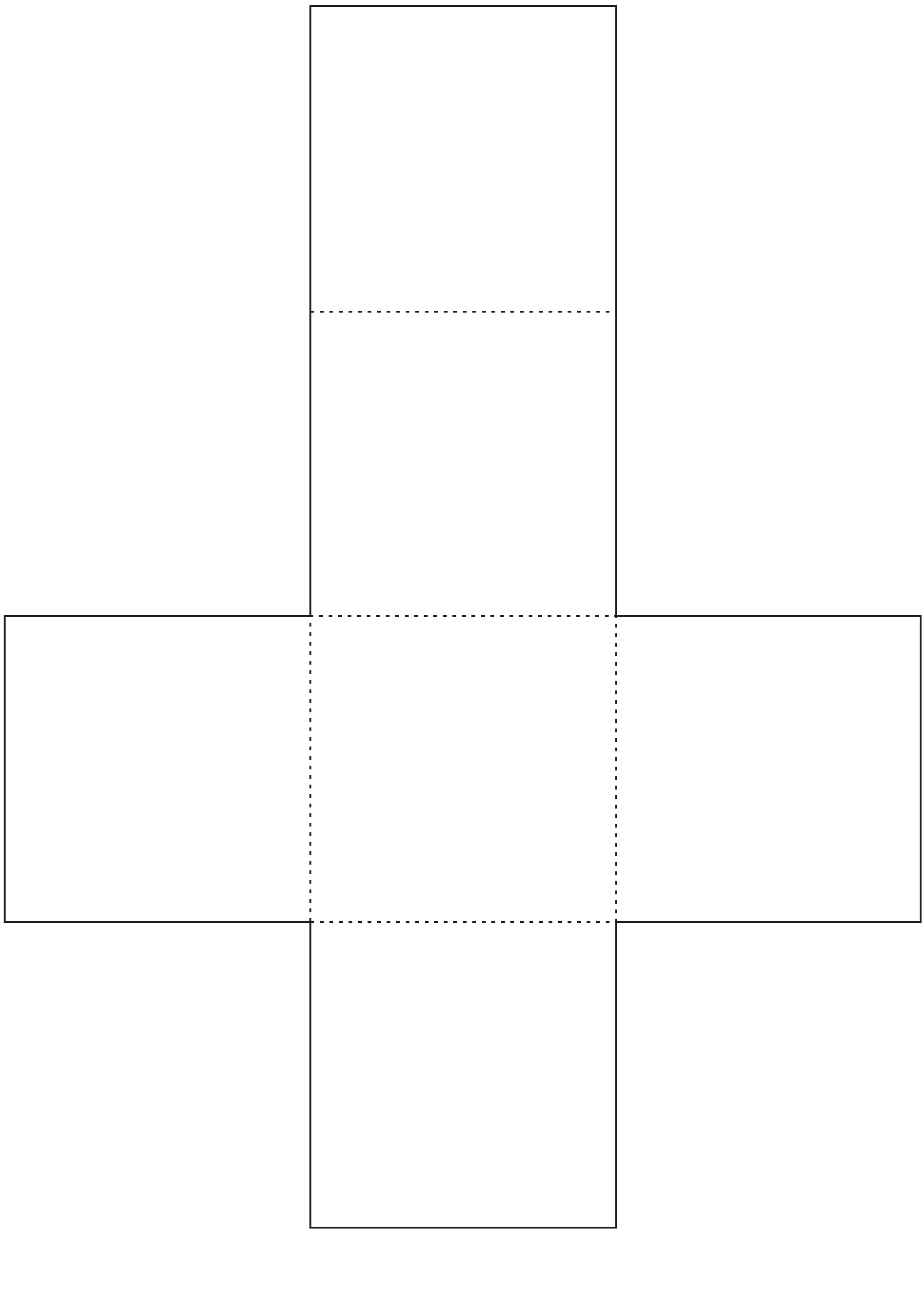
11.1 Calculate the length of AT. [3]

11.2 Calculate:

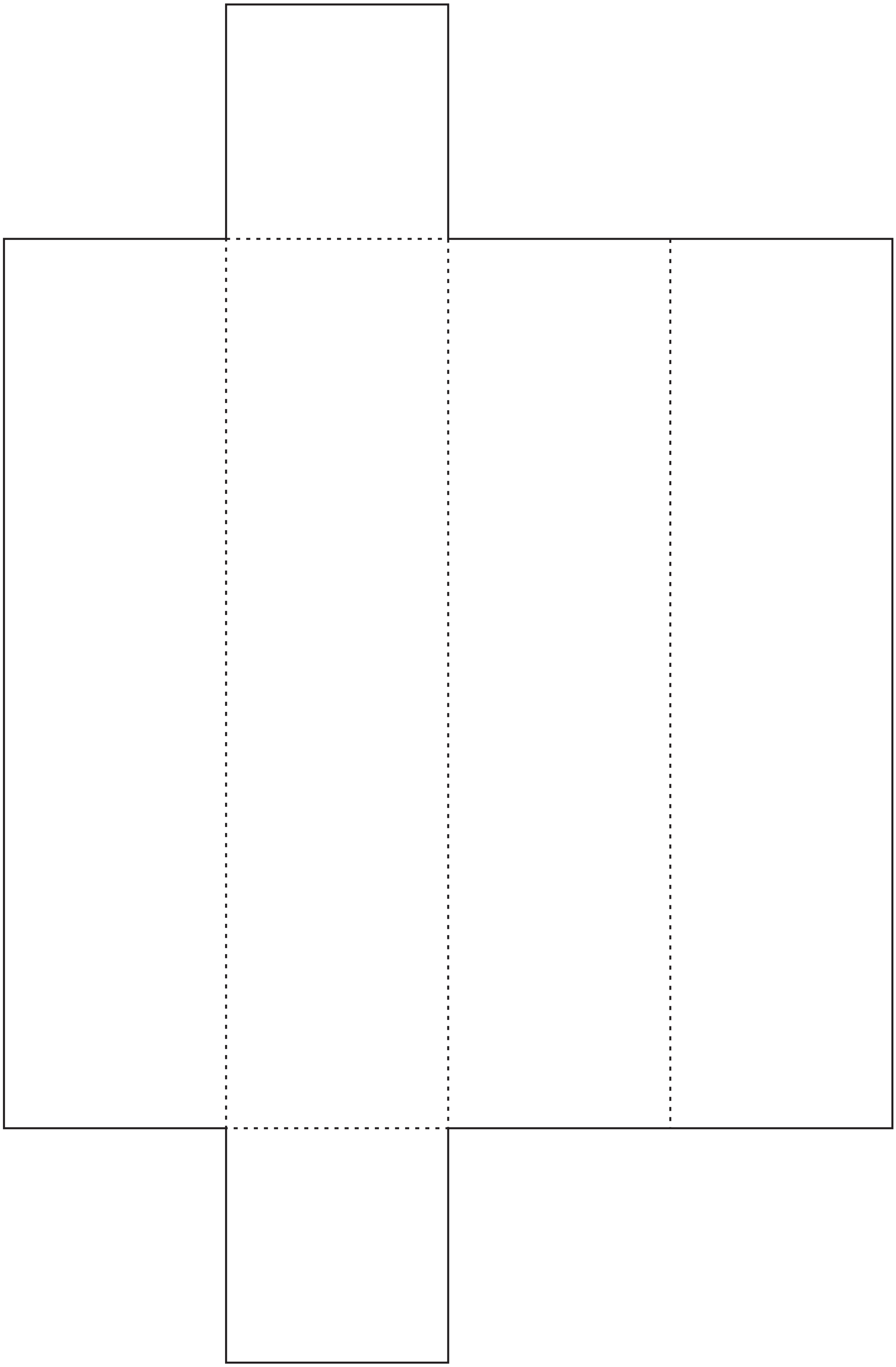
11.2.2 the area of trapezium $ADCT$. [3]

Notes:

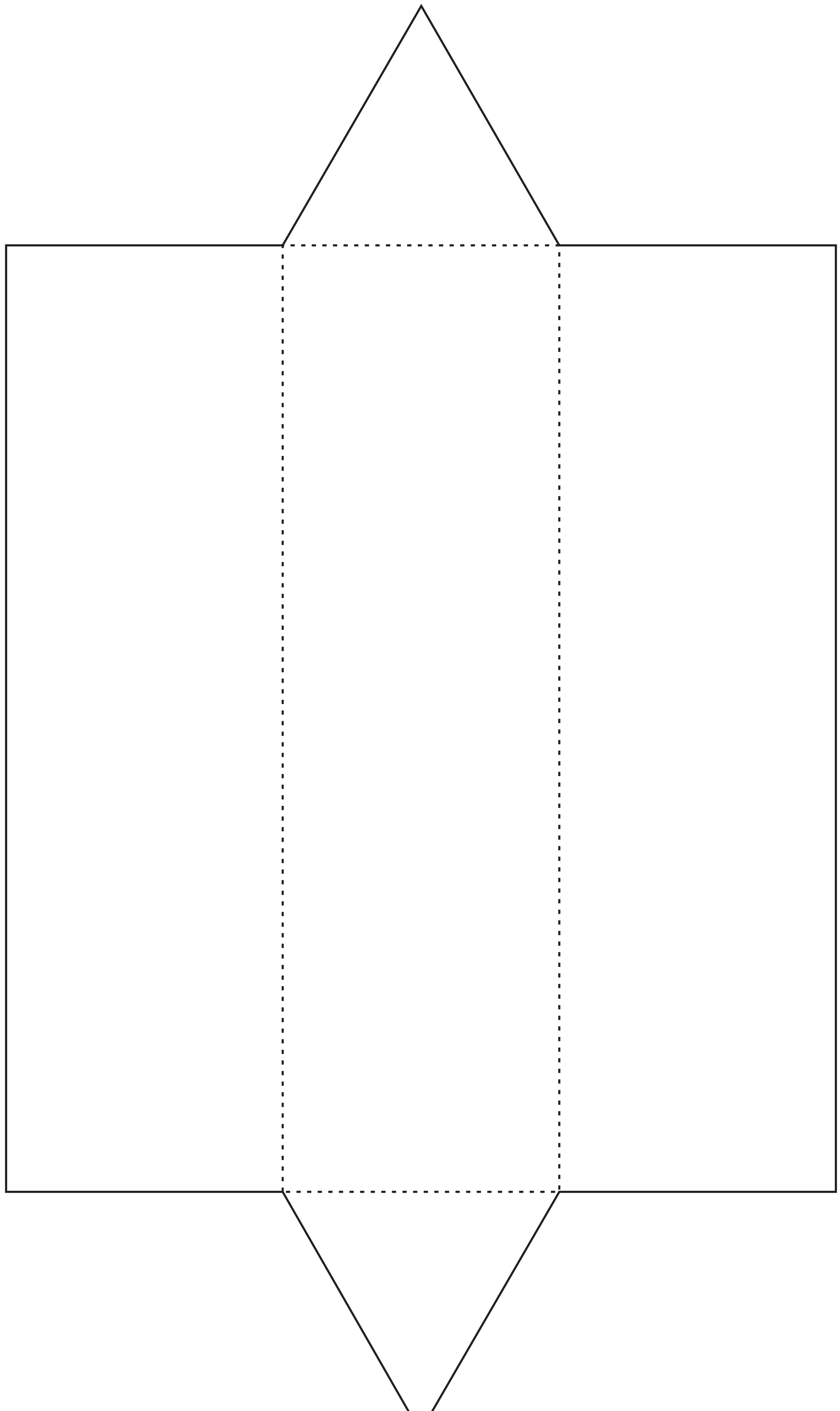
Printable: Net of Cube



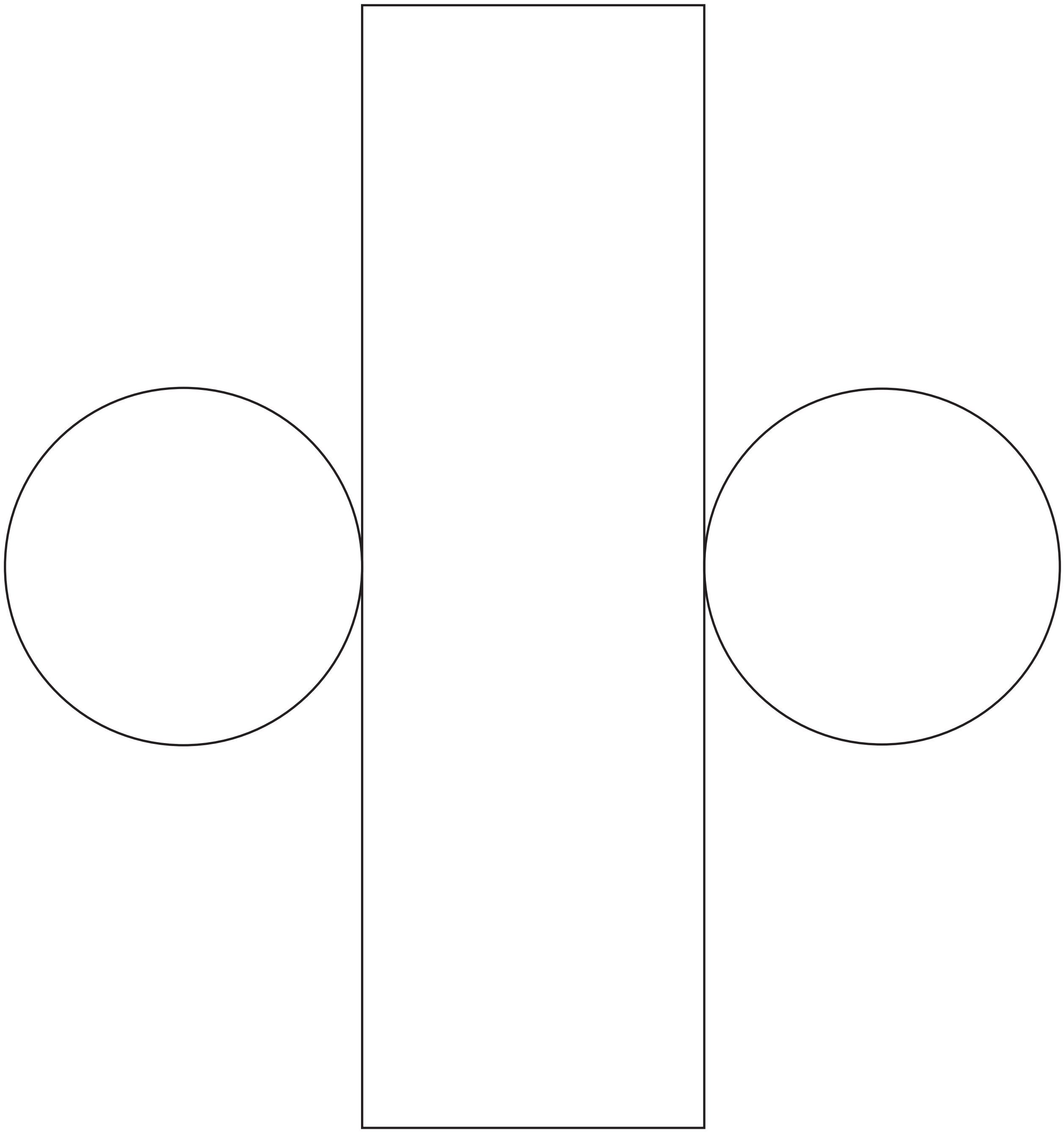
Printable: Net of Rectangular prism



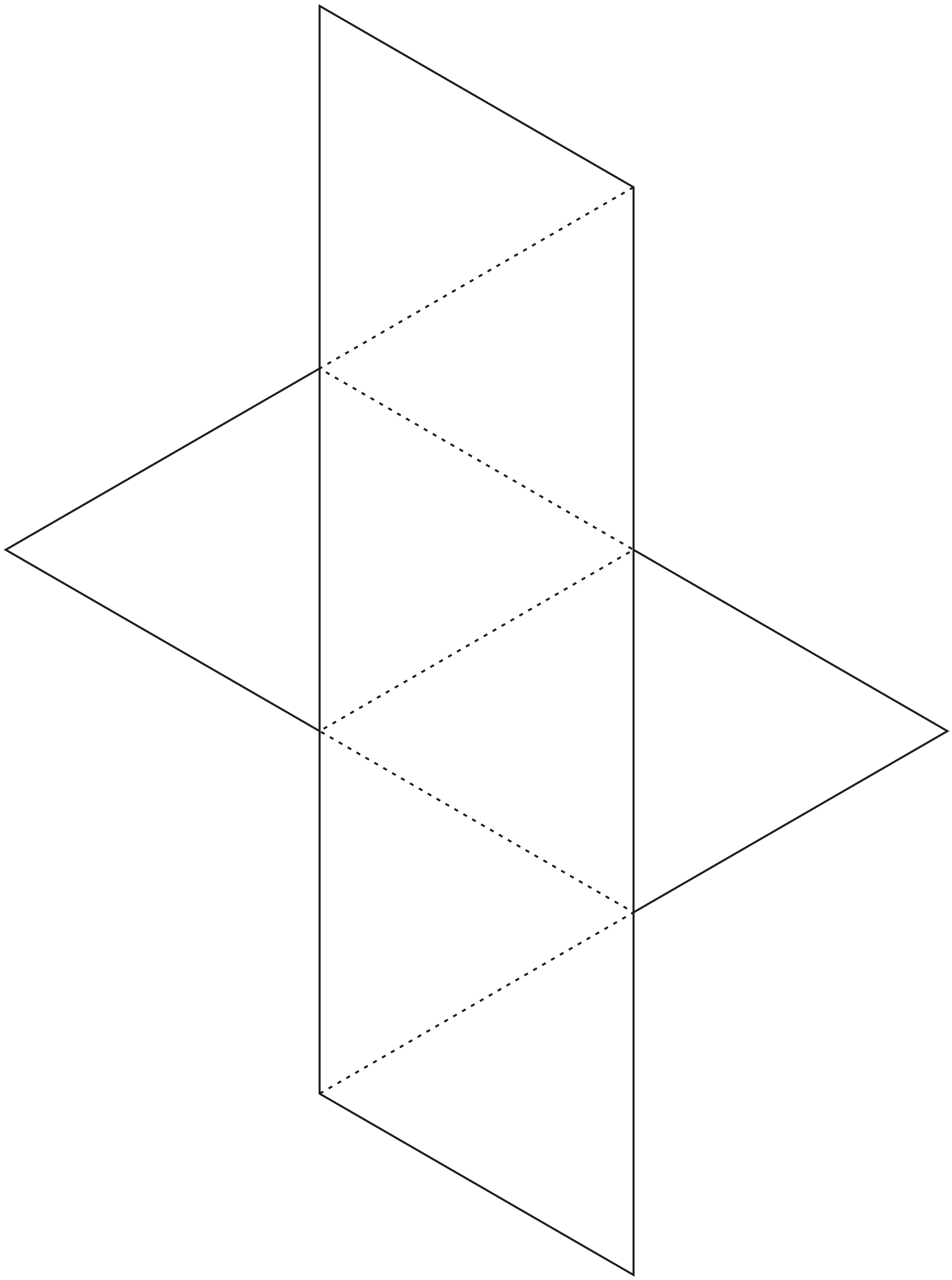
Printable: Net of Triangular prism



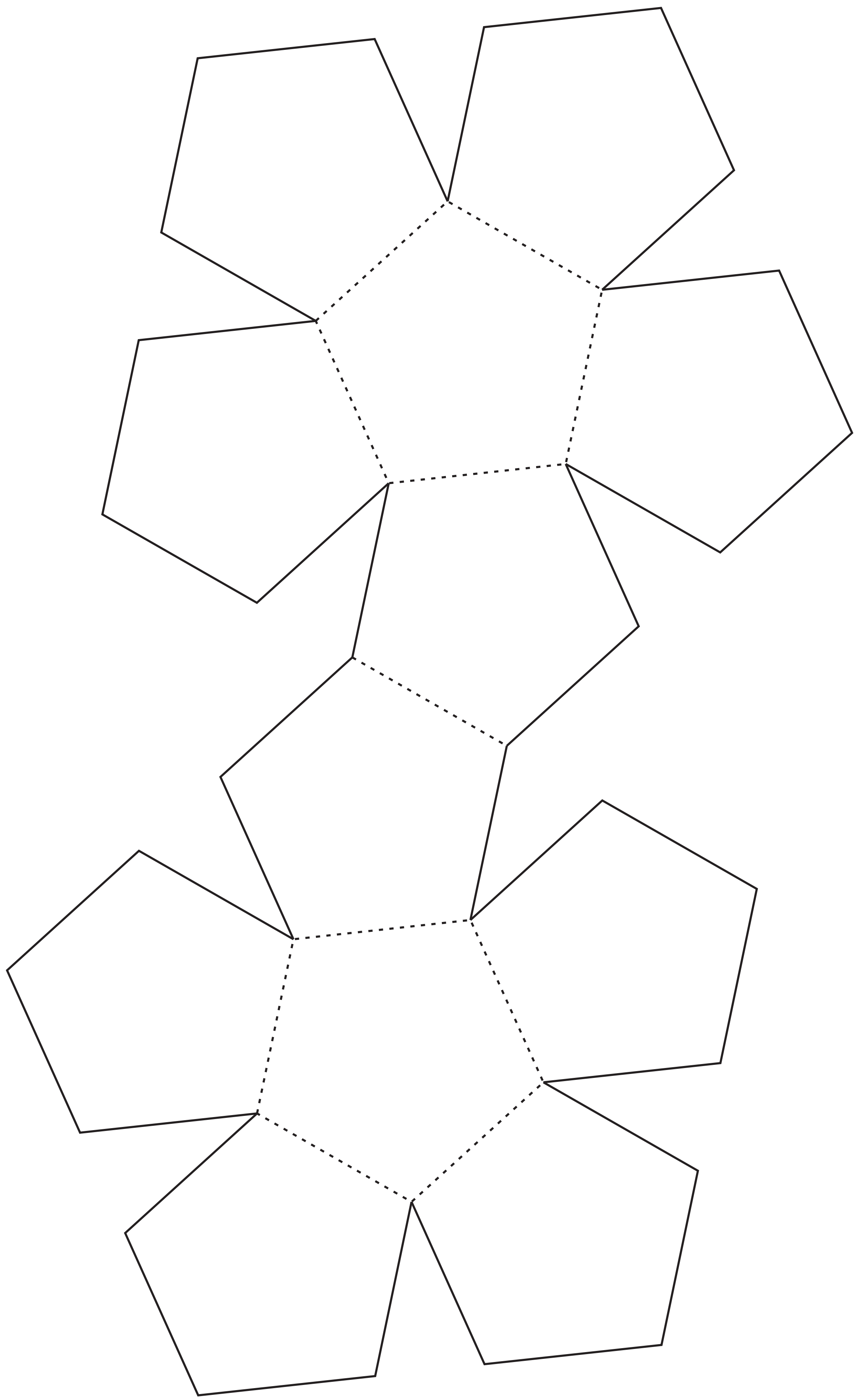
Printable: Net of Cylinder



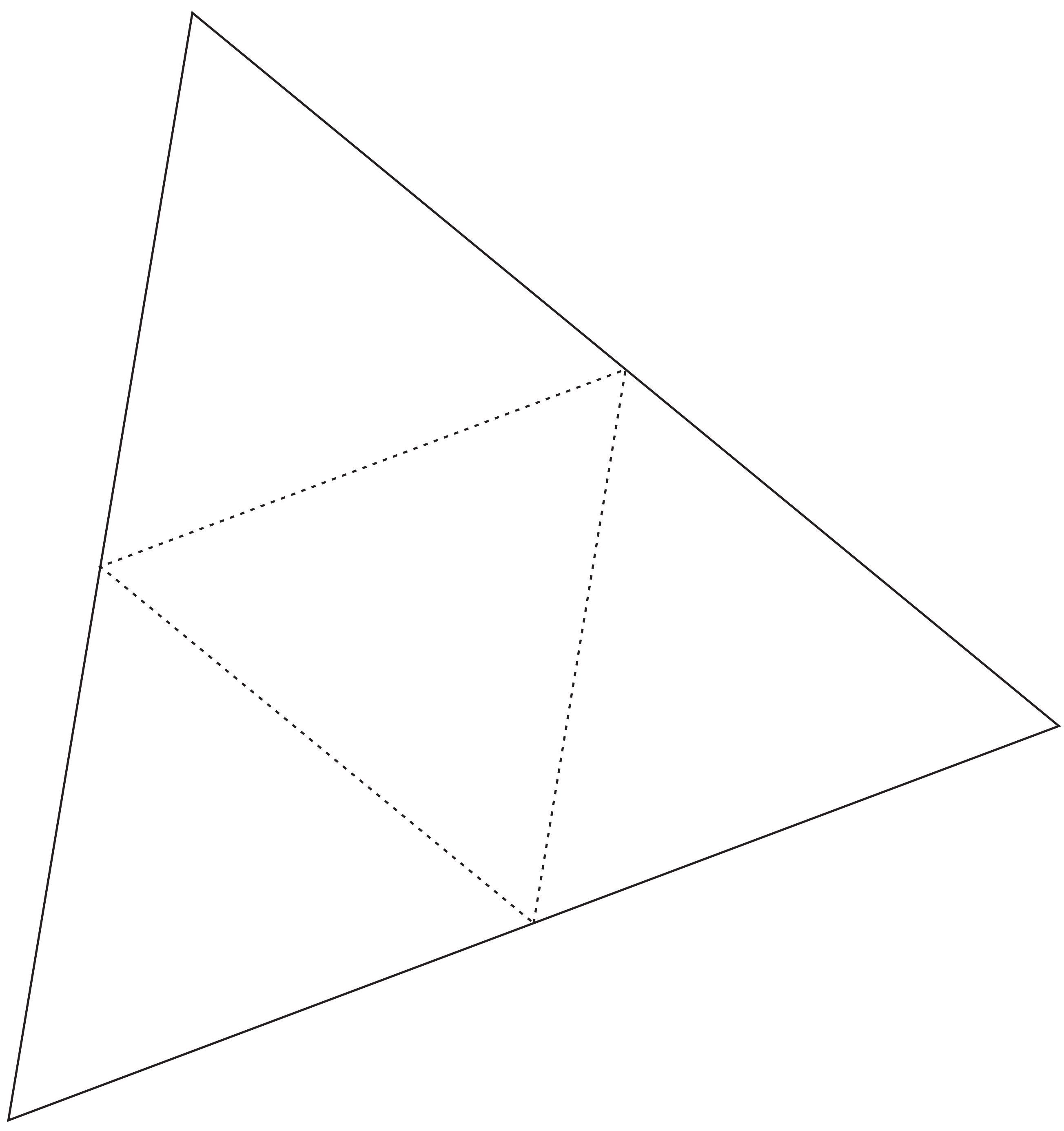
Printable: Net of Octahedron



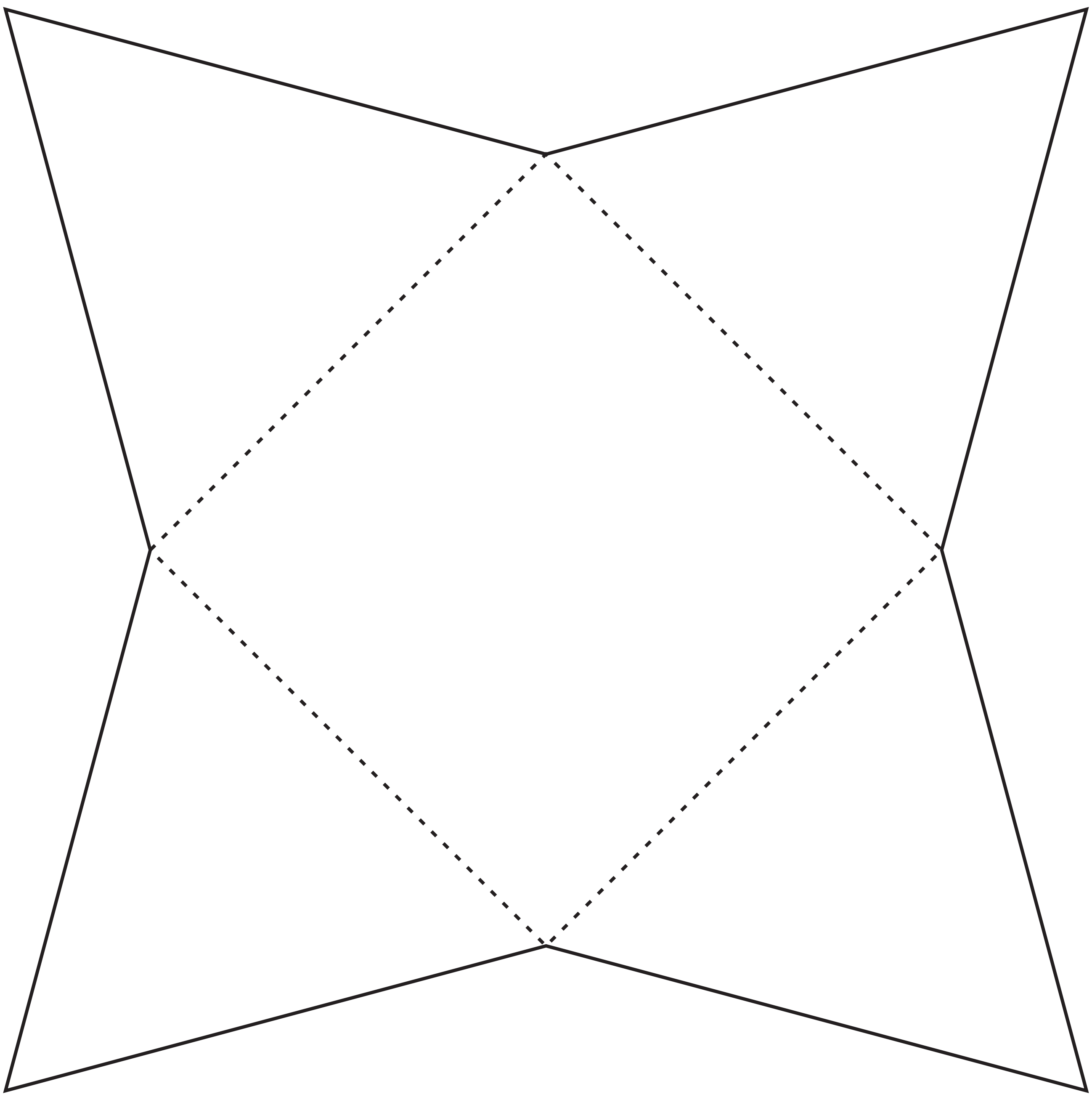
Printable: Net of Dodecahedron



Printable: Net of Triangular pyramid

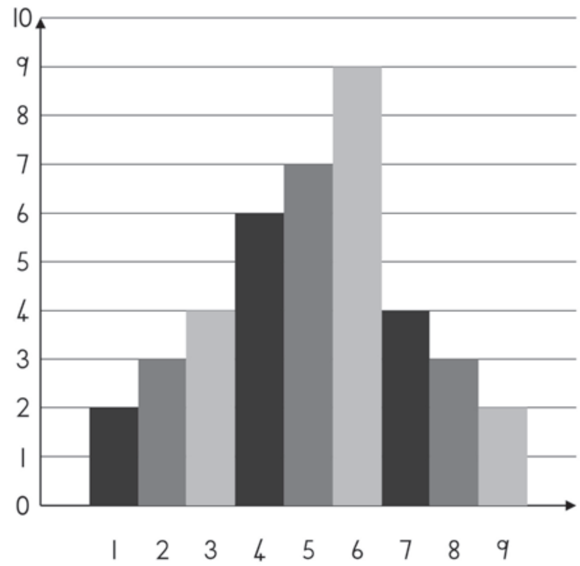


Printable: Net of Square based pyramid



Data Handling: Analyse, interpret and report data

ANA 2013 Grade 9 Mathematics Items 11.1, 11.2, 11.3 and 11.4



11.1. Complete the frequency table for the given histogram. [4]

Mark x	Frequency f	Product $f.x$
1	2	2
2		

11.2. How many learners were tested? [1]

11.3. Calculate the mean test mark. [3]

The mean mark =

11.4. What percentage of the learners obtained 7 or more out of 10 for the test? [2]

What should a learner know to answer these questions correctly?

Learners should be able to:

Item 11.1

- Critically read and interpret data represented in a variety of ways.

Item 11.2, 11.3 and 11.4

- Critically analyse data;
- Answer questions related to summary statistics of data.

Where is this topic located in the curriculum? Grade 9 Term 4

Content area: Data handling.

Topic: Analyse, interpret and report data.

Concepts and skills:

- Critically read and interpret data;
- Critically analyse data by answering questions related to:
- Data collection methods;
- Summary statistics of data;
- Sources of error and bias in data.

What would show evidence of full understanding?

If the learner obtained the correct solution by using an appropriate mathematical strategy to answer the question.

Item 11.1

11.1 Complete the frequency table for the given histogram.

Mark x	Frequency f	Product $f.x$
1	2	2
2	3	6
3	4	12
4	6	24
5	7	35
6	9	54
7	4	28
8	3	24
9	2	18

Item 11.2

11.2 How many learners were tested?

40 learners

Item 11.3

11.3 Calculate the mean test mark.

The mean mark = $\frac{\sum fx}{\sum f}$

= $\frac{203}{40}$

= 5,075

Item 11.4

11.4 What percentage of the learners obtained 7 or more out of 10 for the test?

percentage of learners = $\frac{9}{40} \times 100$

= 22,5

What would show evidence of partial understanding?

Item 11.1

If the learner completed part of the table correctly: in the example the learner has completed the frequency column correctly but has not found the product correctly.

11.1 Complete the frequency table for the given histogram.

Mark x	Frequency f	Product $f.x$
1	2	2
2	3	3
3	4	4
4	6	0
5	7	7
6	9	9
7	4	4
8	3	3
9	2	2

Item 11.2

If the learner followed the correct mathematical procedures, but calculated incorrectly: the examples that follow indicate that the learners added the number of learners, but made careless errors, since the total number is not 40 learners, but close to 40 learners.

11.2 How many learners were tested?

39 learners

11.2 How many learners were tested?

32

Item 11.3

- If the learner used the correct formula but did not substitute the correct values;

11.3 Calculate the mean test mark.

The mean mark = sum of marks / № of marks

= 40/9

= 4,4

- If the learner used the correct two numbers (f and f(x)) but did not use the correct formula and thus did not obtain the correct answer (this learner had f=39 which needs to be acknowledged when marking)

11.3 Calculate the mean test mark.

The mean mark = 39 + 203

= 242 ÷ 10

= 24.2

Item 11.4

- If the learner used the correct formula but substituted the incorrect values.

11.4 What percentage of the learners obtained 7 or more out of 10 for the test?

$\frac{11 \times 100}{40}$ = 225%

What would show evidence of no understanding?

Item 11.1

- If the learner did not attempt the question;
- If the learner followed an incorrect number pattern and gave no indication of having understood what the question entails.

11.1 Complete the frequency table for the given histogram.

Mark x	Frequency f	Product $f.x$
1	2	2
2	3	3
3	4	4
4	6	6
5	7	7
6	9	9
7	4	4
8	3	3
9	2	2

Item 11.2

- If the learner did not attempt the question;
- If the learner gave the number corresponding to the highest frequency as the answer.

11.2 How many learners were tested?

9

Item 11.3

- If the learner did not attempt the question;
- If the learner confused measures of dispersion with measures of central tendency;

11.3 Calculate the mean test mark.

The mean mark = Biggest Number - Smallest number

= 9-1

= 8

- If the learner used tally marks to work out the mean of the data set.

11.3 Calculate the mean test mark.

The mean mark = ||||| 𝍸

= 𝍸 𝍸

= ____________

Item 11.4

- If the learner did not attempt the question;
- If the learner used the values provided in the question to work out the percentage.

11.4 What percentage of the learners obtained 7 or more out of 10 for the test?

$7/10 \times 100/1$.

$= 70\%$.

What do the item statistics tell us?

Item 11.1

14 % of learners answered the question correctly.

Item 11.2

12 % of learners answered the question correctly.

Item 11.3

3 % of learners answered the question correctly.

Item 11.4

3 % of learners answered the question correctly.

Factors contributing to the difficulty of the items

- Learners may have poor understanding of the concepts and skills tested in this item;
- Learners may be unable to critically read and interpret data;
- Learners may be unable to critically analyse data by answering questions related to:
  - Data collection methods;
  - Summary statistics of data;
  - Sources of error and bias in data.

Teaching strategies

Organising and summarising data

- In order for learners to solve problems effectively they are required to know the terminology used in this topic. Learners must become acquainted with the terminology used and learners must know the meaning of the terms appropriate for this section.
- You can use the following glossary of terms to assist your learners.
- You need to explain the meaning of these terms but work through examples while you do so.
- The explanations of the terms are given here and following these explanations are some worked examples that you can use in order to explain what is involved in organising and summarising data.

Glossary of terms: Organising and summarising data

Central tendency

- The **mean**, **median** and **mode** are called the measures of central tendency:
- Mean $= \frac{\textit{sum of all values}}{\textit{number of values}}$;
- Median is the number that is found in the middle of the data set when the data is arranged in ascending order (smallest number to largest number). When there are an even numbers of values in the data set there will be two numbers in the middle. To find the median of this data set you need to add the two middle values together and then divide the sum by 2;
- Mode is the number that occurs most often in a data set. It is possible for a data set to have more than one mode.

Dispersion

- The measures of dispersion are the **range** and **outliers**:
- The range of the data is equal to the highest value minus the lowest value.
- Extreme data values that lie far above or far below the other values are called outliers.

Once learners are familiar with the terminology provide examples for them to work with so that they can consolidate their knowledge. The following example may be used.

Example

Organising and summarising data

Given the following data set: 6; 7; 8; 9; 9; 11; 12; 34 answer the questions that follow:

1). Determine the mean
2). Determine the median
3). Write down the mode
4). Determine the range
5). Write down the outlier

6). Determine the mean without the outlier
7). Determine the median without the outlier
8). Write down the mode without the outlier
9). Determine the range of the data set without the outlier
10). Which scores are affected by the outlier?

Solutions

1). To find the mean you have to find the number of scores and the total value of all the scores added together.
Then use the formula for the mean:
Mean $= \frac{sum\ of\ all\ values}{number\ of\ values}$

Mean $= \frac{6+7+8+9+9+11+12+34}{8}$
$= \frac{96}{8} = 12$

2). To find the median you have to sort the data set in ascending order (smallest number to largest number). The median is the number in the middle when the data has been sorted. In this example, there is an even numbered data set and so there is not one number in the middle, there are two. To find the median of this data set you need to add the two middle values together and then divide the sum by 2:

Median $= \frac{9+9}{2} = 9$

3). The mode is the number that occurs most often in a data set. When you have sorted the data set (which you did to find the median) it is easier to see which numbers in the data set are repeated.

Mode $= 9$

4). To find the range of the data set you need to subtract the lowest value from the highest value.

Range $= 34 - 6 = 28$

5). A data set has an outlier if it has a value that is very different in size to the general data value. Extreme data values that lie far above or far below the other values are called outliers.

Outlier $= 34$

The next few questions call for calculations that exclude the outlier. We do this to see what difference the outlier makes on the data set.

6). Mean (excluding outlier) $= \frac{6+7+8+9+9+11+12}{7} = 8{,}9$

7). Median (excluding outlier) $= 9$

8). Mode (excluding outlier) $= 9$

9). Range (excluding outlier) $= 12 - 6 = 6$

10). The mean and range are affected by the outlier.

Provide learners with an activity sheet to consolidate their knowledge. The following activity sheet may be used.

Activity

1). Given the following data set which comprises the mathematics test marks out of 25 of Grade 9 learners:

24; 12; 24; 5; 13; 14; 25; 24;15; 16; 19; 21; 23; 22; 17; 24; 18; 20; 24

a). Determine the mean

b. Determine the median

c). Write down the mode

d). Determine the range

e). Write down the outliers

2). Given the following data set:

56; 55; 50; 35; 53; 54; 55; 51;55; 56; 52; 55; 53; 58; 57; 55; 50; 54

a). Determine the mean

b). Determine the median

c). Write down the mode

d). Determine the range

e). Write down the outliers

Solutions

1). Rearrange values in ascending order:

5; 12; 13; 14; 15; 16; 17; 18; 19; 20; 21; 22; 23; 24; 24; 24; 24; 24; 25

a). Mean $= \frac{5+12+13+14+15+16+17+18+19+20+21+22+23+24+24+24+24+24+25}{19}$

$= 18{,}9$

b). The median is the middle value = 20

c). The mode is the value that occurs most frequently = 24

d). Range = 25 – 5 = 20

e). The outlier is 5

2). Rearrange values in ascending order:

35; 50; 50; 51; 52; 53; 53; 54; 54; 55; 55; 55; 55; 55; 56; 56; 57; 58

a). Mean $= \frac{35+50+50+51+52+53+53+54+54+55+55+55+55+55+56+56+57+58}{18}$

$= 53$

b). Median is the middle value $= \frac{54+55}{2} = 55$

c). The mode is the value that occurs most frequently = 55

d). Range = 58 – 35 = 23

e). The outlier is 35.

Representing data

- Once learners are familiar with the terminology that is important for organising and summarising data they can progress to representing this data.
- Allow learners to work with tally tables before representing data on graphs.
- Ensure learners know relevant details and facts about representing data.
- Use the following summary regarding bar graphs, double bar graphs and histograms to assist your learners with the relevant facts about representing data using these types of graphs.
- Summary: Bar graphs, double bar graphs and histograms:
- Data may be represented in a number of ways using different types of graphs.

Bar graphs are used to represent discrete data.

- Discrete data refers to data that is finite or has countable values, for example, the number of learners in a class.
  - Vertical or horizontal bars may be used when representing data on a bar graph.
  - Bar graphs have spaces between the bars.
  - The bars of a bar graph ought to be the same width.
  - When drawing bar graphs the bars should be an equal distance apart from each other.

- When comparing two sets of data you may use a double bar graph to compare the data sets.

- Histograms are used to represent continuous data.
  - Continuous data is represented by the set of real or rational numbers, for example, the numbers representing the mass or the height of learners. The values change over time.
  - Horizontal bars are used to draw a histogram.

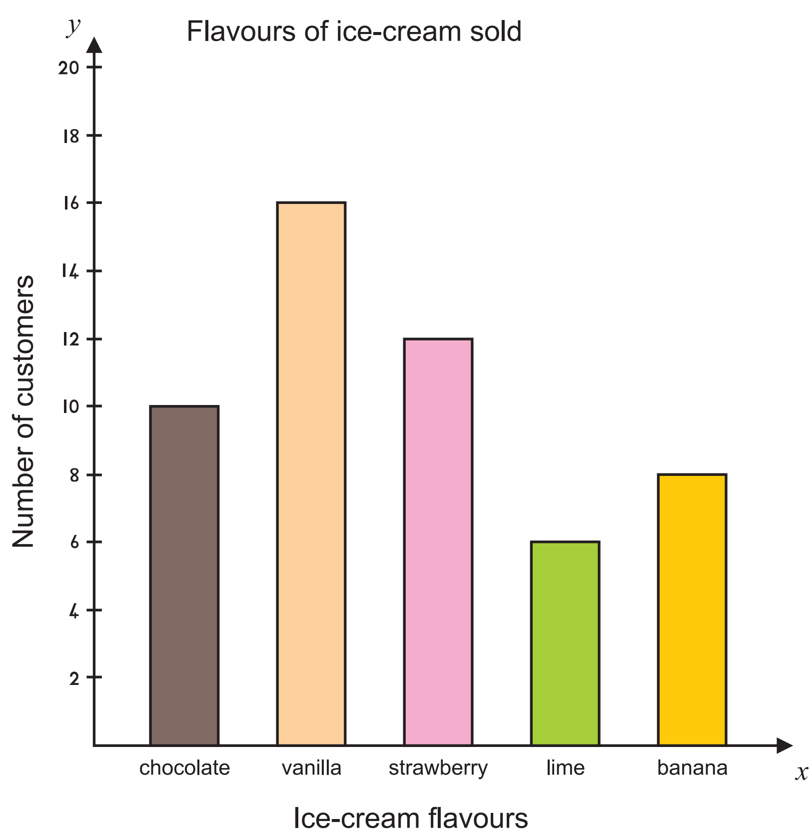
The following examples may be used to assist learners with representing data using the bar graph, double bar graph and histogram.

Examples

1). Lihle was conducting a survey to find out which was the most popular ice-cream flavour. She sat in the ice-cream shop and recorded the flavours of all the ice-cream that was purchased. She recorded her data in a tally table. Use Lihle's tally table that follows and represent her data using a bar graph.

Ice-Cream Flavour	Tally	Frequency
Chocolate	𝍸 𝍸	10
Vanilla	𝍸 𝍸 𝍸 I	16
Strawberry	𝍸 𝍸 II	12
Lime	𝍸 I	6
Banana	𝍸 II	7

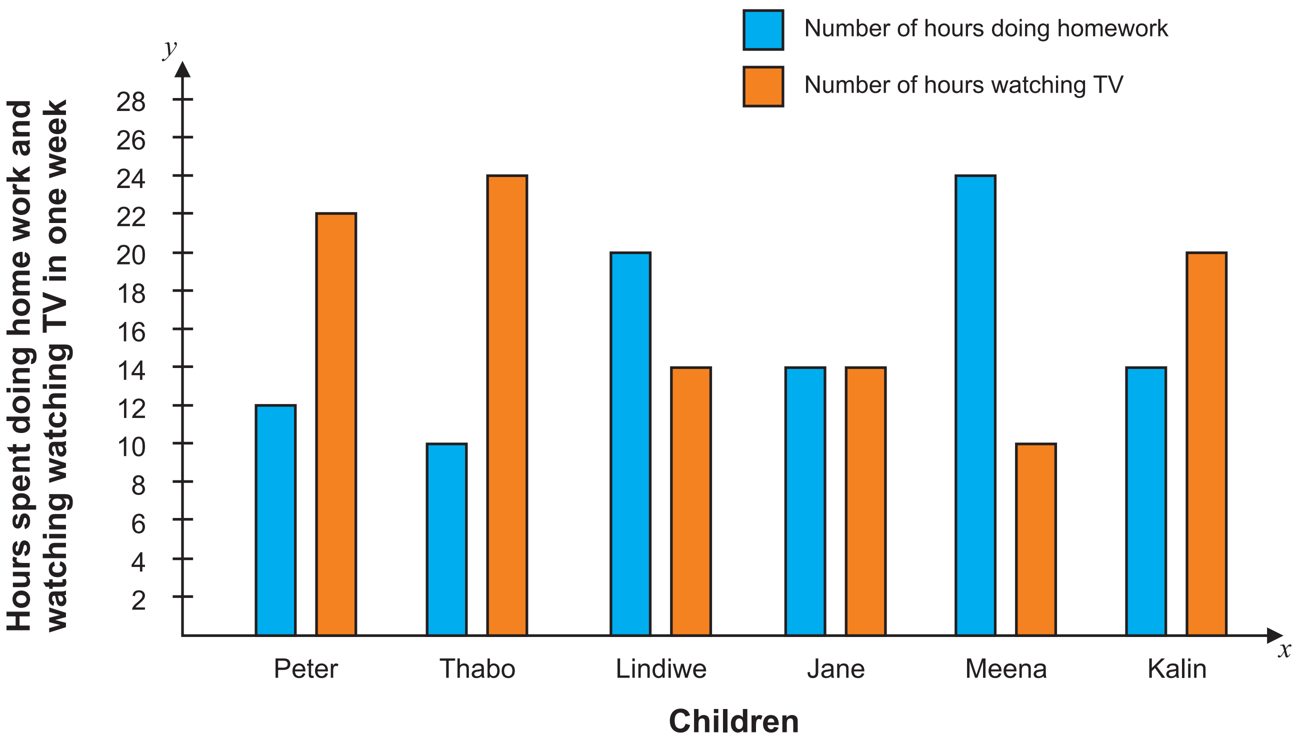
Solution



2). The table that follows represents the number of hours learners spent doing homework and watching television per week. Represent the data displayed in the table on a double bar graph.

Children	Number of hours doing homework be week	Number of hour watching TV per week
Peter	12	22
Thabo	10	24
Lindiwe	20	14
Jane	14	14
Meena	24	10
Kalin	14	20

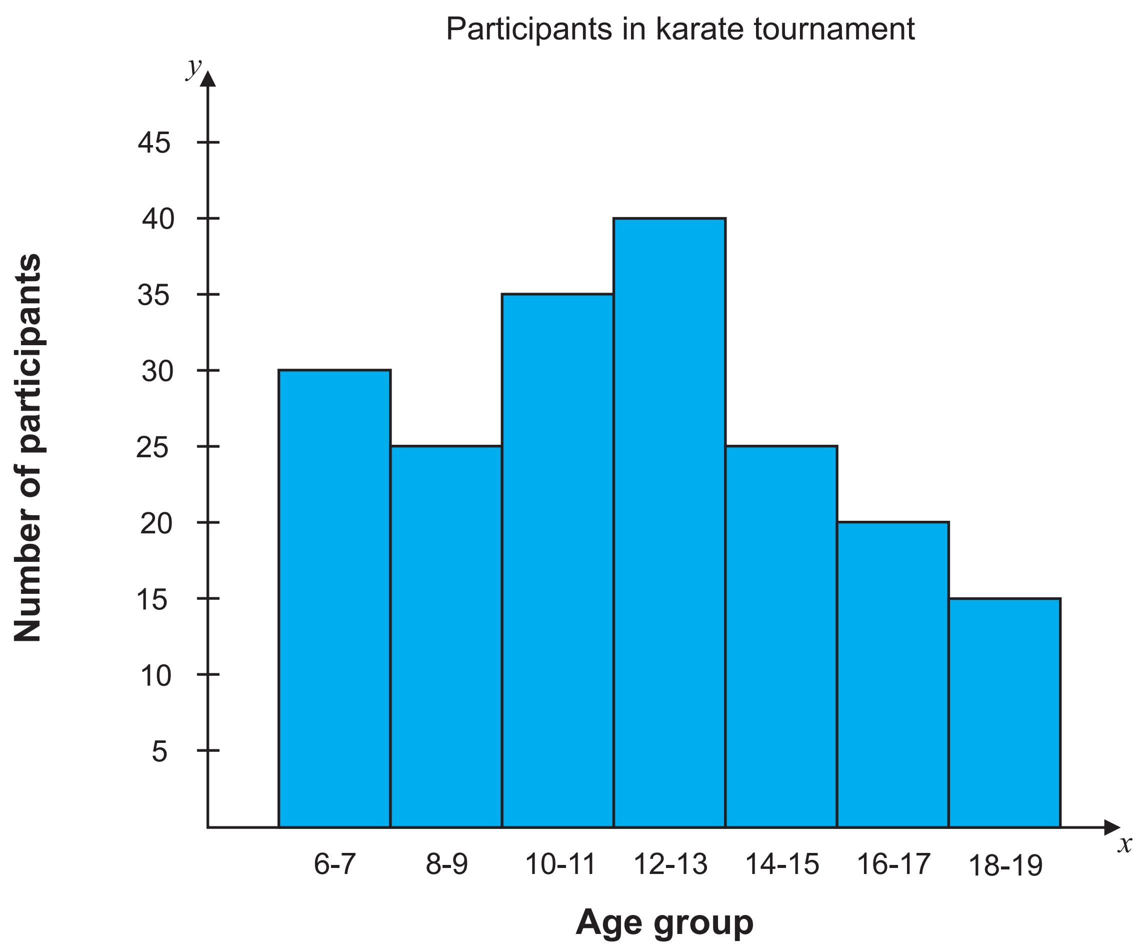
Solution



3). In a Karate tournament the participants are divided into age groups. The table that follows represents the number of participants in the tournament. Represent the data that follows on a histogram.

Age group	Number of participants
6-7	30
8-9	25
10-11	35
12-13	40
14-15	25
16-17	20
18-19	15

Solution



Consolidate learners' knowledge by allowing learners to work on an activity.
The following activities may be used.

Activities

1). Thembi recorded the colours of cars passing her school in a tally table. Use the tally table to sketch a bar graph to represent the data collected.

Colour of car	Tally	Frequency
Red	~~IIII~~ I	6
Blue	~~IIII~~ ~~IIII~~ II	12
White	~~IIII~~ ~~IIII~~ ~~IIII~~ I	16
Black	~~IIII~~ ~~IIII~~ IIII	14
Silver	~~IIII~~ III	8

2). Jayshree conducted a survey in her school to find out what types of food children enjoy eating. She used a table to display her data. Use a double bar graph to represent the data collected.

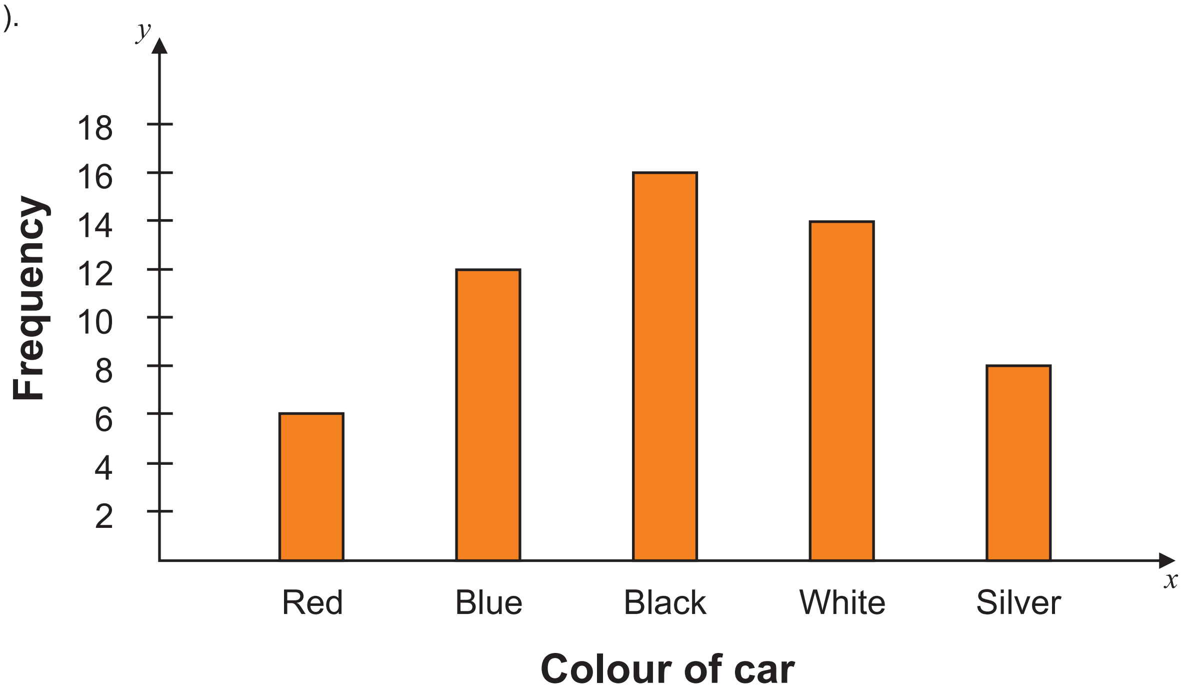
Type of food	Girls	Boys
Pizza	10	15
Burger	10	25
Hotdogs	5	20
Sandwich	20	10

3). John collected data on percentages the Grade 9 learners in his class achieved in their final mathematics test. He drew up a table to display the data collected. Use the table that follows to represent the data on a histogram.

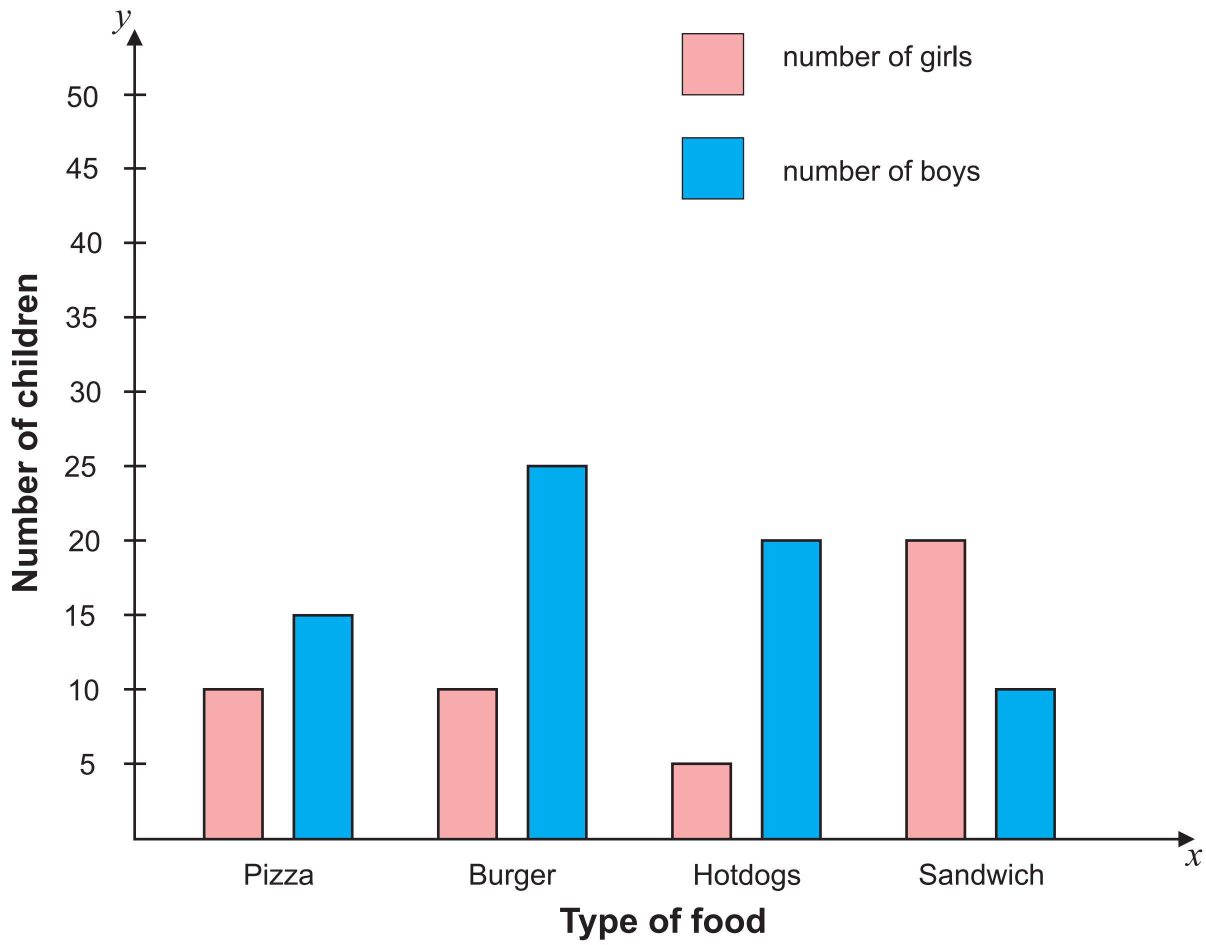
Percentage	Number of learners
90-100	20
80-89	25
70-79	30
60-69	60
50-59	55
40-49	40
30-39	10
20-29	20
10-19	10

Solutions

1).

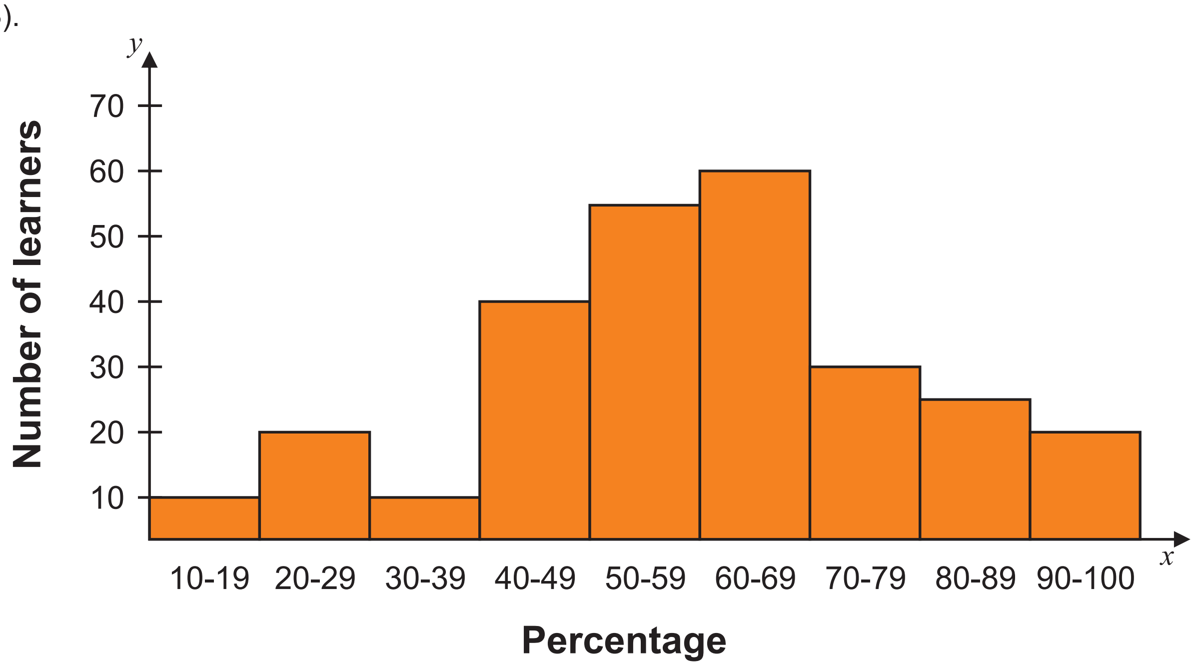


2).



y
50
45
40
35
30
25
20
15
10
5
Number of children
number of girls
number of boys
Pizza
Burger
Hotdogs
Sandwich
x
Type of food

3).



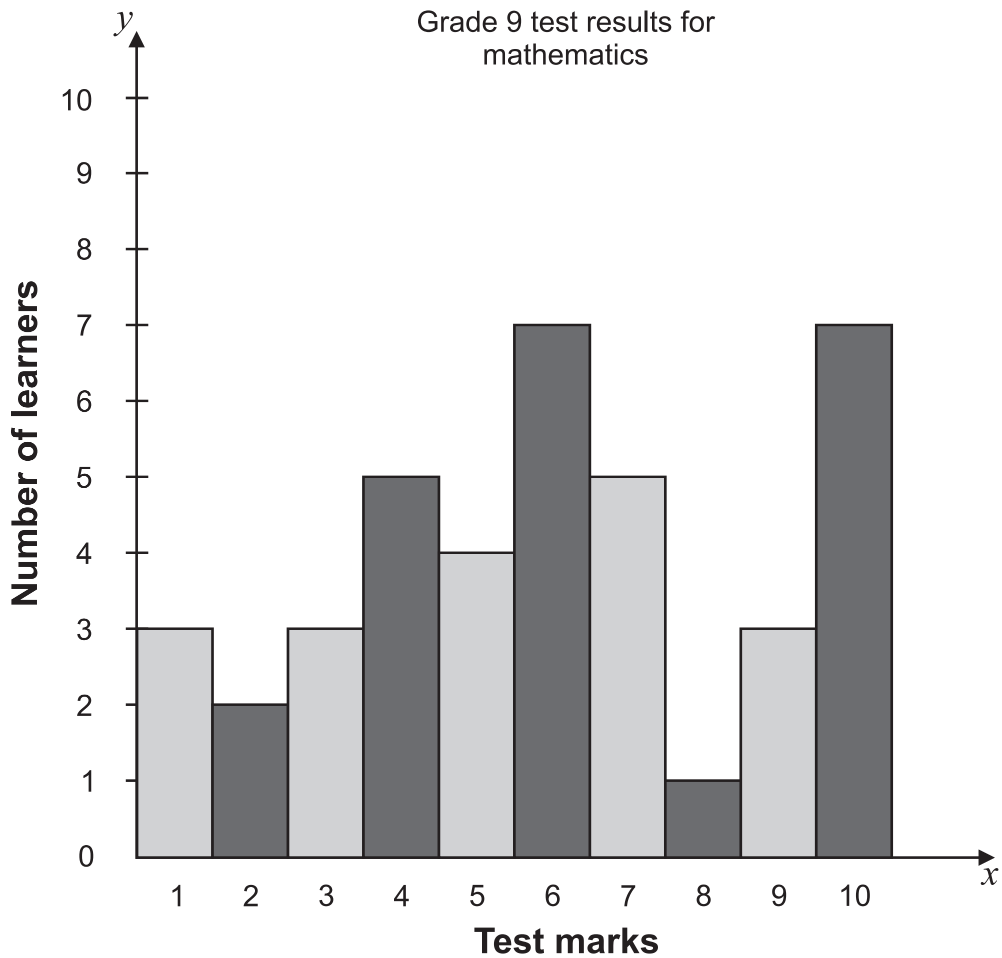
y
70
60
50
40
30
20
10
Number of learners
10-19
20-29
30-39
40-49
50-59
60-69
70-79
80-89
90-100
x
Percentage

Analysing and interpreting data

- Once learners are confident organising, summarising and representing data, provide them with opportunities to practice analysing and interpreting data represented on graphs.
- The following example may be used.

Example

The histogram that follows represents the mathematics test marks out of 10 obtained by one Grade 9 class.



Use the histogram to answer the questions that follow.

1). Complete the frequency table for the given histogram.

Mark x	Frequency f	Product $f. x$
1	3	3
2	2	4
3		
4		
5		
6		
7		
8		
9		
10		

2). If the pass mark is 40% (4 out of 10) how many Grade 9 learners passed the test?

3). How many learners wrote the test?

4). What percentage of learners obtained 8 or more out of 10?

5). Calculate the mean test mark.

Solution

1).

Mark x	Frequency f	Product $f. x$
1	3	3
2	2	4
3	3	9
4	5	20
5	4	20
6	7	42
7	5	35
8	1	8
9	3	27
10	7	70

2). 5 + 4 + 7 + 5 + 1 + 3 + 7 = 32 learners passed the test.

3). 3 + 2 + 3 + 5 + 4 + 7 + 5 + 1 + 3 + 7 = 40 learners wrote the test.

4). 1 + 3 + 7 = 11 learners obtained 8 or more out of 10. The percentage is $\frac{11}{40} \times 100 = 27,5 \approx 28\%$

5). Mean test mark

$= \frac{\text{sum of all values}}{\text{number of values}}$

$= \frac{3+4+9+20+20+42+35+8+27+70}{40}$

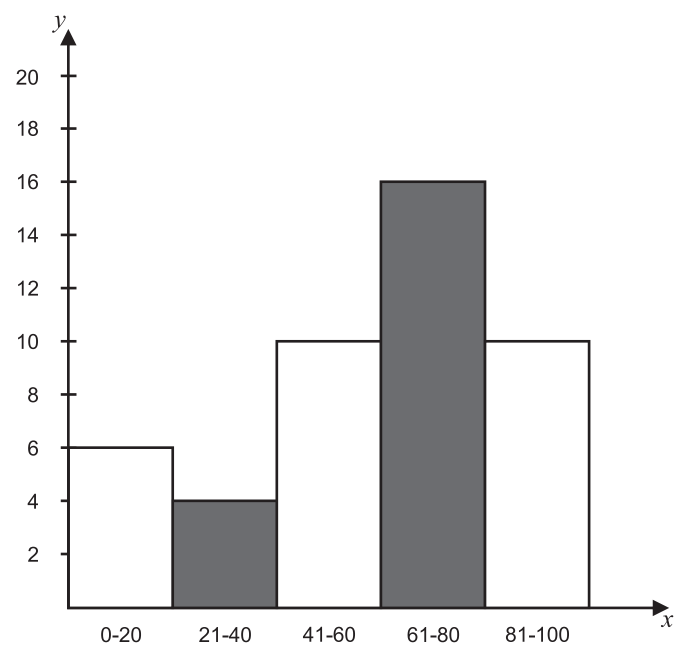
$= \frac{238}{40}$

$= 5,95 \approx 6$

After working through the example with your learners provide them with an activity for practice. The following activities may be used.

Activities

1). The histogram that follows represents Grade 9 learners' examination results for mathematics.



Use the histogram to answer the questions that follow.

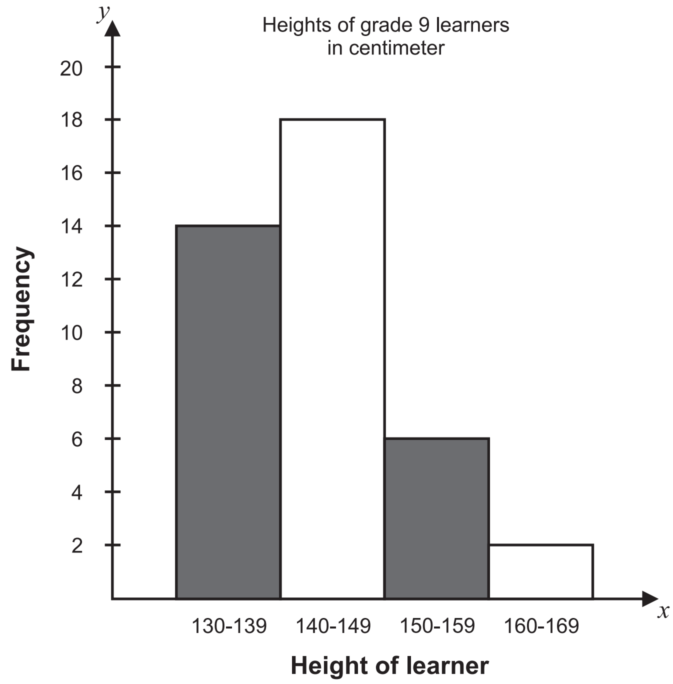
a). How many learners wrote the examination?

b). In which interval are the majority of the learners' marks?

c). What percentage of learners obtained 81% more?

d). What percentage of learners obtained between 0% and 20% for this examination?

2). The histogram below represents the heights of Grade 9 learners in a class.



Use the histogram to answer the questions that follow.

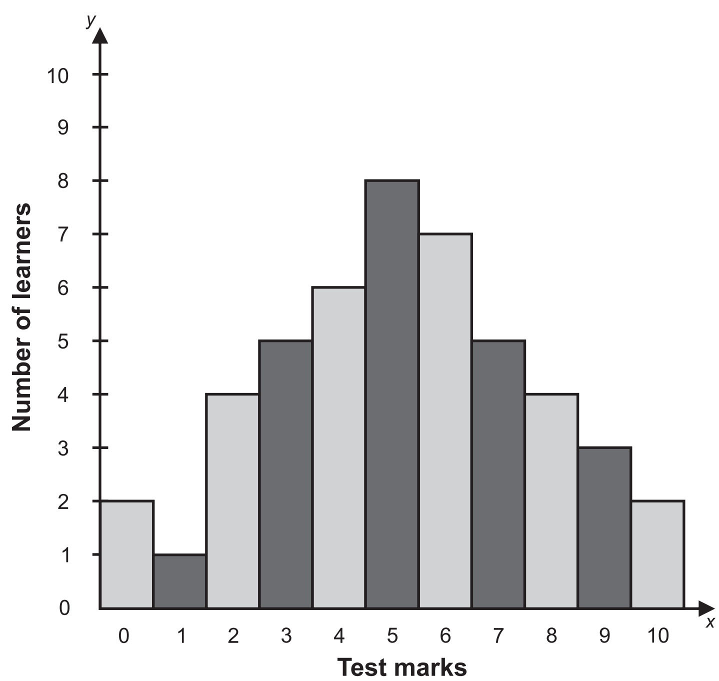
a). Determine the class interval with the highest frequency.

b). How many learners are taller than 139 cm?

c). What percentage of learners is between 150 cm and 159 cm tall?

d). What percentage of learners is shorter than 150 cm?

3). The following histogram represents Grade 9 learners' marks in a mathematics test out of 10.



Use the histogram to answer the questions that follow.

a). Complete the frequency table for the given histogram.

Mark x	Frequency f	Product $f.x$
0	2	0
1	1	1
2	4	8
3		
4		
5		
6		
7		
8		
9		
10		

b). If the pass mark for this test is 50% (5 out of 10) how many Grade 9 learners passed the test?

c). How many learners wrote the test?

d). What percentage of learners obtained 7 or more out of 10?

e). Calculate the mean test mark.

Solutions

1). a). 6 + 4 + 10 + 16 + 10 = 46 learners wrote the test.

b). 61 – 80% interval.

c). $\frac{10}{46} \times 100 = 21{,}7 \approx 22\,\%$

d). $\frac{6}{46} \times 100 = 13{,}04 \approx 13\,\%$

2). a). 140 cm – 149 cm

b). 18 + 6 + 2 = 26 learners are taller than 139 cm

c). Total number of learners: 14 + 18 + 6 + 2 = 40 learners

Total number of learners between 150 cm and 159 cm tall: 6

$\frac{6}{40} \times 100 = 15$ % of the learners are between 150 cm and 159 cm tall.

d). Total number of learners: 14 + 18 + 6 + 2 = 40 learners

Number of learners that are shorter than 150 cm: 18 + 14 = 32

$\frac{32}{40} \times 100 = 80\,\%$ of the learners are shorter than 150 cm

3). a).

Mark x	Frequency f	Product $f.x$
0	2	0
1	1	1
2	4	8
3	5	15
4	6	24
5	8	40
6	7	42
7	5	35
8	4	32
9	3	27
10	2	20

b). 8 + 7 + 5 + 4 + 3 + 2 = 29 learners passed the test.

c). 2 + 1 + 4 + 5 + 6 + 8 + 7 + 5 + 4 + 3 + 2 = 47 learners wrote the test.

d). 5 + 4 + 3 + 2 = 14 learners obtained 7 or more out of 10.

The percentage is $\frac{14}{47} \times 100 = 29{,}8 \approx 30\,\%$

e). Mean test mark

$$= \frac{\textit{sum of all values}}{\textit{number of values}}$$

$$= \frac{0+1+8+15+24+40+42+35+32+27+20}{47} = \frac{244}{47}$$

$$= 5{,}2 \approx 5$$

Notes:

Data handling: Representing data

ANA 2013 Grade 9 Mathematics Items 12.1 and 12.2

The following are the heights in centimetres of a group of Grade 9 learners.

156	147	173	165	170
145	153	165	149	158
163	156	153	157	137
177	146	150	153	158

12.1. Draw a stem-and-leaf plot to illustrate the data. [5]

Stem	**Leaves**
13	
14	
15	
16	
17	

12.2. Use the data to complete each of the following:

12.2.1. The range = ________________________ [1]

12.2.2. The mode = ________________________ [1]

12.2.3. The median = ________________________ [1]

12.2.4. The number of learners who are shorter than 160 cm = ___ [1]

What should a learner know to answer these questions correctly?

Learners should be able to:

Item 12.1

- Draw a stem-and-leaf plot to illustrate given data.

Item 12.2

- Use the given data to determine the range, mode and median;
- Interpret and analyse the data provided.

Where is this topic located in the curriculum? Grade 9 Term 4

Content area: Data handling.

Topic: Representing data.

Concepts and skills:

- Draw a variety of graphs by hand or using technology to display and interpret data including:
- Bar graphs and double bar graphs;
- Histograms with given and own intervals;
- Pie charts;
- Broken line graphs; and
- Scatter plots.

What would show evidence of full understanding?

- If the learner obtained the correct solution by using an appropriate mathematical strategy.

Item 12.1

12.1 Draw a stem-and-leaf plot to illustrate the data.

Stem	Leaves
13	7
14	5, 6, 7, 9
15	0, 3, 3, 3, 6, 6, 7, 8, 8
16	3, 5, 5
17	0, 3, 7

12.1 Draw a stem-and-leaf plot to illustrate the data.

Stem	Leaves
13	~~3~~ 7
14	5 679
15	0 3 3 3 66~~8~~ 7 88
16	3 5 5
17	0 3 7

Item 12.2

12.2 Use the data to complete each of the following:

12.2.1 The range = 177 − 137 = 40.

12.2.2 The mode = 153 cm.

12.2.3 The median = ~~14 learners~~ 156 cm

12.2.4 The number of learners who are shorter than 160 cm = 14 learners

What would show evidence of partial understanding?

Item 12.1

- If the learner completed the table with the correct values for the stems, but did not fill in the correct values for the leaves;
- If the learner did not arrange the values in ascending order.

12.1 Draw a stem-and-leaf plot to illustrate the data.

Stem	Leaves
13	137
14	145, 146, 149, 147,
15	150, 153, 153, 153, 158, 158, 157, 156.
16	163, 165, 165.
17	170, 173, 177

Item 12.2.1

- If the learner gave the number with the largest value only.

12.2.1 The range = 177.

Items 12.2.2 and 12.2.3

- If the learner confused the measures of central tendency.

12.2.2 The mode = 156.

12.2.3 The median = 153 cm.

Item 12.2.4

- If the learner knew what needed to be calculated, but calculated incorrectly.

12.2.4 The number of learners who are shorter than 160 cm = 16.

What would show evidence of no understanding?

Item 12.1

- If the learner did not attempt the question;
- If the learner completed the table by indicating the number of values for each stem;

12.1 Draw a stem-and-leaf plot to illustrate the data.

Stem	Leaves
13	One
14	Four
15	Eight
16	Three
17	Three

- If the learner used tally marks to represent the numbers given in the 'stem' column;

12.1 Draw a stem-and-leaf plot to illustrate the data.

Stem	Leaves
13	~~IIII~~ ~~IIII~~ III
14	~~IIII~~ ~~IIII~~ IIII
15	~~IIII~~ ~~IIII~~ ~~IIII~~
16	~~IIII~~ ~~IIII~~ ~~IIII~~ I
17	~~IIII~~ ~~IIII~~ ~~IIII~~ II

- If the learner completed the table using each column of the data to fill each row of the table: this has no logical mathematical process or reasoning in relation to drawing up a stem-and-leaf plot;

12.1 Draw a stem-and-leaf plot to illustrate the data.

Stem	Leaves
13	156; 145; 163; 177
14	146; 147; 153; 156
15	150; 153; 165; 173
16	~~153; 157~~; 149; 153; 157; 165
17	137; 158; 158; 170

- If the learner completed the table incorrectly with no logical mathematical process or reasoning.

12.1 Draw a stem-and-leaf plot to illustrate the data.

Stem	Leaves
13	~~137~~ 177
14	163
15	156
16	145
17	137

Items 12.2.1, 12.2.2, 12.2.3 and 12.2.4

- If the learner did not attempt the question;
- If the learner solved the question incorrectly with no logical mathematical process or reasoning.

12.2.1 The range = 5.

12.2.2 The mode = 24.

12.2.3 The median = 2,4.

12.2.4 The number of learners who are shorter than 160 cm = 137.

What do the item statistics tell us?

Item 12.1

27 % of learners answered the question correctly.

Item 12.2.1

20 % of learners answered the question correctly.

Item 12.2.2

26 % of learners answered the question correctly.

Item 12.2.3

13 % of learners answered the question correctly.

Item 12.2.4

23 % of learners answered the question correctly.

Factors contributing to the difficulty of the items

- Learners may have poor understanding of the concepts and skills tested in this item.
- Learners may be unable to represent data.
- Learners may be unable to answer questions based on represented data.
- Learners may be unable to interpret statistics based on represented data.

Teaching strategies

Representing data: Pie Graphs and stem-and-leaf plots

- Remind the learners of the different ways in which data may be represented (Refer to Teaching strategy 2 in Item 11).
- Allow learners to work with examples based on pie graphs and stem-and-leaf plots.
- You may use the following examples to assist you.

Examples

1). The following table represents the type of vehicle passing a stop street in a 4 hour period.

a). Complete the table

Types of vehicle	Number of vehicles	Number of degrees
Cars	60	
Moterbikes	20	
Vans	30	
Trucks	10	
Buses	10	
Taxis	20	

b). Using the completed table, draw a pie graph to represent the data collected.

Solution

a). Remind learners how to calculate the number of degrees when drawing pie graphs:

Cars: $\frac{60}{150} \times 360 = 144°$

Motorbikes: $\frac{20}{150} \times 360 = 48°$

Vans: $\frac{30}{150} \times 360 = 72°$

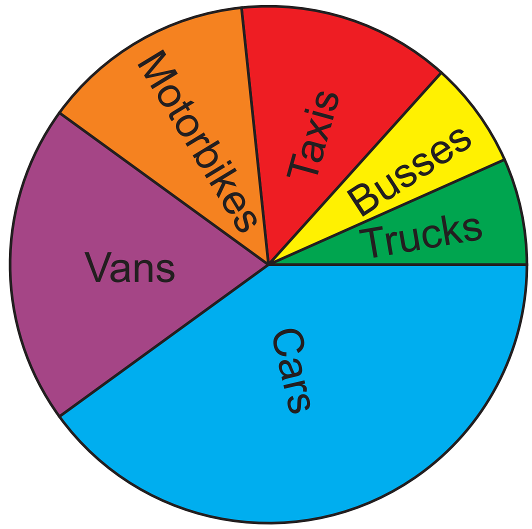
Trucks: $\frac{10}{150} \times 360 = 24°$

Buses: $\frac{10}{150} \times 360 = 24°$

Taxis: $\frac{20}{150} \times 360 = 48°$

Types of vehicle	Number of vehicles	Number of degrees
Cars	60	144°
Moterbikes	20	48°
Vans	30	72°
Trucks	10	24°
Buses	10	24°
Taxis	20	48°
Total	150	360°

b).



Type of vehicle passing a stop street during a 4 hour period

2). The following data set is the mathematics test scores of a group of Grade 9 learners.

35	28	68	36	88
68	43	72	45	87
42	23	75	58	86
13	15	83	67	84

a). Draw a stem-and-leaf plot to illustrate the data.

b). Use the data to complete each of the following:

- The range =
- The mode =
- The median =

Solution

a).

Stem	Leaves
1	3; 5
2	3; 8
3	5; 6
4	2; 3; 5
5	8
6	7; 8; 8
7	2; 5
8	3; 4; 6; 7; 8

b). The range = 88 – 13 = 75

The mode = 68

The median = $\frac{58 + 67}{2}$ = 62,5 $\approx$ 63

Allow learners to consolidate their knowledge by working on an activity. You may use the activities that follow to assist you.

Activities

1). Jabu is conducting a survey on what snacks learners at his school purchase during their tea break. He draws a table to display his data.

a). Complete Jabu's table:

Type of snack	Number purchased	Degrees
Cup cakes	40	
Chips	30	
Peanuts	30	
Chocolates	20	

b). Use the completed table to sketch a pie graph to represent the data collected.

2). Roshini is conducting a survey at her school on learners' favourite types of food. She asks the Grade 9 learners what their favourite type of food is. She records her data in a table.

a). Complete Roshini's table.

Favourite type of food	Number of learners	Degrees
Pizza	80	
Burger	60	
Hot dogs	30	
Fresh chips	10	
Sandwiches	20	

b). Use the completed table to sketch a pie graph to represent the data collected.

3). Consider the following percentages obtained by Grade 9 learners in a mathematics test.

49	33	50	92	65
86	81	25	44	51
80	78	39	18	35
42	28	25	74	52
92	82	92	87	92

a). Draw a stem-and-leaf plot to illustrate the data.

b). Use the data to complete each of the following:

- The range =

- The mode =
- The median =

c). How many learners achieved more than 50% for the test?

Solutions

1). a). Cupcakes: $\frac{40}{120} \times 360 = 120°$

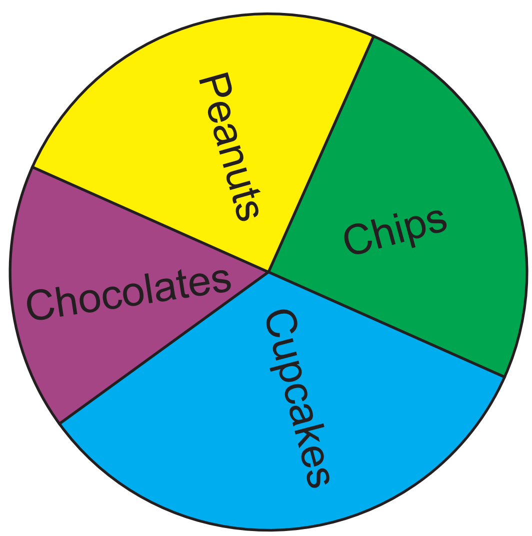
Chips: $\frac{30}{120} \times 360 = 90°$

Peanuts: $\frac{30}{120} \times 360 = 90°$

Chocolates: $\frac{20}{120} \times 360 = 60°$

Type of snack	Number purchased	Degrees
Cup cakes	40	120°
Chips	30	90°
Peanuts	30	90°
Chocolates	20	60°
Total	120	360°

b).



Snacks purchased during break at school

2). a). Pizza: $\frac{80}{200} \times 360 = 144°$

Burgers: $\frac{60}{200} \times 360 = 108°$

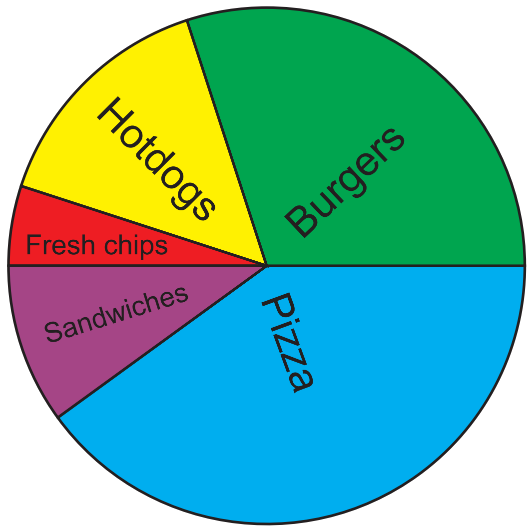
Hot dogs: $\frac{30}{200} \times 360 = 54°$

Fresh chips: $\frac{10}{200} \times 360 = 18°$

Sandwiches: $\frac{20}{200} \times 360 = 36°$

Favourite type of food	Number of learners	Degrees
Pizza	80	144°
Burger	60	108°
Hot dogs	30	54°
Fresh chips	10	18°
Sandwiches	20	36°
Total	200	360°

b).



Favourite type of food

3). a).

Stem	Leaves
1	8
2	5; 5; 8
3	3; 5; 9
4	2; 4; 9
5	0; 1; 2
6	5
7	4; 8
8	0; 1; 2 6; 7
9	2; 2; 2; 2

b) The range = 92 – 18 = 74

The mode = 92

The median = 52

c). 14 learners achieved more than 50% for the test.

Analysing and interpreting data: Pie graphs and stem-and-leaf plots

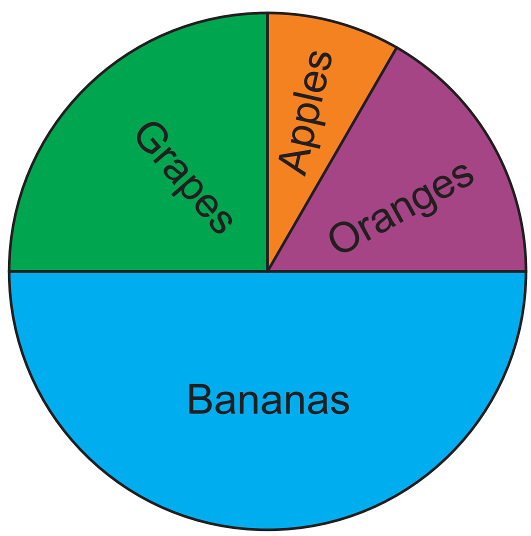
- Once learners are confident representing data, provide them with opportunities to analyse and interpret data represented on pie graphs and stem-and-leaf plots.
- The following examples may be used to assist learners with analysing and interpreting data represented on pie graphs and stem-and –leaf plots.

Examples

1). Nomsa conducted a survey with a group of Grade 9 learners. She asked them to name only one of their favourite fruits. She recorded all the data collected in a table and drew a pie chart to represent the data collected.

Favourite Fruit	Number of Grade 9 learners	Degrees
Apples	6	30°
Oranges	12	60°
Grapes	18	90°
Bananas	36	180°

The pie chart shown here represents the data collected. Look at the pie graph and answer the questions that follow:



Favourite fruit

a). How many Grade 9 learners took part in the survey?
b). Which was the least favourite fruit amongst the learners?
c). Which was the second favourite fruit?
d). Which fruit was preferred by most learners?

Solution

a). 6 + 12 + 18 + 36 = 72: therefore 72 learners took part in the survey.
b). The smallest number of learners (6 learners) preferred apples: this was the least favorite

fruit.

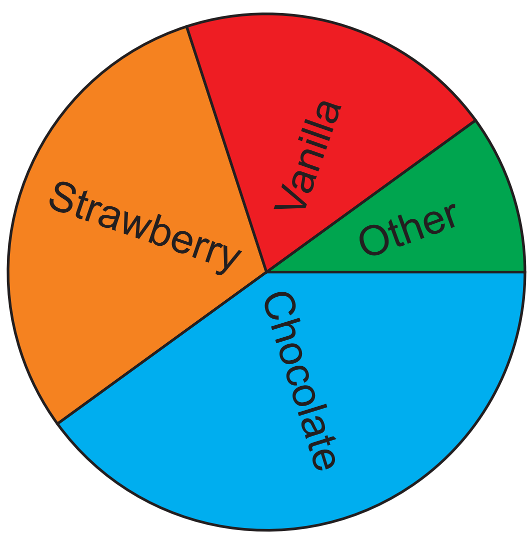
c). The second largest number of learners (18 learners) preferred grapes: this was the second favorite fruit.

d). The largest number of learners (36 learners) preferred bananas: this was the fruit preferred by learners.

2). Krishna conducts a survey outside an ice-cream shop. He wants to find out which ice-cream flavour is most popular. He records the different flavoured ice-cream purchased during a 1 hour period in a table.

Ice-cream flavour	Number of participants	Degrees
Chocolate	20	144°
Strawberry	15	108°
Vanilla	10	72°
Other	5	36°

Using the data from the table he drew the following pie graph to represent the data collected. Look at the pie graph that follows and answer the questions that follow:



Most popular ice-cream flavours

a). How many people participated in the survey?

b). What the most popular flavour of ice-cream?

c). Which was the third most popular flavour of ice-cream?

d). Did the participants only buy chocolate, vanilla and strawberry flavoured ice-cream? Give a reason for your answer.

Solution

a). 20 + 15 + 10 + 5 = 40 : 40 people participated in the survey.

b). The most popular flavour of ice-cream was chocolate: half of the participants preferred this flavour.

c) Vanilla was the third most popular ice-cream flavour.

d). No: 5 participants bought other flavours so Krishna placed this information under the other flavours column.

Allow learners to consolidate their knowledge by working on an activity. You may use the activities that follow to assist you.

Activities

1). Consider the following percentages obtained by Grade 9 learners in a mathematics test.

34	63	80	68	65
45	82	91	49	54
64	98	65	56	55
70	46	72	60	88
65	74	83	73	93

a). Draw a stem-and-leaf plot to illustrate the data.

b). Use the data to complete each of the following:

- The range =
- The mode =
- The median =

c). How many learners achieved more than 40% for the test?

2). Consider the following percentages obtained by Grade 9 learners in a mathematics test.

67	85	74	86	89
49	68	56	74	90
45	84	32	51	48
54	74	50	69	92
74	87	74	59	66

a). Draw a stem-and-leaf plot to illustrate the data.

b). Use the data to complete each of the following:

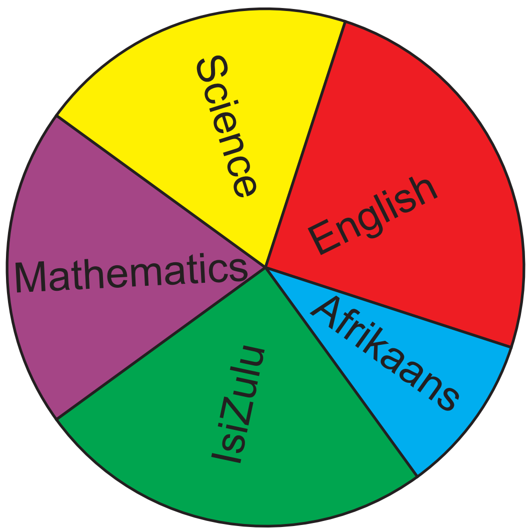
- The range =
- The mode =
- The median =

c). How many learners achieved more than 60% for the test?

3). Mr Naidoo conducted a survey with a group of Grade 9 learners. He asked the participants in the group to select only one school subject as their favourite. He recorded the information collected in a table and drew a pie graph to represent the data collected.

Favourite subject	Number of learners	Degrees
Mathematics	20	72°
Natural Science	20	72°
English	25	90°
Afrikaans	10	36°
IsiZulu	25	90°

Look at the table and the pie graph and answer the questions that follow:



Favourite school subjects

a). How many learners participated in the survey?

b). Which subject or subjects were most popular?

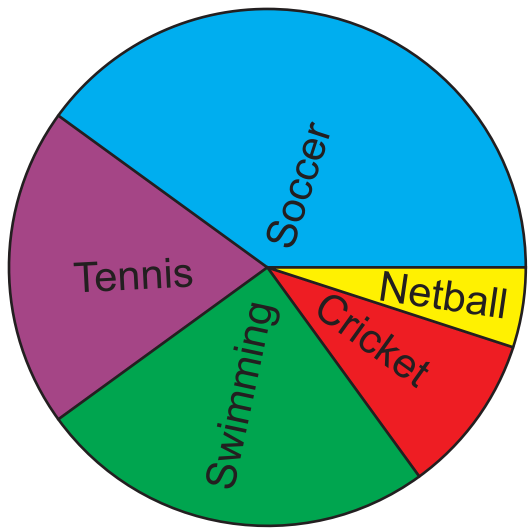
c). Which subject was selected by the same number of participants that chose IsiZulu as their favourite subject?

d). Which subject was selected by the same number of participants that chose mathematics as their favourite subject?

4). Mrs. Nandi conducted a survey with a group of Grade 9 learners. She asked each participant to select only one sport as their favourite sport. She recorded the information collected in a table and drew a pie graph to represent the data collected.

Favourite sport	Number of learners	Degrees
Netball	5	18°
Cricket	10	36°
Soccer	40	144°
Tennis	20	72°
Swimming	25	90°

Look at the table and the pie graph and answer the questions that follow:



Favourite school sport

a). How many learners participated in the survey?

b). Which sport was the least popular?

c). Which sport was the most popular?

d). Which sport was selected by twice the number of participants that chose netball as their favourite sport?

e). Which sport was selected by half the number of participants that chose soccer as their favourite sport?

Solutions

1. a).

Stem	Leaves
3	4; 9
4	5; 5; 6
5	4; 5
6	0; 3; 4; 5; 5; 5; 8
7	0; 2; 3; 4
8	0; 2; 3; 8
9	1; 3; 8

b). The range = 98 – 34 = 64
The mode = 65
The median = 65

c). 23 learners achieved more than 40% for the test.

2. a).

Stem	Leaves
3	2
4	5; 8; 9
5	0; 1; 4; 6; 9
6	6; 7; 8; 9
7	4; 4; 4; 4; 4
8	4; 5; 6; 7; 9
9	0; 2

b). The range = 92 – 32 = 60
The mode = 74
The median = 69

c). 16 learners achieved more than 60% for the test.

3. a). 100 learners participated in the survey.
b). English and IsiZulu were the most popular
c). English
d). Science

4. a). 100 learners participated in the survey
b). Netball
c). Soccer
d). Cricket
e). Tennis

Data Handling: Probability

ANA 2013 Grade 9 Mathematics Item 13

13.	A box contains 3 blue, 4 white and 5 green marbles of the same size.	
13.1.	If you take out 1 marble, what is the probability that you will take out a green marble?	[1]
13.2.	What is the probability of then taking out a white marble if you replace the marble you took out of the box previously?	[1]
13.3.	If you take out a white marble and do not replace it, what is the probability of taking out another white marble?	[1]

What should a learner know to answer this question correctly?

Learners should be able to:

- Determine the probabilities of outcomes of events;
- Predict the relative frequency of outcomes of events in simple experiments.

Where is this topic located in the curriculum? Grade 9 Term 4

Content area: Data handling.

Topic: Probability.

Concepts and skills:

- Consider situations with equally probable outcomes and:
  - Determine probabilities of compound events using two-way tables and tree diagrams;
  - Determine the probabilities of outcomes of events and predict their relative frequency in simple experiments.
  - Compare relative frequency with probability and explain possible differences.

What would show evidence of full understanding?

- If the learner obtained the correct solution by using an appropriate mathematical strategy.

Item 13.1

13.1 If you take out 1 marble, what is the probability that you will take out a green marble?

P(a) = 5/12

Item 13.2

13.2 What is the probability of then taking out a white marble if you replace the marble that you took out of the box previously?

$P(W) = \frac{4}{12} = \frac{1}{3}$

Item 13.3

13.3 If you take out a white marble and do not replace it, what is the probability of taking out another white marble?

$P(W) = \frac{3}{11}$

What would show evidence of partial understanding?

Item 13.1

- If the learner worked out the probability, but confused the colour of the marble that was taken out;

13.1 If you take out 1 marble, what is the probability that you will take out a green marble?

$P(W) = \frac{7}{12}$

13.1 If you take out 1 marble, what is the probability that you will take out a green marble?

$P(G) = \frac{7}{12}$

- If the learner worked out the probability using only the green marbles.

13.1 If you take out 1 marble, what is the probability that you will take out a green marble?

~~$\frac{1}{5}$~~ ~~$\frac{1}{12}$~~ $\frac{1}{5}$

Item 13.2

- If the learner worked out the probability, but confused the colour of the marble that was taken out;

13.2 What is the probability of then taking out a white marble if you replace the marble that you took out of the box previously?

$P(G) = \frac{8}{12} = \frac{2}{3}$

13.2 What is the probability of then taking out a white marble if you replace the marble that you took out of the box previously?

$P(W) = 8/12$

- If the learner worked out the probability using only the white marbles;

13.2 What is the probability of then taking out a white marble if you replace the marble that you took out of the box previously?

$\frac{1}{4}$

- If the learner worked out the probability, but simplified incorrectly.

13.2 What is the probability of then taking out a white marble if you replace the marble that you took out of the box previously?

$P(W) = \frac{4}{12} = \frac{1}{2}$

Item 13.3

- If the learner worked out the probability, but confused the colour of the marble that was taken out;

13.3 If you take out a white marble and do not replace it, what is the probability of taking out another white marble?

$P(W) = 8/11$

13.3 If you take out a white marble and do not replace it, what is the probability of taking out another white marble?

$P(G) = \frac{8}{11}$

- If the learner worked out the probability using only the white marbles.

13.3 If you take out a white marble and do not replace it, what is the probability of taking out another white marble?

$\frac{1}{3}$

What would show evidence of no understanding?

Item 13.1

- If the learner did not attempt the question;
- If the learner solved the problem incorrectly with no mathematical reasoning or logic.

13.1 If you take out 1 marble, what is the probability that you will take out a green marble?

$\frac{4}{5}$

13.1 If you take out 1 marble, what is the probability that you will take out a green marble?

$P(G) = \frac{5}{11}$

Item 13.2

- If the learner did not attempt the question;
- If the learner solved the problem incorrectly with no mathematical reasoning or logic.

13.2 What is the probability of then taking out a white marble if you replace the marble that you took out of the box previously?

$\frac{5}{4}$

Item 13.3:

- If the learner did not attempt the question;
- If the learner solved the problem incorrectly with no mathematical reasoning or logic.

13.2 What is the probability of then taking out a white marble if you replace the marble that you took out of the box previously?

$\frac{1}{4}$

13.3 If you take out a white marble and do not replace it, what is the probability of taking out another white marble?

$P(W) = \frac{3}{12}$

What do the item statistics tell us?

Item 13.1

10% of learners answered the question correctly.

Item 13.2

10% of learners answered the question correctly.

Item 13.3

10% of learners answered the question correctly.

Factors contributing to the difficulty of the items 13.1, 13.2 and 13.3

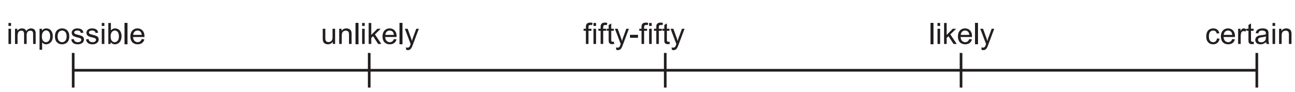
- Learners may have poor understanding of the concepts and skills tested in these items.
- Learners may be unable to determine the probabilities of outcomes of events and predict the relative frequency of the outcomes in simple experiments.

Teaching strategies

Finding probabilities of simple events

- Probability is the part of mathematics that studies **chance** or **likelihood**. For example, when you see on the weather that there is a 60% chance of rain, they are talking about a probability.
- Discuss with your learners what probability entails: the chance of an event occurring is impossible, certain or somewhere in-between.
- Some things will never happen. We say that they are **impossible**. For example, if you throw an ordinary die – it is impossible that it will land on 8.
- Other things will definitely happen and we say that they are **certain** to happen. For example, it is certain for an ordinary die to land on 1, 2, 3, 4, 5, or 6.
- Some things are not certain or impossible. We judge them on how likely they are to happen.
  For example:
  - It is likely to rain when the sky is very cloudy.
  - It is unlikely to snow in Johannesburg in September.
  - There is a 50-50 chance of a coin landing on heads when it is tossed.

- You can use the printable for the following probability scale **(see printables)** to assist you with your discussions.
- If the learners have copies of the scale, or if you have drawn it on the board, you can fill in events along the scale.



- To help us determine the probability of an event we can use a formula:

$$\text{Probability} = \frac{\textit{number of favourable outcomes}}{\textit{number of possible outcomes}}$$

- To assist you with your teaching of probability, provide learners with a glossary of terms that they will encounter while working with probability. The following glossary of terms may be used. These are brief definitions that will be further explained and consolidated in the context of teaching examples in the strategies that follow.

Glossary of terms: Probability

Event: An event is something that may or may not occur, for example: the rolling of a die, the tossing of a coin, the spinning of a spinner and so on.

Outcome: This is the result of an event, for example: rolling a die and the die landing on a 4.

Probability: The likelihood of something happening or not happening.

- Rather than using words to describe the chance of an event happening, you can give the probability as a number, usually written as a fraction, percentage or decimal from 0 to 1.
- If it is impossible for an event to happen, the probability is 0.
- If an event is certain to happen, the probability is 1.
- All other probabilities are greater than 0, but less than 1.
- The more likely an event is to happen, the higher the probability, so the larger the fraction, decimal or percentage will be.
- The less likely an event is to happen, the lower the probability, so the smaller the fraction, decimal or percentage will be.

Experiment: An experiment is a series of trials, for example if a die is rolled 10 times, the result of each roll is recorded. Each roll of the die is called a trial.

Relative frequency: This is a measure of the experimental probability of an outcome, for example: to find the relative frequency of obtaining a 4 when rolling a die we need to perform an experiment. We need to use a formula to work out the relative frequency:

$$\text{Relative frequency} = \frac{\textit{number of favourable outcomes}}{\textit{number of possible outcomes}}$$

Two way table: Two way tables are used to record the outcomes of two events occurring at the same time in a compound event, for example: tossing a coin and rolling a die simultaneously.

Tree diagram: Tree diagrams are used when events take place one after the other or when a compound event contains more than two events, for example: tossing a coin, rolling a die and then tossing a second coin.

Learners need to work with examples involving simple events in order to consolidate their knowledge of probability. The following examples may be used:

Examples

1). Use the probability scale **(see printables)** given to find the probability of the following events occurring:

a). Tuesday following directly after Friday

b). Wednesday following directly after Tuesday

c). A coin landing on tails after a toss

2). A card is drawn from a standard pack of 52 playing cards. Calculate the probability that the card drawn will be:

a). A black card

b). A heart

hearts

c). A club

clubs

d). A number 6 card

3). A four-number spinner **(see printables)** is spun once. What is the probability of the outcome being a 2?

Solutions

1). a). Impossible. (Tuesday comes after Monday: it will never come after Friday).

b). Certain. (This is true and will certainly be the case).

c). Fifty-fifty. (There are two possible events that can occur when you toss a die. It could land on heads or on tails. Each event has an equal chance of occurring. Thus each event has a 50% chance of occurring).

2). The probabilities in this question are all worked out based on the number of cards (52) in a standard pack of cards.

a). $\frac{26}{52} = \frac{1}{2}$

b). $\frac{13}{52} = \frac{1}{4}$

c). $\frac{13}{52} = \frac{1}{4}$

d). $\frac{4}{52} = \frac{1}{13}$

3). The probability of the outcome being a 2 is $\frac{1}{4}$. (We can write the probability as a fraction (in this case $\frac{1}{4}$) or as a percentage (in this case 25%). This probability is calculated based on the chance of this event occurring. It has a 1 out of 4 chance of occurring, since the possible event when this spinner turns is that it will stop on 1, 2, 3 or 4).

Once learners have worked with the examples provide an activity for learners to work with. The following activities may be used to consolidate your discussions on finding probabilities of simple events.

Activities

1). Find the probability of the following events occurring:
a). Tuesday following directly after Monday
b). Wednesday following directly after Thursday
c). A coin landing on heads after a toss

2). A card is drawn from a standard pack of 52 playing cards. Calculate the probability that the card drawn is:
a). A red card
b). A diamond
c). A spade
d). An 8
e). A king
f). A queen

3). A five-number spinner is spun once (see printables), what is the probability of obtaining the outcome of 3?

Solutions

1. a). Certain

   b). Impossible

   c). Fifty-fifty

2. a). $\frac{26}{52} = \frac{1}{2}$

   b). $\frac{13}{52} = \frac{1}{4}$

   c). $\frac{13}{52} = \frac{1}{4}$

   d). $\frac{4}{52} = \frac{1}{13}$

   e). $\frac{4}{52} = \frac{1}{13}$

   f). $\frac{4}{52} = \frac{1}{13}$

3). The probability of obtaining a 3 is $= \frac{1}{5}$ or 20%.

Comparing relative frequency and probability

- After learners have worked through examples and an activity of finding the probability of simple events, discuss the relationship between relative frequency and probability.
  - Relative frequency is found by performing an experiment or collecting information from past records.
  - The relative frequency of an event is calculated using the formula:

Relative frequency $= \frac{\textit{number of favourable outcomes}}{\textit{number of possible outcomes}}$

Example

In an experiment a drawing pin is dropped 100 times. The drawing pin lands point up 37 times. What is the relative frequency of the drawing pin landing point up?

Solution

Relative frequency $= \frac{\textit{number of favourable outcomes}}{\textit{number of possible outcomes}}$

$= \frac{37}{100}$

$= 0{,}37$ or 37%

Explain to your learners that we can use relative frequency to **predict** the probability of an event. This means that we use the same formula to calculate probability that we use to calculate relative frequency. The prediction is better when the experiment is based on a lot of trials.

$$\text{Probability} = \frac{\textit{number of favourable outcomes}}{\textit{number of possible outcomes}}$$

Use examples to assist you with your discussions. The following examples may be used.

Examples

1). What is the probability of selecting a spade out of a pack of 52 playing cards?

2). If after 20 selections you picked a spade 10 times, determine the relative frequency of selecting a spade.

3). A letter is chosen from the word FREQUENCY

a). What is the probability of choosing an *E*?

b). After 10 selections the letter *E* was chosen 6 times. Calculate the relative frequency of selecting the letter *E*.

4). A die was rolled 10 times and the results are recorded in the table that follows:

Roll	1	2	3	4	5	6	7	8	9	10
Outcome	2	1	5	3	2	4	1	6	2	3

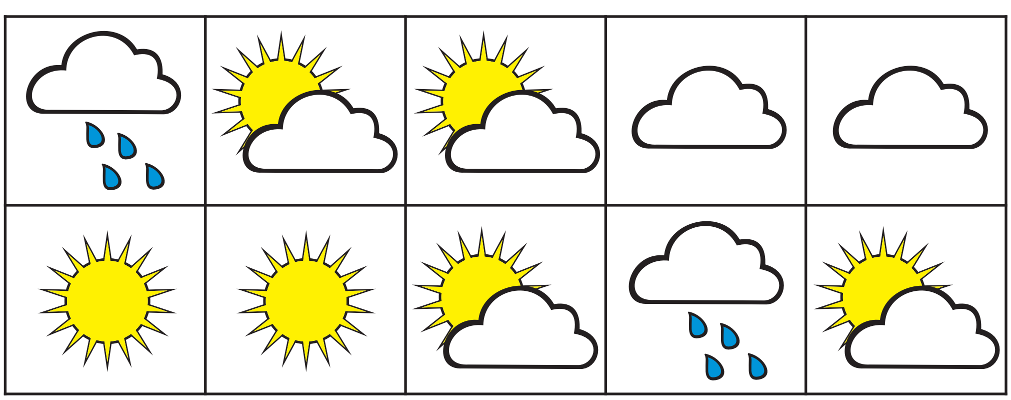
Calculate the relative frequency of obtaining a 2.

5). A coin was tossed 10 times and the results are recorded in the table that follows:

Toss	1	2	3	4	5	6	7	8	9	10
Outcome	H	T	T	H	T	H	H	T	H	H

Calculate the relative frequency of obtaining a tails.

6). The pictures that follow show the weather patterns **(see printables)** for the next ten days. Calculate the relative frequency of each type of weather.



Solutions

1). $\frac{13}{52} = \frac{1}{4} = 25\%$

2). $\frac{10}{20} = \frac{1}{2} = 50\%$

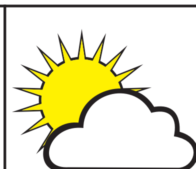
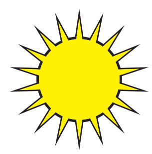
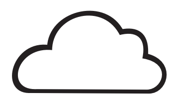
3). a). $\frac{2}{9}$

b). $\frac{6}{10} = \frac{3}{5} = 60\%$

4). $\frac{3}{10} = 30\%$

5). $\frac{4}{10} = 40\%$

6).

			
$\frac{2}{10} = \frac{1}{5} = 20\%$	$\frac{4}{10} = \frac{2}{5} = 40\%$	$\frac{2}{10} = \frac{1}{5} = 20\%$	$\frac{2}{10} = \frac{1}{5} = 20\%$

Once learners have worked through examples allow learners to work on an activity to consolidate their knowledge. The following activities may be used:

Activities

1). When using a pack of normal playing cards:

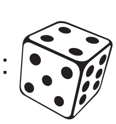
a). What is the probability of selecting a card of clubs out of a pack of 52 playing cards?

b). If after 20 selections you picked a card of clubs 12 times, determine the relative frequency of selecting a clubs.

2). A letter is chosen from the word DIVIDEND

a). What is the probability of choosing a ***D***?

b). After 15 selections the letter ***D*** was chosen 5 times. Calculate the relative frequency of selecting the letter ***D***.

3). A die was rolled 16 times and the results are recorded in the table that follows: 

Roll	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Outcome	5	2	3	5	1	5	4	5	6	2	1	4	5	6	6	5

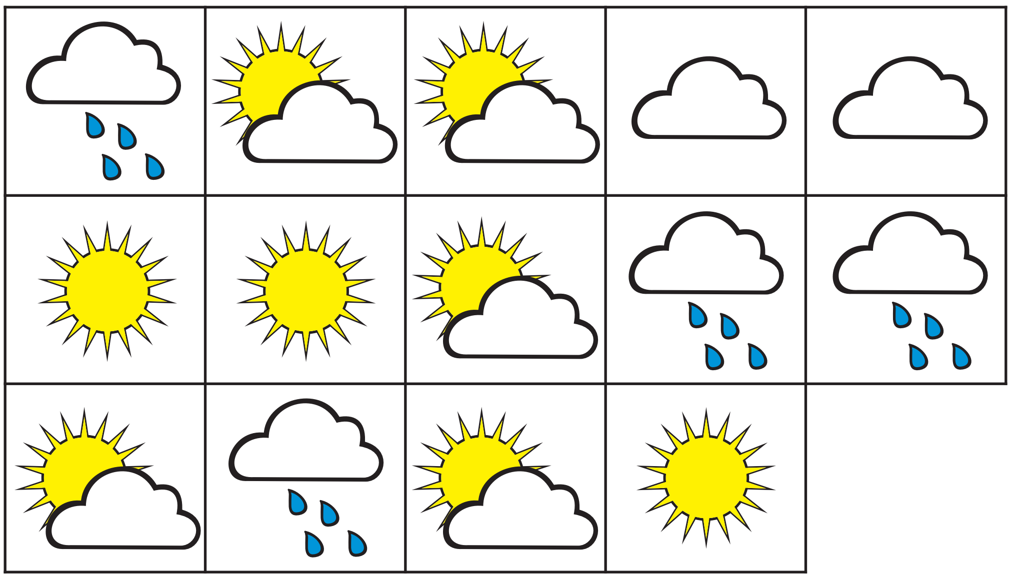
Calculate the relative frequency of obtaining a 5.

4). A coin was tossed 12 times and the results are recorded in the table that follows:

Toss	1	2	3	4	5	6	7	8	9	10	11	12
Outcome	H	H	T	T	H	T	T	H	T	H	H	T

Calculate the relative frequency of obtaining heads

5). The pictures that follow show the weather patterns for the next two weeks.



Calculate the relative frequency of each type of weather.

Solutions

1. a). $\frac{13}{52} = \frac{1}{4} = 25\%$

b). $\frac{12}{20} = \frac{3}{5} = 60\%$

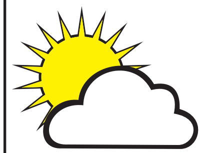
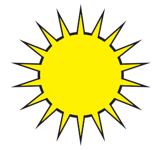
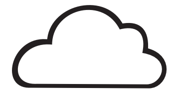
2. a). $\frac{1}{8} = 12{,}5\%$

b). $\frac{5}{15} = \frac{1}{3} = 33{,}3\%$

3). $\frac{6}{16} = \frac{3}{8} = 37{,}5\%$

4). $\frac{6}{12} = \frac{1}{2} = 50\%$

5).

			
$\frac{4}{14} = \frac{2}{7} = 28{,}6\%$	$\frac{5}{14} = 35{,}7\%$	$\frac{2}{14} = \frac{1}{7} = 14{,}3\%$	$\frac{3}{14} = 21{,}4\%$

Finding probabilities for compound events using two-way tables and tree diagrams

- Once learners know the difference between relative frequency and probability introduce the idea of working with two way tables and tree diagrams.
- Two way tables are generally used when two events in a compound event occur at the same time.
- Tree diagrams are generally used when events take place one after the other or when a compound event contains more than two events.
- Use examples to assist you when discussing two way tables and tree diagrams with your learners.
- The following examples may be used.

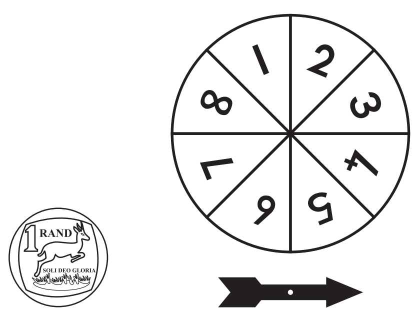
Examples

1). Two dice are rolled at the same time. Determine the following:

a). The probability of 2 and 4

b). The probability of at least one 3

2). One coin is tossed and a three-number spinner **(see printables)** is spun at the same time.



Determine the following:

a). The probability of obtaining a tails and a 3

b). The probability of obtaining at least a heads

3). Two coins are tossed at the same time. Determine the following:

a). The probability of obtaining a tails and a tails (TT)

b). The probability of obtaining at least a heads

4). Two coins are tossed one after the other.

a). Draw a tree diagram for this compound event

b). Determine all possible outcomes for the compound event

c). Determine the probability of scoring two heads (HH)

Solutions

1).

	Die 1					
Die 2	**1**	**2**	**3**	**4**	**5**	**6**
1	1 & 1	2 & 1	3 & 1	4 & 1	5 & 1	6 & 1
2	1 & 2	2 & 2	3 & 2	4 & 2	5 & 2	6 & 2
3	1 & 3	2 & 3	3 & 3	4 & 3	5 & 3	6 & 3
4	1 & 4	2 & 4	3 & 4	4 & 4	5 & 4	6 & 4
5	1 & 5	2 & 5	3 & 5	4 & 5	5 & 5	6 & 5
6	1 & 6	2 & 6	3 & 6	4 & 6	5 & 6	6 & 6

a). $\frac{2}{36}$ (Only two of the possible 36 outcomes shows 2 and 4)

b). $\frac{11}{36}$ (Only 11 of the possible 36 outcomes show at least one 3)

2).

	Spinner		
Coin	**1**	**2**	**3**
Heads(H)	H 1	H 2	H 3
Tails (T)	T 1	T 2	T 3

a). $\frac{1}{6}$

b). $\frac{3}{6} = \frac{1}{2} = 50\%$

3).

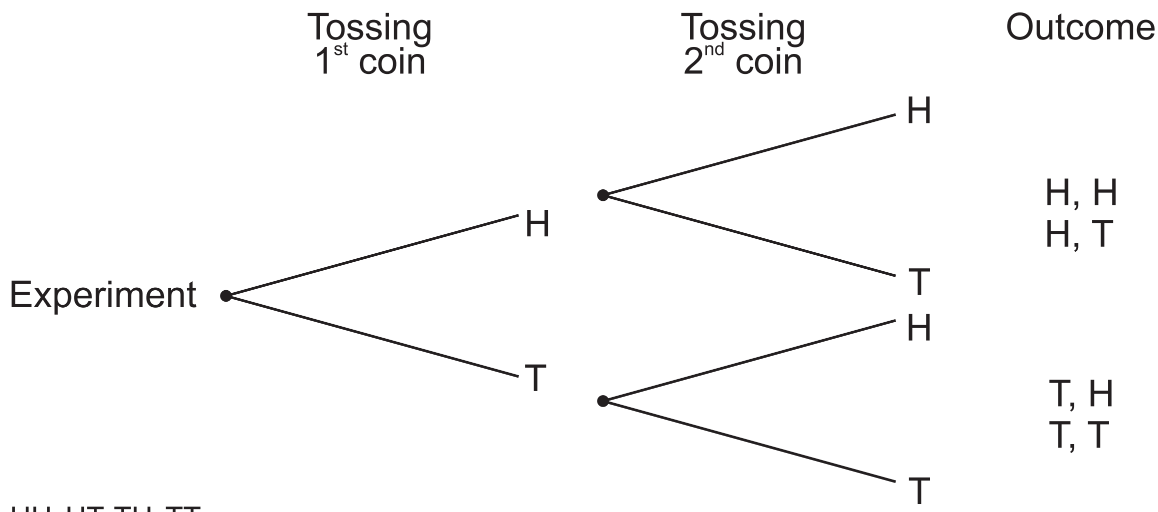
	Coin 2	
Coin 1	**Heads (H)**	**Tails (T)**
Heads(H)	H H	H T
Tails (T)	T H	T T

a). $\frac{1}{4} = 25\%$

b). $\frac{3}{4} = 75\%$

4).

a).



b). HH, HT, TH, TT

c). $\frac{1}{4} = 25\%$

Once learners have worked through examples allow them to work on activities to consolidate their knowledge. The following activities may be used.

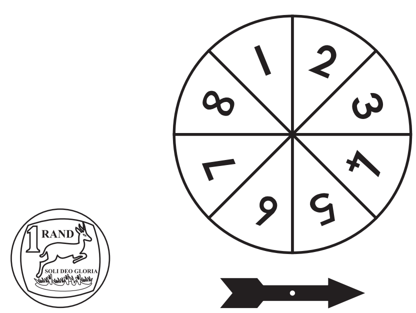
Activities

1). Two dice are rolled at the same time. Determine the following:

a). The probability of obtaining a 3 and a 6

b). The probability of obtaining at least one 5

2). One coin is tossed and an eight-number spinner **(see printables)** is spun at the same time.



Determine the following:

a). The probability of obtaining a heads and a 6

b). The probability of obtaining at least a tails

3). Two coins are tossed at the same time. Determine the following:

a). The probability of obtaining a heads and a heads (HH)

b). The probability of obtaining at least a tails

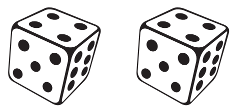
4). A coin is tossed and a die is rolled.

a). Draw a tree diagram for this compound event

b). Determine all possible outcomes for the compound event

c). Determine the probability of scoring a heads and a 6 (H, 6)

5). A die is rolled and then a second die is rolled.



a). Draw a tree diagram for this compound event

b). Determine all possible outcomes for the compound event

c). Determine the probability of scoring a 4 and a 3 (4, 3)

Solutions

1).

	Die 1					
Die 2	**1**	**2**	**3**	**4**	**5**	**6**
1	1 & 1	2 & 1	3 & 1	4 & 1	5 & 1	6 & 1
2	1 & 2	2 & 2	3 & 2	4 & 2	5 & 2	6 & 2
3	1 & 3	2 & 3	3 & 3	4 & 3	5 & 3	6 & 3
4	1 & 4	2 & 4	3 & 4	4 & 4	5 & 4	6 & 4
5	1 & 5	2 & 5	3 & 5	4 & 5	5 & 5	6 & 5
6	1 & 6	2 & 6	3 & 6	4 & 6	5 & 6	6 & 6

a). $\frac{2}{36}$

b). $\frac{11}{36}$

2).

	Spinner							
Coin	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Heads(H)	H 1	H 2	H 3	H 4	H 5	H 6	H 7	H 8
Tails (T)	T 1	T 2	T 3	T 4	T 5	T 6	T 7	T 8

a). $\frac{1}{16}$

b). $\frac{8}{16} = \frac{1}{2}$

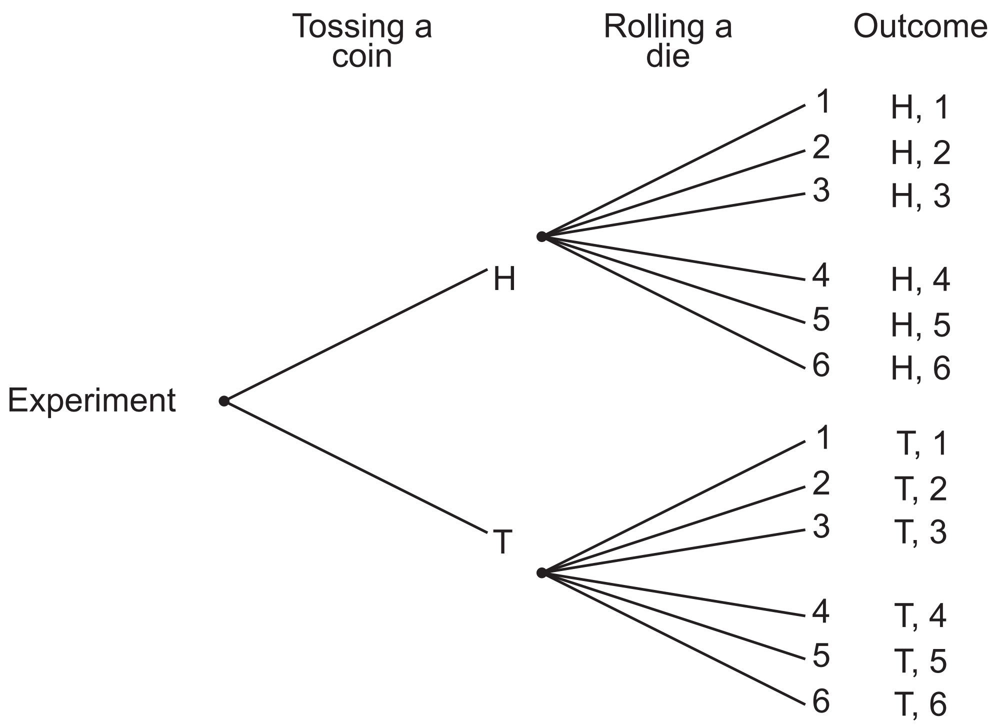
3).

	Coin 2	
Coin 1	**Heads (H)**	**Tails (T)**
Heads(H)	H H	H T
Tails (T)	T H	T T

a). $\frac{1}{4} = 25\%$

b). $\frac{3}{4} = 75\%$

4). Tree diagram for this compound event :



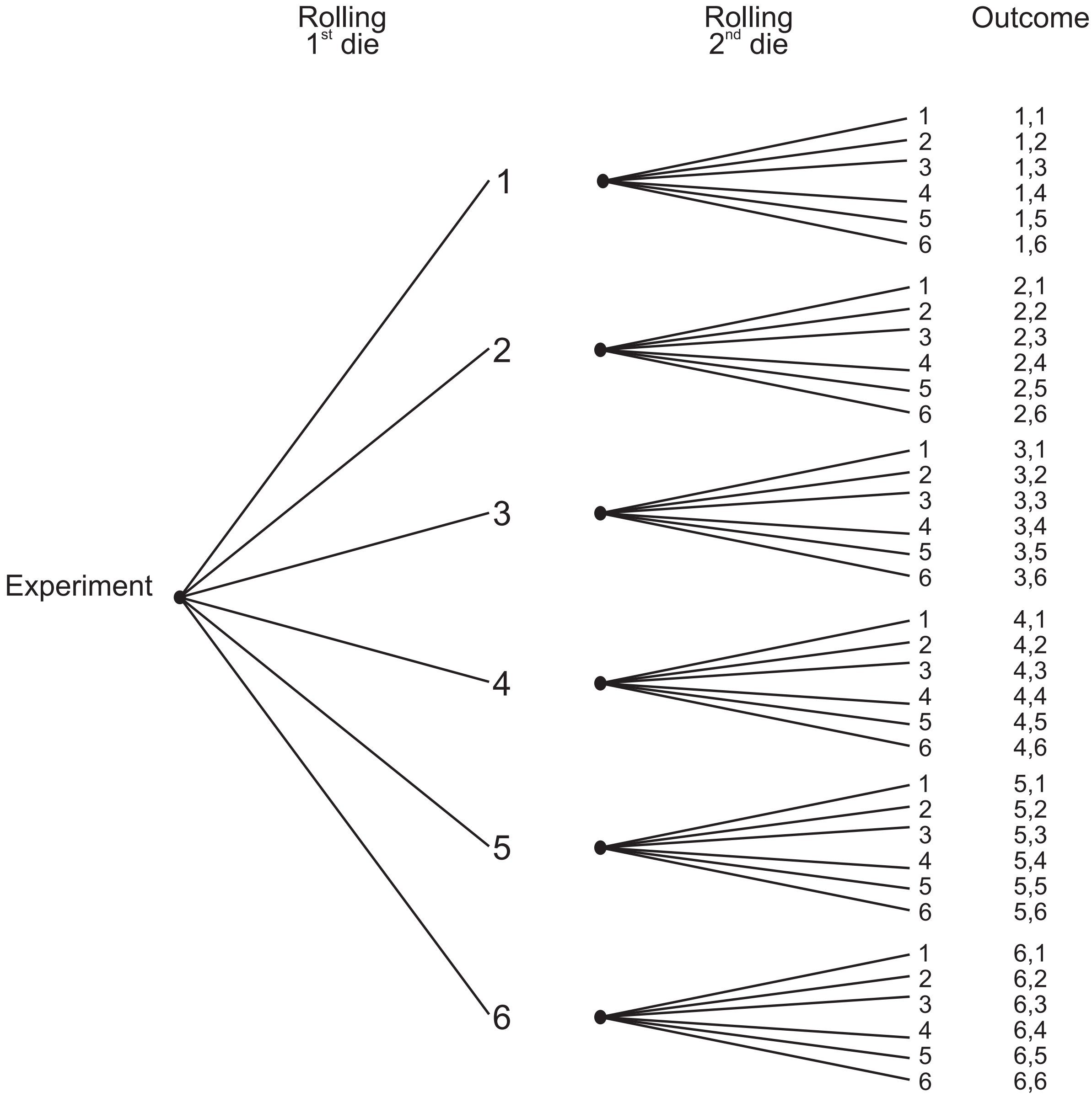
a). All possible outcomes for the compound event:

H,1; H,2; H,3; H,4; H,5; H,6; T,1; T,2; T,3; T,4; T,5; T,6

b). $\frac{1}{12}$

5).

a). Tree diagram for this compound event

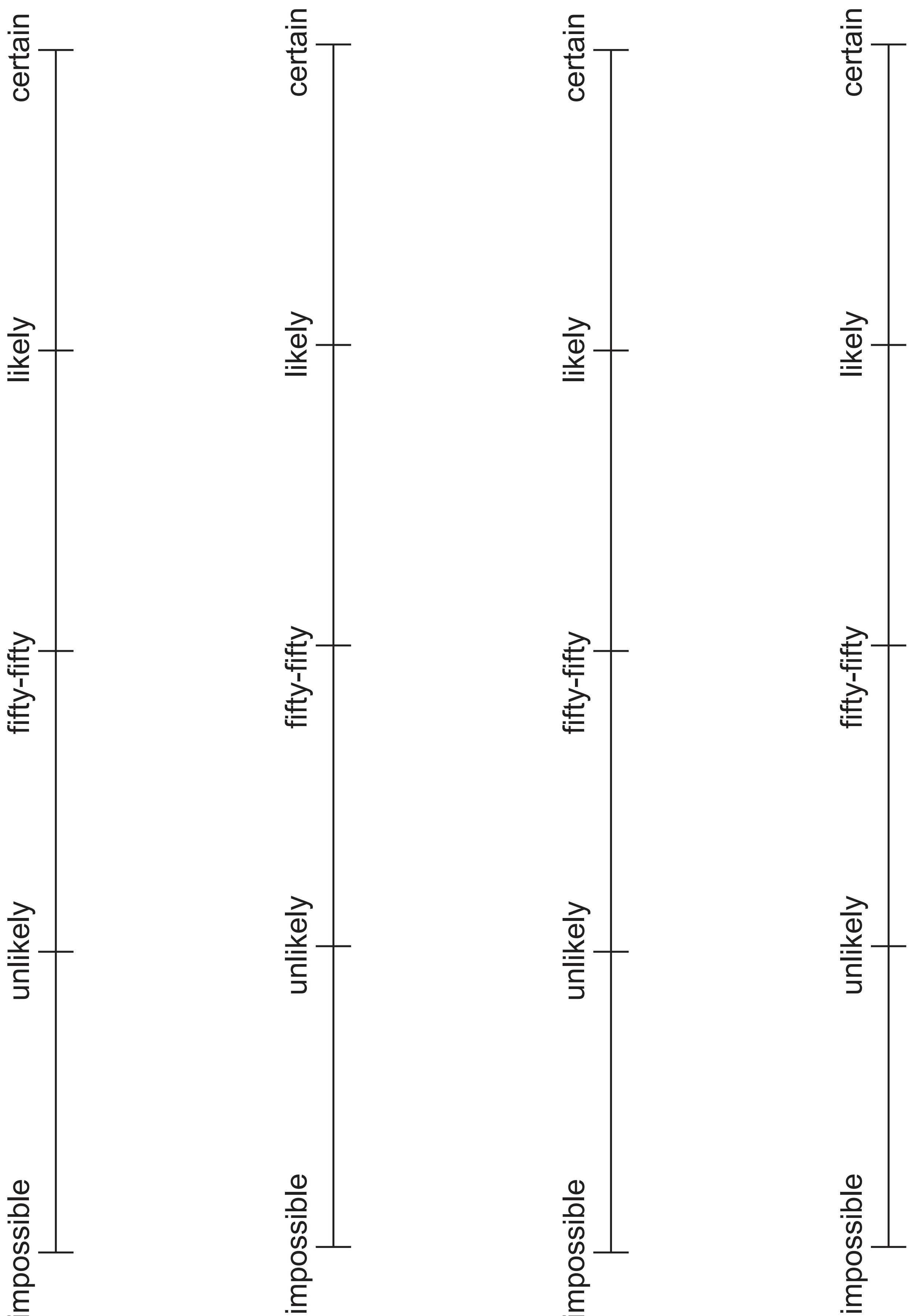


b). All possible outcomes for the compound event

Outcome					
1,1	2,1	3,1	4,1	5,1	6,1
1,2	2,2	3,2	4,2	5,2	6,2
1,3	2,3	3,3	4,3	5,3	6,3
1,4	2,4	3,4	4,4	5,4	6,4
1,5	2,5	3,5	4,5	5,5	6,5
1,6	2,6	3,6	4,6	5,6	6,6

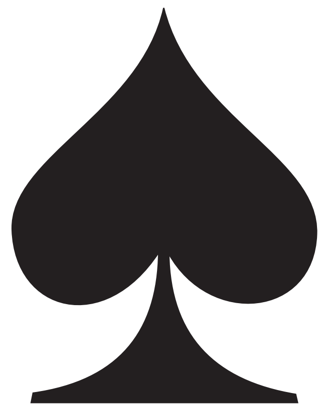
c). $\frac{2}{36}$

Printable: Probability scale



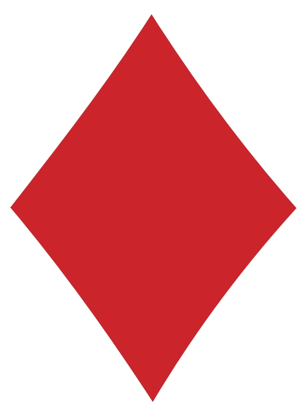
Printable: Playing Cards

hearts



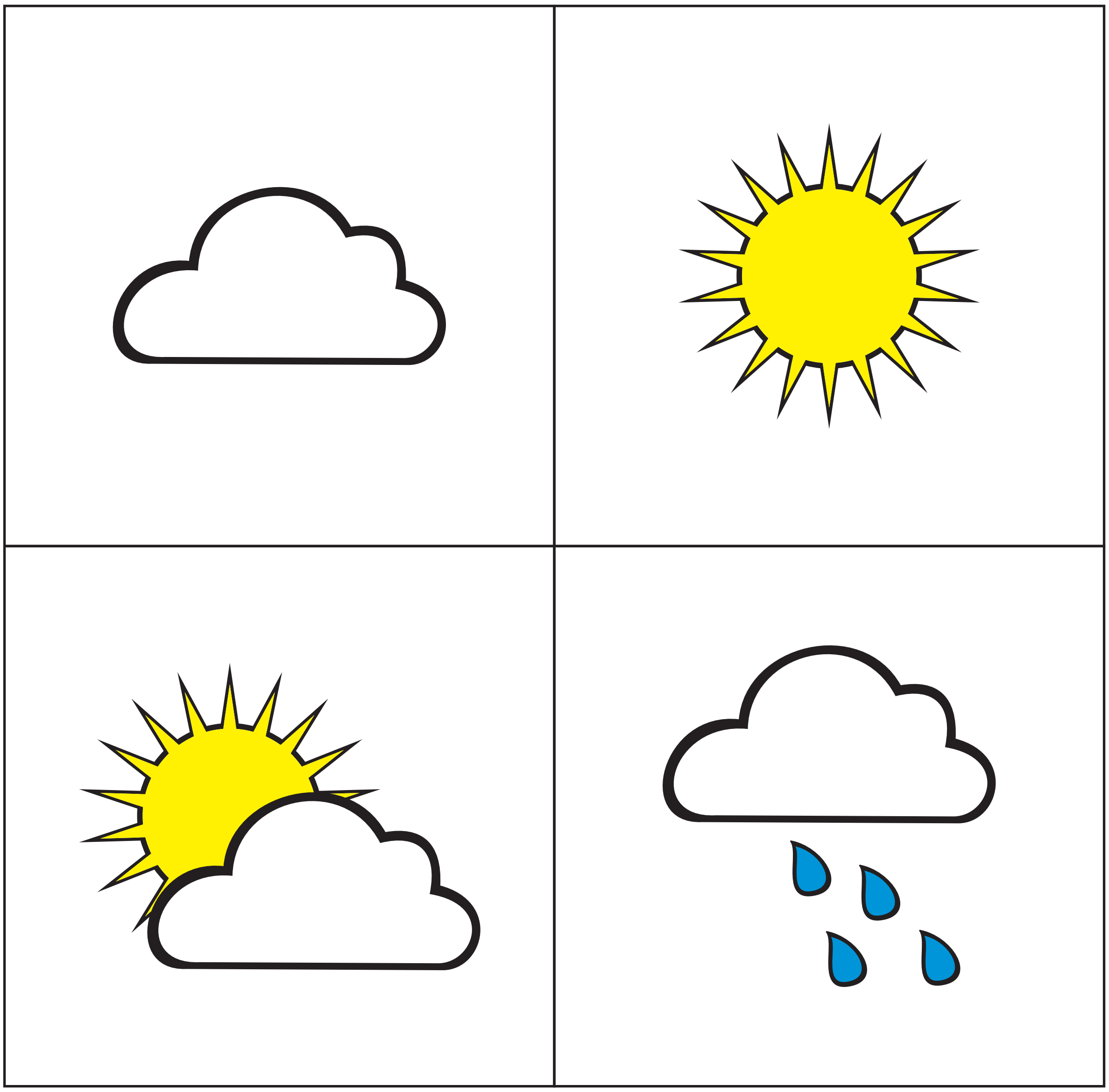
spades

clubs

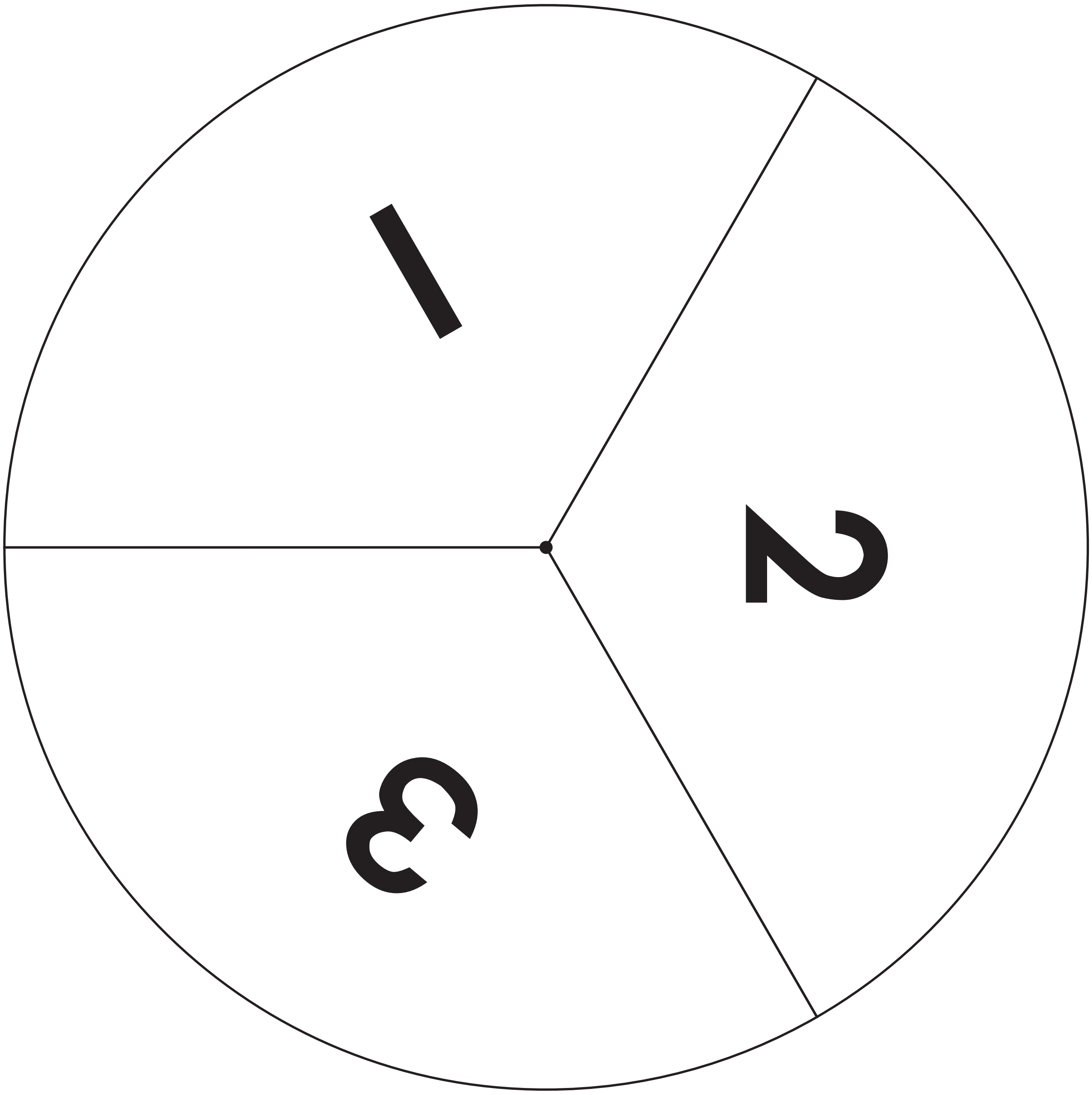


diamonds

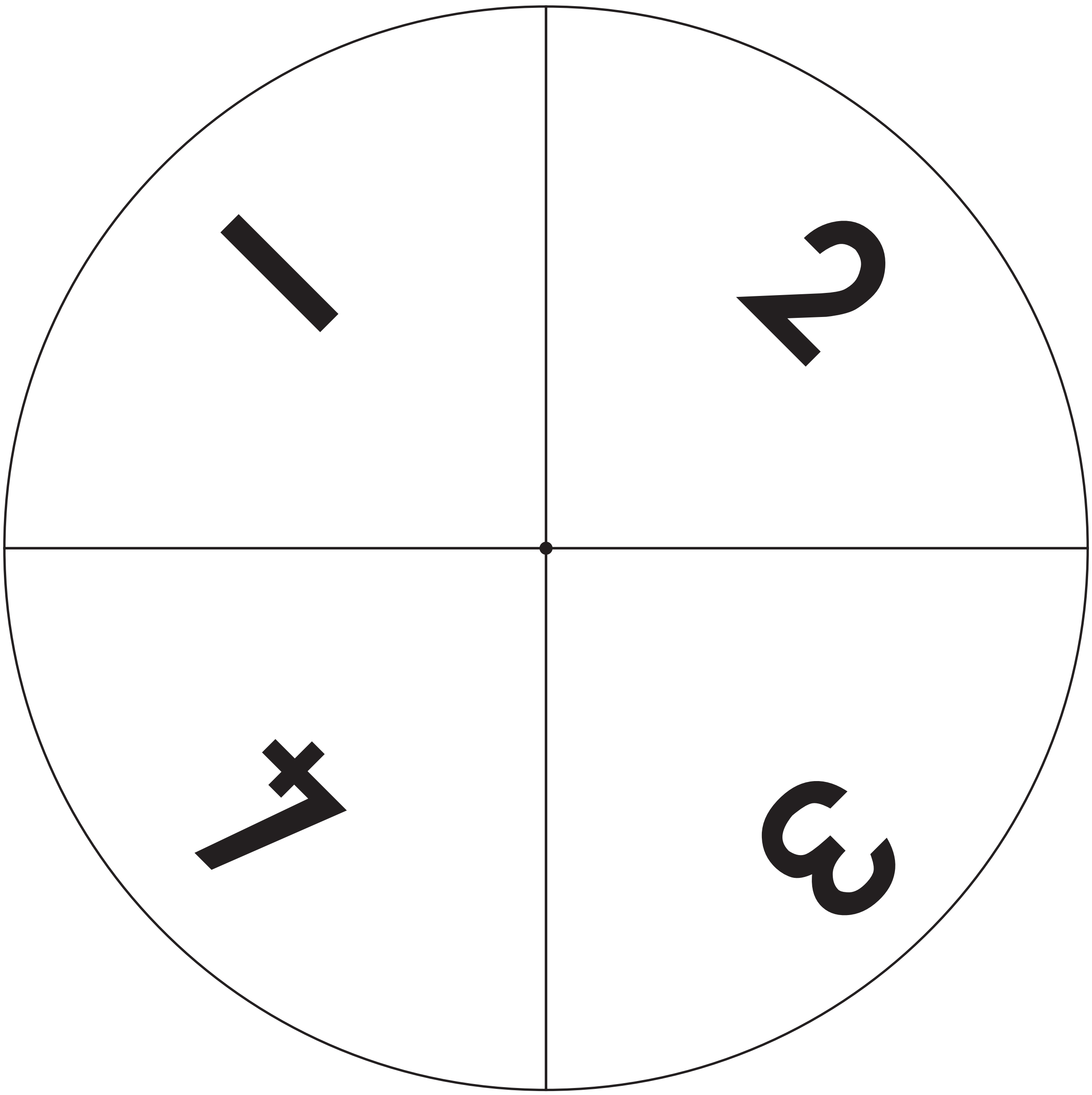
Printable: Weather patterns



Printable: Spinner 1 to 3



Printable: Spinner 1 to 4



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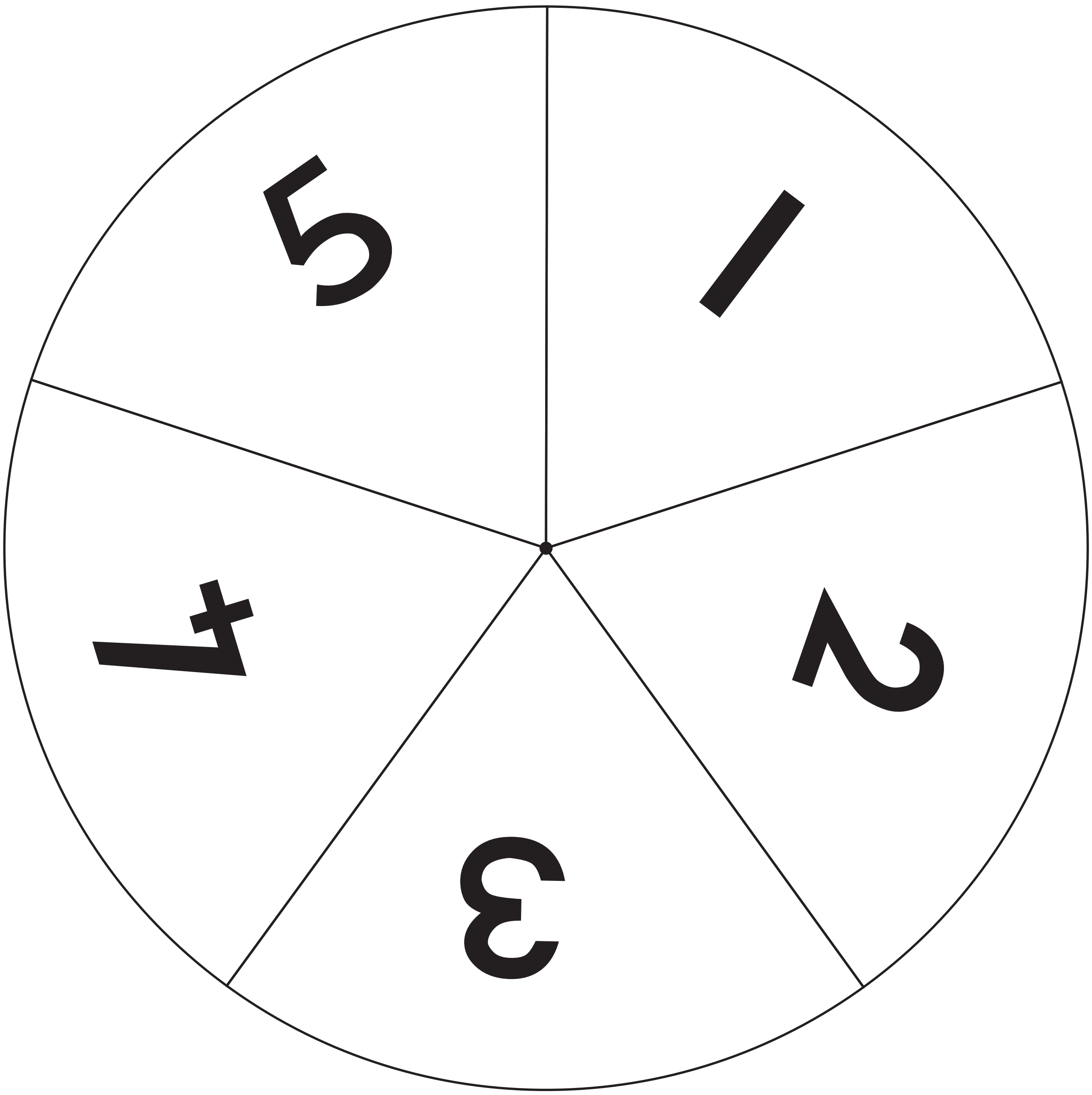

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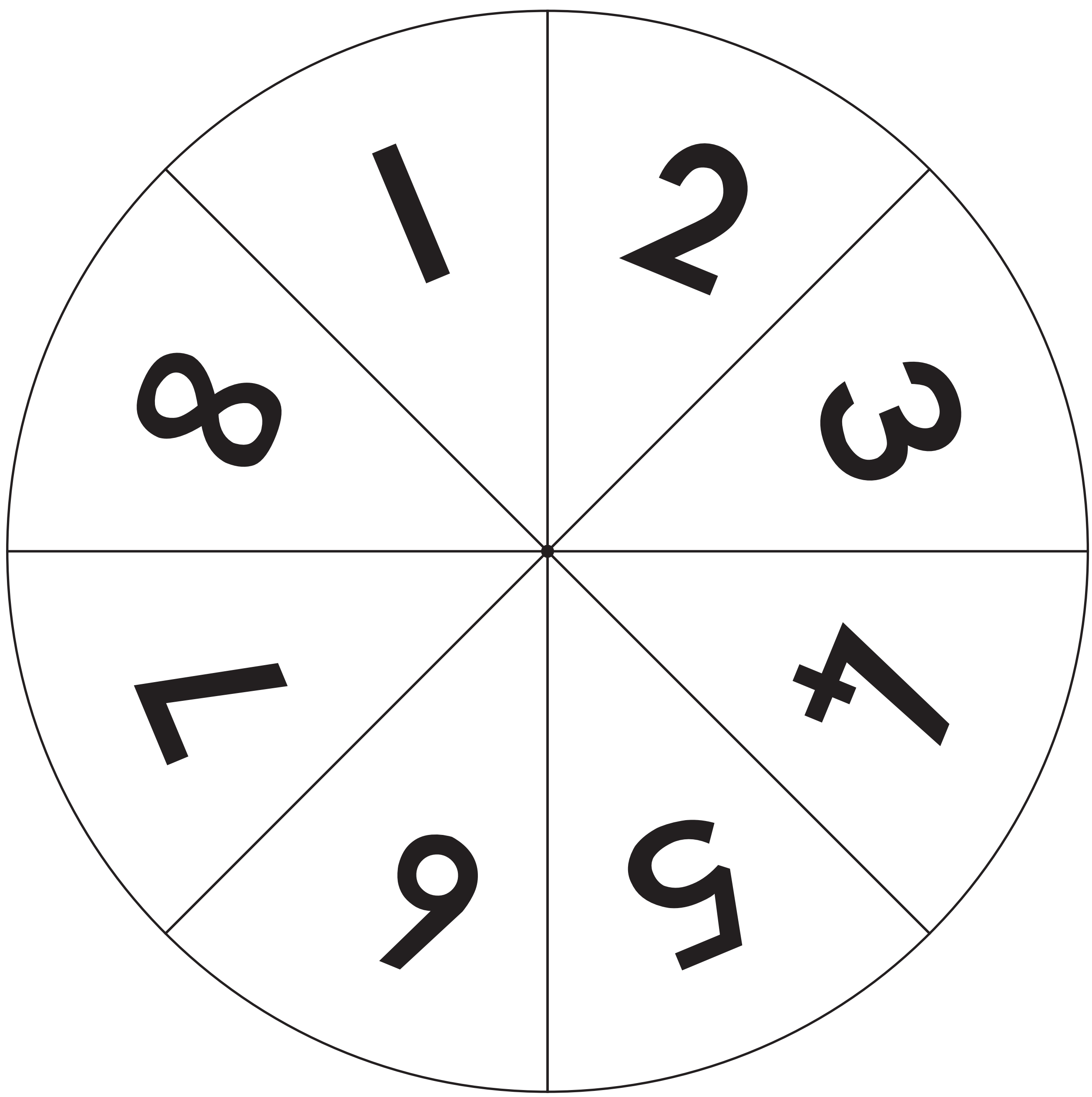

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Department of Basic Education
222 Struben Street, Pretoria, 0001
Private Bag X895, Pretoria, 0001, South Africa
Tel: (012) 357 3000 • Fax: (012) 323 0601

Private Bag X9035, Cape Town, 8000, South Africa
Tel: (021) 486 7000 • Fax: (021) 461 8110

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